The study of the earth and the environment requires an understanding of the physical processes within and at the surface of the earth. This book will allow the student to develop a broad working knowledge of mechanics and its application to the earth and environmental sciences, providing the background necessary to study the professional literature. The only mathematical background required is that given by a first course in calculus; all other mathematical concepts are introduced in the context of their application to the earth and environmental sciences.

For each topic within mechanics, the physics is developed to give an insight into a wide range of natural phenomena. For example, the theory of stress and strain is applied to landslides, rock fractures, and earthquakes; the theory of flow through porous media is applied to the movement of groundwater, and the compaction of sediments; and the properties of Newtonian and non-Newtonian fluids are discussed in the context of the flow of lava, ice, and debris. The breadth of applications described and the many worked examples will help both students and professionals to understand the physical systems at work in the earth’s crust and at its surface.
Mechanics in the Earth and Environmental Sciences

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Preface

This book is the product of our conviction that the earth and environmental science curriculum, as generally taught in departments of geology, geography, and environmental science, must provide better training in the mathematical and physical aspects of the earth sciences. A quantitative approach using models is increasingly important for all research in these disciplines, even in supposedly “qualitative” fields, such as stratigraphy or environmental design. More and more, a working knowledge of physical processes is needed to understand the earth’s surface, or near-surface, environment. Environmental scientists need to understand the flow of fluids over or within surface materials, the stability of slopes, and the effects of storms, earthquakes, and volcanic eruptions. A good grasp of physical processes is also increasingly important for traditional applications in the mineral and petroleum industries.

The training necessary to understand natural physical processes is not covered in the freshman mathematics and physics that is traditionally required of graduates in earth and environmental science programs. Indeed, it is difficult to find this material in an accessible and relevant form in any upper level classes. In contrast, most earth science curricula require at least one formal course in chemistry, and further study of chemical principles is an essential part of the training provided by most courses in crystallography, mineralogy, petrology, and environmental processes. The student who wishes to pursue such applications further will find many suitable courses offered by departments of chemistry or environmental engineering. The earth science student who needs to understand more mathematics or physics is not generally so lucky. Courses in linear algebra (or vectors) and ordinary differential equations are required for most introductory courses in mechanics, but even a first course in mechanics does not cover the topics that are most important for the earth sciences, namely the mechanics of continua, including aspects
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of elasticity, fluid mechanics, and rheology. It is true that these are large fields that cannot be mastered in a single course – but it is better that a student know something about them than nothing at all. In this book, we aim to provide a basic working knowledge of the principles underlying these fields, without attempting a thorough survey of any of them.

The range of knowledge touched on in this book is vast, and extends far beyond the expertise of the two authors, who are a sedimentologist and a geomorphologist respectively. We have chosen appropriate topics and examples and have deliberately avoided confining these to our own fields of specialist knowledge. We have tried to write a book that will be useful as a text for an upper level undergraduate or beginning graduate course in a department of geology, geography, or environmental science. The course can be taught by anyone with a good background in the physical aspects of one of these disciplines. We have provided carefully selected references partly because we feel an obligation to give sources in areas where we go beyond our own expertise. We hope, however, that other instructors will take up the challenge of teaching similar material, and that they and at least some students will make use of the references provided in order to pursue topics that particularly interest them.

The main aims of the book are to introduce some topics in the mechanics of continua, and their applications in the earth sciences. To do this, however, we need to teach some mathematical material – vectors, tensors, and some aspects of ordinary and partial differential equations. These are all topics that have extensive applications in other areas of physics: in gravitation and in electricity and magnetism, for example. Such mathematics is generally considered to be relatively advanced, yet it is our strong belief that earth scientists need to have at least some acquaintance with these topics, even though they may never have the time or opportunity to take rigorous courses in them. We have introduced all mathematical concepts using no more knowledge than is learned in a freshman course in calculus.

In this book, breadth is stressed at the expense of depth. Yet we do strive for as complete an understanding of the basic physics as is possible under the circumstances. Breadth can be achieved because the underlying mechanics is common to many different natural processes. For example, the concepts of stress and strain find applications to rocks, soils, groundwater, rivers, oceans, and atmosphere. Diffusion is a process operating at many scales, from the molecular diffusion of momentum or chemical species to large-scale turbulent diffusion of heat or sediment in the ocean or atmosphere. Stress, deformation, flow, diffusion are examples of the broad concepts whose understanding is critical for all of the geophysical sciences. We hope to give
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some understanding of these ideas, and the notation and methods that make use of them – to the point where the student will be able to read original papers in, for example, the *Journal of Geophysical Research*, without simply skipping all of the equations.

We believe that, depending on the students’ background, this book contains more material than can be taught in a single semester. It would be easy to expand the subject matter to a year-long treatment by including more thorough discussion of examples. In order to shorten the course some topics can fairly readily be omitted: they include some of the later parts of Chapter 2, some topics in Chapters 5 and 6, and all of Chapter 12. Our views on the main topics that must be developed by the instructor and understood by the student are explained more fully in Chapter 1.

The beginnings of this book can be traced to 1979, when Gerry Middleton and Paul Clifford introduced a course like this in the Department of Geology at McMaster University. Over several years, Middleton developed a set of course notes, which served as a text for that course and its successors. In 1987, Peter Wilcock joined the Department of Geography and Environmental Engineering at The Johns Hopkins University and began to teach a similar course to a mixed class of students in geology and environmental science. He heard about the course at McMaster and began using a set of the course notes as a text. The book developed out of the long-distance collaboration that resulted. After accumulating four years of modifications and additions from the experience of teaching the material at both universities, the notes were completely rewritten by Middleton during a sabbatical year at the University of Washington. Large parts of the text have been added, deleted or rewritten by Wilcock, to the point where we now find it hard to remember exactly who wrote what.

We thank our universities for supporting us while we struggled with this relatively new teaching and scholarly initiative. Middleton received a McMaster Teaching and Learning grant one summer, which enabled him to experiment with the use of personal computers in this course. He is indebted to the University of Washington, which provided fine facilities and a most stimulating intellectual environment during the year of writing (1991–2): thanks go particularly to Darrel Cowen, Tom Dunne, Bernard Hallet, and Jody Bourgeois. The book has been prepared using the \LaTeX\ system developed by Donald Knuth and Leslie Lamport. The particular implementation used in the later stages has been \TeX\, a Shareware program for personal computers, made available without charge by its author, Eberhard Mattes. The authors of this book express their gratitude to those who have made available such a sophisticated typesetting system,
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without taking any personal profit. Drafts of various chapters, and of
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Susan Parkinson, whose contributions to this book went far beyond the
normal work of a copy-editor. Last, but certainly not least, we thank the
students who took our courses over the years.

A set of computer programs designed to accompany this text, together with
a brief set of instructions for their use, may be obtained at a nominal cost
from the Computer Oriented Geological Society, P.O. Box 371046, Denver
CO 80237-0246, USA. Tel. (303) 751-8553, Email: cogs@mines.colorado.edu.
This set of programs includes both source code (in Turbo Pascal) and exec-
utable code and should run on most IBM-compatible computers. Computer
programs referred to in the text (e.g., EJECTA, LAPLACE, EHEAT) are all
part of this set.