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Calculations on lens systems are often marred by the unjustifiable use of the small angle approximation. This book describes in detail how the ray and wave pictures of lens behavior can be combined and developed into a theory capable of dealing with the large angles encountered in real optical systems. A distinct advantage of this approach is that Fourier optics, a topic of key importance in modern imaging science, appears naturally, in a form valid for arbitrarily large angles.

The book begins with extensive reviews of geometrical optics, eikonal functions, and the theory of wave propagation. The propagation of waves through lenses is then treated by exploiting the close connection between eikonal function theory and the stationary phase approximation. Aberrations are discussed next, first geometrically, and then from the wave point of view. The book concludes with various applications in lens design and analysis, including chapters on laser beam propagation and diffractive optical elements. Throughout, special emphasis is placed on the intrinsic limitations of lens performance.

The many practical insights it contains, as well as the exercises with their solutions, will make this book of great interest to gradute students taking courses in this field, as well as to anyone working in optical design and engineering.

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# The Ray and Wave Theory of Lenses

A. WALTHER

*Department of Physics*
*Worcester Polytechnic Institute*
*Worcester, Massachusetts*

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# Preface

Many books on modern optics confine the treatment of lens theory to the paraxial approximation. Aberrations are treated casually as an afterthought, and it usually remains unexplained whether they are due to the laws of physics or due to the limited art of the lens designer. The reader is often left with the notion that lenses must be designed to obey the laws of Gaussian optics as closely as possible. This is regrettable, because it has been known since the eighteenth century that paraxial optics used with finite heights and angles leads to projective geometry, a valuable branch of mathematics which is, however, a poor representation of the behavior of actual lenses.

The development of Fourier optics over the last forty years has brought lens theory and physics much closer together, but again many insights are lost because most authors, in spite of the ubiquity of high aperture lenses in the laboratory, are content with the small angle approximation when dealing with the theory of image formation. Either a clear and convincing demonstration should be given that the small angle approximation can be used with impunity for very large angles, or the theory should be developed honestly, without the small angle approximation. This honest theory exists, but is buried in books and papers providing so much detail that beginners are apt to get lost in the mathematical intricacies.

The primary goal of this monograph is to show how the old and well established theory of eikonal functions can be used to develop a theory of lenses that can deal with large angles not only in geometrical optics, but also in the wave theory of image formation. I have tried to abide by three ground rules:

- The connection between geometrical optics and physical optics must be presented not as a sterile mathematical derivation, but as an efficient



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engineering tool which allows the user to switch back and forth as needed.

- Energy conservation must be an integral part of the theory; mysterious unspecified amplitude factors must be avoided.
- Merely elegant theory should be left out; any result obtained should help to carry the argument forward or should be of some use in a practical application.

In addition to eikonal function theory, the primary mathematical tools used to accomplish these goals are plane wave expansions and the stationary phase technique.

To whet the reader's appetite, I briefly show in part one how some important results of geometrical optics can be derived straight from the wave equation. Part two deals with the basics of geometrical optics and eikonal theory. A chapter on radiometry is included as an application. Part three is a summary of Gaussian optics, with brief chapters on pupils and chromatic aberrations. Part four deals with various aspects of wave propagation in free space, as needed later. Plane wave expansions are used extensively, and the stationary phase method is used to show how rays can play a role in wave propagation.

In part five the stationary phase technique is used to relate the propagation of waves through lenses to geometrical optics in its eikonal form. Transformations between eikonals turn out to be intimately related to Fourier transforms, which is a great help in finding explicit expressions for the various wave propagation kernels. Fourier optics, in a form valid for arbitrarily large angles, appears as a natural product of the theory. In part six the mathematical tools developed in the previous parts are used to discuss lens aberrations.

In the last part of the book I show some, one may hope, non-trivial applications of the theory. As the theory of eikonal functions is currently not very popular in the optical community, some of the applications are chosen to demonstrate their use in purely geometrical optics, such as the chapter on mock ray tracing. An interesting application that needs both rays and waves is the theory of diffractive optical elements, which can be incorporated into the theory of lenses quite naturally. As a consequence the fundamental limitations on image formation derived for common glass lenses apply to diffractive lenses just as well. Another application at the borderline between geometrical optics and the wave theory is the propagation of Gaussian beams along a skew ray.

Most chapters end with a few problems, sometimes to suggest a minor

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extension of the theory, and sometimes because a numerical calculation gives greater insight than a theoretical discussion. Drill problems have not been provided.

Much of this material started out as lectures to graduate students and workers in the field of optics interested in developing a better understanding of the lenses they use in their daily work. The minimum background knowledge assumed is an undergraduate course in optics at the level of, for example, Hecht [1], and a solid course in mathematical physics including Fourier transforms, linear systems theory, a little variational calculus, and the rudiments of boundary value problems. I have not aimed for high standards of mathematical rigor; most engineers and physicists need to develop a basic physical understanding of the material before they can appreciate the mathematical niceties.

I owe a great debt to the many people who, over the years, have contributed to my understanding of this material. Many readers will recognize the influence of Max Herzberger's work, especially his book *Strahlenoptik*, and of T. Smith's publications. Many others, students, colleagues, and critics, must regrettably remain unnamed. I do, however, wish to express my special gratitude to three people who have profoundly influenced my thinking. The first is my thesis advisor A.C.S. van Heel, a true gentleman of physics who was a lasting inspiration to all the students who had the privilege of working at his laboratory. The second is W. Brouwer, another student of Van Heel, who provided at Diffraction Limited, Inc. a unique opportunity to test the practical value of theoretical ideas. And then there was always Edward L. O'Neill, good friend and colleague for well over thirty years, never too busy to listen and to encourage and to ask penetrating questions. It is a great sorrow that he passed away just before this book was finished. Peter Clark deserves special mention for reviving my interest in lenses after a long stretch of elementary physics teaching and work in other corners of optics.

But above all else I am grateful for the unfailing support of my wife Trudy. Without her selfless encouragement this work would never have been completed at all.

*A. Walther*
Acton, August 1993

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