

Contents

Preface	ix
Thanks	x
1 Introduction	1
1.1 The Statistical Description	1
1.2 Some Typical Problems and Concepts	2
 I Fundamentals	 17
2 Probability Theory	19
2.1 Introduction	19
2.2 Probability	20
2.3 Discrete Random Variables	21
2.4 Examples of Discrete Random Variables	23
2.5 Discrete Multivariate Distributions	26
2.6 Continuous Random Variables	29
2.7 Example of Continuous Random Variables	33
2.8 Random Vectors	38
2.9 Extreme Value Distributions	45
3 Distributions of Climate Variables	51
3.1 Atmospheric Variables	52
3.2 Some Other Climate Variables	63
4 Concepts in Statistical Inference	69
4.1 General	69
4.2 Random Samples	74
4.3 Statistics and Sampling Distributions	76
5 Estimation	79
5.1 General	79
5.2 Examples of Estimators	80
5.3 Properties of Estimators	84
5.4 Interval Estimators	90
5.5 Bootstrapping	93
 II Confirmation and Analysis	 95
Overview	97

6	The Statistical Test of a Hypothesis	99
6.1	The Concept of Statistical Tests	99
6.2	The Structure and Terminology of a Test	100
6.3	Monte Carlo Simulation	104
6.4	On Establishing Statistical Significance	106
6.5	Multivariate Problems	108
6.6	Tests of the Mean	111
6.7	Test of Variances	118
6.8	Field Significance Tests	121
6.9	Univariate Recurrence Analysis	122
6.10	Multivariate Recurrence Analysis	126
7	Analysis of Atmospheric Circulation Problems	129
7.1	Validating a General Circulation Model	129
7.2	Analysis of a GCM Sensitivity Experiment	131
7.3	Identification of a Signal in Observed Data	133
7.4	Detecting the 'CO ₂ Signal'	136
III	Fitting Statistical Models	141
	Overview	143
8	Regression	145
8.1	Introduction	145
8.2	Correlation	146
8.3	Fitting and Diagnosing Simple Regression Models	150
8.4	Multiple Regression	160
8.5	Model Selection	166
8.6	Some Other Topics	168
9	Analysis of Variance	171
9.1	Introduction	171
9.2	One Way Analysis of Variance	173
9.3	Two Way Analysis of Variance	181
9.4	Two Way ANOVA with Mixed Effects	184
9.5	Tuning a Basin Scale Ocean Model	191
IV	Time Series	193
	Overview	195
10	Time Series and Stochastic Processes	197
10.1	General Discussion	197
10.2	Basic Definitions and Examples	199
10.3	Auto-regressive Processes	203
10.4	Stochastic Climate Models	211
10.5	Moving Average Processes	213
11	Parameters of Univariate and Bivariate Time Series	217
11.1	The Auto-covariance Function	217
11.2	The Spectrum	222
11.3	The Cross-covariance Function	228
11.4	The Cross-spectrum	234
11.5	Frequency–Wavenumber Analysis	241

<i>CONTENTS</i>	vii
12 Estimating Covariance Functions and Spectra	251
12.1 Non-parametric Estimation of the Auto-correlation Function	252
12.2 Identifying and Fitting Auto-regressive Models	255
12.3 Estimating the Spectrum	263
12.4 Estimating the Cross-correlation Function	281
12.5 Estimating the Cross-spectrum	282
V Eigen Techniques	289
Overview	291
13 Empirical Orthogonal Functions	293
13.1 Definition of Empirical Orthogonal Functions	294
13.2 Estimation of Empirical Orthogonal Functions	299
13.3 Inference	301
13.4 Examples	304
13.5 Rotation of EOFs	305
13.6 Singular Systems Analysis	312
14 Canonical Correlation Analysis	317
14.1 Definition of Canonical Correlation Patterns	317
14.2 Estimating Canonical Correlation Patterns	322
14.3 Examples	323
14.4 Redundancy Analysis	327
15 POP Analysis	335
15.1 Principal Oscillation Patterns	335
15.2 Examples	339
15.3 POPs as a Predictive Tool	345
15.4 Cyclo-stationary POP Analysis	346
15.5 State Space Models	350
16 Complex Eigentechniques	353
16.1 Introduction	353
16.2 Hilbert Transform	353
16.3 Complex and Hilbert EOFs	357
VI Other Topics	367
Overview	369
17 Specific Statistical Concepts in Climate Research	371
17.1 The Decorrelation Time	371
17.2 Potential Predictability	374
17.3 Composites and Associated Correlation Patterns	378
17.4 Teleconnections	382
17.5 Time Filters	384
18 Forecast Quality Evaluation	391
18.1 The Skill of Categorical Forecasts	392
18.2 The Skill of Quantitative Forecasts	395
18.3 The Murphy–Epstein Decomposition	399
18.4 Issues in the Evaluation of Forecast Skill	402
18.5 Cross-validation	405

viii	CONTENTS
VII Appendices	407
A Notation	409
B Elements of Linear Analysis	413
C Fourier Analysis and Fourier Transform	416
D Normal Density and Cumulative Distribution Function	419
E The χ^2 Distribution	421
F Student's t Distribution	423
G The F Distribution	424
H Table-Look-Up Test	431
I Critical Values for the Mann–Whitney Test	437
J Quantiles of the Squared-ranks Test Statistic	443
K Quantiles of the Spearman Rank Correlation Coefficient	446
L Correlations and Probability Statements	447
M Some Proofs of Theorems and Equations	451
References	455