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978-0-521-43064-7 - Numerical Recipes in Fortran 77: The Art of Scientific Computing: Second Edition:

Volume 1 of Fortran Numerical Recipes

William H. Press, Saul A. Teukolsky, William T. Vetterling and Brian P. Flannery

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