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Jan Krajicek

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SYMBOL INDEX

- 2.1 L_{PA} , Q , IND, LNP, PA, \mathcal{N} , $\text{Th}(\mathcal{N})$, IE_1 , \subset , \subseteq , \neg , \vee , \wedge , \equiv , \rightarrow ,
 $O(g(n))$, $\Omega(g(n))$, $\Theta(g(n))$, $o(g(n))$
- 2.2 $\text{Time}(f)$, $\text{Space}(f)$, $\text{NTime}(f)$, $\text{NSpace}(f)$, LinTime, P, NP, L,
 PSpace, LinSpace, E, EXP, LinH, Σ_i^{lin} ,
 Σ_i^p , Π_i^{lin} , Π_i^p , coX , $\#R(x)$, $\#P$, P/poly, NP/poly, L/poly, \square_i^p
- 3.1 $C_\Omega(f)$, $\text{Depth}_\Omega(f)$, $L_\Omega(f)$, CLIQUE, $C^+(f)$,
 $C^m(f)$, \oplus , MAJ, $S_d(x_{i_1}, \dots, x_{i_d})$, MOD_p , $\text{CC}(f)$
- 3.2 L , $\lfloor \frac{x}{2} \rfloor$, $|x|$, $x\#y$, E_i , U_i , Δ_o , $\Delta_o(M)$, Λ , \neg ,
 \subseteq_p , RUD, SRUD, RUD^+ , strRUD, $\text{TimeSpace}(f, g)$, $V_k(\mathcal{N})$, Σ_i^b ,
 Π_i^b , Σ_∞^b , Δ_i^b
- 4.1 TAUT, $P \leq Q$, $P \leq_p Q$
- 4.2 $\text{Ext}(\theta)$, $\neg\text{PHP}_n^m$, R
- 4.3 LK
- 4.4 F, $k_P(\tau)$, $\ell_P(\tau)$
- 4.5 SF, EF, ER
- 4.6 Σ_i^q , Π_i^q , Σ_∞^q , G , G_i , G_i^* , TAUT_i
- 5.1 $I\Delta_0$, PA^- , \subseteq_e , IOpen, Ω_1 , $\omega(x)$, Exp, $\omega_k(x)$,
 $I\Delta_0 + \Omega_1$, $I\Delta_0 + \text{Exp}$
- 5.2 L^+ , BASIC, T_2^i , T_2 , S_2^i , S_2 , PIND, LIND,
 $\#_k$, MIN, LENGTH-MIN, MAX, LENGTH-MAX,
 Δ_i^b -IND, $\mathcal{B}(\Gamma)$, $B\Sigma_i^b$, $BB\Sigma_i^b$, BASIC^+ , $S_2^i(L^+)$, $\text{strict}\Sigma_i^b$

- 5.3 PV, $S_2^1(\text{PV})$, PV_i
- 5.4 Numones(x), $\langle a, b \rangle$, $\text{bit}(a, i)$, $i \in a$, $\text{Seq}(w)$
- 5.5 $\text{Log}(M)$, α^t , L_2 , $\Sigma_i^{1,b}$, $\Pi_i^{1,b}$, $\Delta_i^{1,b}$, $I\Sigma_0^{1,b}$, V_j^i , V_j ,
 U_j^i , U_j , SEP, AC, DC, R_2^i , R_2 , R_3^i , Enum(f, u, α), 1-Exp
- 6.1 $L(\alpha)$, $\Sigma_i^b(\alpha)$, $\Pi_i^b(\alpha)$, $\Delta_i^b(\alpha)$, $S_2^i(\alpha)$,
 $T_2^i(\alpha)$, Comp_M , UNIV_i
- 6.2 $\text{P}^{\text{NP}}[O(\log n)]$, $\text{P}^{\Sigma_i^p}[O(\log n)]$, L^{NP} , \leq_{tt} (NP), $\Sigma_0^b(\Sigma_i^b)$
- 6.3 WitComp_M , $\text{FP}^{\Sigma_i^p}[\text{wit}, q(n)]$
- 7.1 LKB, BASIC^{LK}
- 7.2 $\text{Witness}_A^{i,\bar{a}}$
- 7.3 ψ_H , $\text{PHP}(\Sigma_\infty^b)$, $\text{WPHP}(f)$, $\text{WPHP}(\text{PV}_1)$
- 7.5 PLS, F_P , N_P , C_P , PPA, PPAD, PPP, PPA(PV_1)
- 8.1 $\#_3$, R_2^i , $L(\#_3)$, TIME^Q , SPACE^Q , EXP^Q , PSPACE^Q
- 8.2 $\text{EXP}^{\Sigma_i^{1,b}}[\text{wit}, \text{poly}]$
- 9.1 $L_{\text{PA}}(R)$, $I\Delta_0(R)$, $\langle \theta \rangle_{(\bar{n})}$
- 9.2 $n(i)$, $\|A\|_{q(m)}^m$, $\|\Gamma\|$
- 9.3 $\text{Fla}(\alpha)$, $\text{Prf}_P(\pi, \alpha)$, $\text{Assign}(\eta, \alpha)$, $\text{Eval}(\eta, \alpha, \gamma)$,
 $\eta \models \alpha$, $\text{TAUT}(\alpha)$, $\text{Fla}_d(\alpha)$, $\tilde{\alpha}$,
 $w \models_i \alpha$, $\text{Taut}_i(A)$, TAUT_i , $P \geq_i Q$, $i\text{-RFN}(P)$, $\text{Con}(P)$
- 10.2 Ω_i
- 10.4 R_q^+ , R_q^- , $\Sigma_{i,m}^{S,t}$
- 10.5 $I\Sigma_i^0$, $\text{Con}(T)$, $\text{BdCon}(T)$, $\text{RCon}(T)$,
 $\text{Tr}_i(x, y)$, $\text{STr}_i(x, y)$
- 11.2 PHP_n^m
- 11.3 $\text{MOD}_k(R, S)_a$
- 11.4 $\text{WPHP}(a, R)$
- 12.1 RAM_n
- 12.2 $\Sigma_d^{S,t}$, $\Pi_d^{S,t}$, Δ_1^t , $S_{i,j}^{d,n}$,
 $R_{i,j}^+(q)$, $R_{i,j}^-(q)$, $\Sigma\text{-depth}$, $\text{MPHP}(A, B, r)$

- 12.3 $\mathcal{M}^{\text{PHP}}, \mathcal{M}^{\text{MOD}_a}, \|H\|, S \times T, S(H),$
 $H \triangleleft S, \alpha^\rho, H^\rho, \text{supp}(\rho)$
- 12.4 $[\alpha], \mathcal{M}^{\text{trivi}}, \phi^\rho, H_\phi, S_\phi$
- 12.5 $\text{Count}_n^a, \text{PHP}_n, g\text{Count}^a$
- 12.6 $Q_a x < t, Q_a \Delta_0, IQ_a \Delta_0,$
 $\text{MOD}_{a,i}, F(\text{MOD}_a), \text{LK}(\text{MOD}_a), \vdash_*, X \perp Y$
- 12.7 $L\exists_1, \mathcal{P}, \|\dashv$
- 13.1 $L_i, L_\omega, \text{CP}, \text{PHP}_n^{\text{CP}}, I\Delta_0(\alpha)^{\text{count}}$
- 13.2 $C(G), D(G)$
- 13.3 $\vdash_\Gamma, \text{BA}, \Gamma_\pi^*, \Gamma^+$
- 14.1 $\text{Taut}(x), c_P(\tau), \text{Prf}_T, \text{Con}_T, \vdash_m$
- 14.2 \geq_P
- 15.1 $\oplus P, \square_\infty^P, \Sigma_\infty^b \text{PHP}, \Delta_0 \text{PHP}, I\Delta_0(\pi)$
- 15.3 $P(M), \Sigma_i^P(M), \Pi_i^P(M), \text{NP}(M), \text{coNP}(M),$
 $\text{PH}(M), P^{\text{stan}}(M), (\Sigma_i^P)^{\text{stan}}(M), (\Pi_i^P)^{\text{stan}}(M), \text{PH}^{\text{stan}}(M),$
 $(P/\text{poly})(M), (\text{NP}/\text{poly})(M), (\text{coNP}/\text{poly})(M), \text{Diag}(M), \Delta_0^{\text{stan}}(M),$
 $\text{Bound}(f)$