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978-0-521-45689-0 - An Introduction to Twistor Theory, Second Edition

S. A. Huggett and K. P. Tod

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Preface

This book is an introduction to Twistor Theory and modern geometrical approaches to space-time structure at the graduate or advanced undergraduate level. The choice of material presented has evolved from graduate lectures given in London and Oxford and we have aimed to retain the informal tone of those lectures.

Topics covered include spinor algebra and calculus; compactified Minkowski space; the geometry of null congruences; the geometry of twistor space; an informal account of sheaf cohomology sufficient to describe the twistor solution of the zero rest mass equations; the active twistor constructions which solve the self-dual Yang–Mills and Einstein equations; and Penrose’s quasi-local-mass constructions. Exercises are included in the text and after most chapters.

The book will provide graduate students with an introduction to the literature of twistor theory, presupposing some knowledge of special relativity and differential geometry. It would also be of use for a short course on space-time structure independently of twistor theory. The physicist could be introduced gently to some of the mathematics which has proved useful in these areas, and the mathematician could be shown where sheaf cohomology and complex manifold theory can be used in physics.

It is a pleasure to acknowledge many useful discussions, comments and corrections from colleagues in London, Oxford and elsewhere. In particular a debt is due to Mike Eastwood, Andrew Hodges, Lane Hughston, Ted Newman, David Robinson, George Sparling, Richard Ward, Ronny Wells and Nick Woodhouse; we are grateful to Val Willoughby for typing the manuscript and dealing patiently with innumerable revisions; but above all we are indebted for teaching and inspiration to Roger Penrose.

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Preface to the second edition

In the eight years since the first edition of this book was published the literature of twistor theory has increased very substantially. There have been several books published whose subject matter overlaps to a greater or lesser extent with ours: Ward and Wells (1990) contains a mathematically more thorough and rigorous account of sheaf cohomology and the machinery from complex analysis necessary for twistor theory than we have found space for; Baston and Eastwood (1989) is a mathematical account of the 'Penrose transform' and its generalisation as part of representation theory; Bailey and Baston (1990) is a collection of commissioned reviews which go on in many directions from topics which we touch on more briefly; finally, Mason and Hughston (1990) follows on from Hughston and Ward (1979) by being a collection of articles which had previously appeared in Twistor Newsletter.

None of these precisely duplicates our book and so we have prepared a new edition taking account of some of the developments of the past eight years. The principal changes for the second edition are as follows:

- chapter 9 has been rewritten with a slightly different emphasis. Our intention is to provide a much clearer and more detailed motivation for the notion of a sheaf, with readers new to the subject in mind. Then (as in the first edition) we go on to discuss the elements of Čech cohomology.
- chapter 13 has been extended to describe the present status of the original quasi-local mass construction. There has been a lot of work in this area, and also a proliferation of other definitions of quasi-local mass. We describe one of them, that of Dougan and Mason, in detail.
- chapter 14 is new. It describes the cohomological framework for studying the kind of multi-linear functionals of zero rest mass fields which arise

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naturally in twistor theory, and which provide a starting point for a twistor approach to quantum field theory.

- chapter 15, the previous chapter 14, has been extended to describe the further 'Further Developments' of the past eight years. These are impressive (even if some of twistor theory's original aims have not yet been fulfilled). Perhaps the browsing mathematical physicist could do worse than start here!

- chapter 16 is new. Part of our aim in the first edition was to smuggle in extra material disguised as exercises, some of which were perhaps unreasonably difficult. With the inclusion of 'Hints, Solutions and Notes to the exercises' we continue this policy but with what we hope is a more accessible set of exercises.

The final major change is that the second edition has been completely reset in \LaTeX and we are very grateful to Domonic Green for undertaking this substantial labour.