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Econophysics: why and what

1.1 Why econophysics?

This is the era of growing financial instability, a new era of worldwide privatization and deregulation made possible by a vast credit expansion based on the Dollar as the worldwide default reserve currency. Derivatives are unregulated and are used as a form of money creation totally beyond the control of any central bank. Standard economic theory completely rules out the possibility of such instability.

Before WWII, the expansion of a currency and consequent inflation was not possible with the Dollar regulated by gold at \$35/oz. The gold standard was finally and completely abandoned by the USA in 1971 after “Euro-dollars” became on the order of magnitude of the US gold supply. On the gold standard, hedging foreign currency bets apparently was not necessary. We can date our present era of inflation, credit, and high level of consumption with increasing finance market instability from the deregulation of the Dollar in 1971, and it’s not accidental that both the Black–Scholes derivatives model and the legalization of large-scale options trading both date from 1973. We can contrast this reality, described in popular books by Stiglitz (2002), Morris (2008), and Soros (2008), with the teaching of equilibrium in standard academic economics texts.

Economists teach market equilibrium as the benchmark in the classroom, even while the real world of economics outside the classroom experiences no stability. There is an implicit assumption in those texts that unregulated markets are stable, as if completely free markets should somehow self-organize in a stable way.

Standard microeconomic theory is based on a deterministic equilibrium model, called neo-classical economics (Chapter 2), where perfect knowledge of the infinite future is assumed on the part of all players. That an equilibrium

exists *mathematically* under totally unrealistic conditions has been proven, but that the hypothetical equilibrium is stable (or computable) or has anything at all to do with reality was never demonstrated. The generalization of the neo-classical model to uncertain but still hypothetically stable markets assumes a stationary stochastic process, and is called “rational expectations”. Standard macroeconomics is based on the assumption of stationary and therefore stable economic variables. Rational expectations emerged as the dominant economic philosophy parallel to deregulation in the 1970s and 1980s, with regression analysis as the tool of choice for modeling. Regression analysis is based on the assumption of stationary noise, *but there is no solid empirical evidence for stationarity of any kind in any known market*. The only scientific alternative is to approach markets as a physicist, and ask the market data what are the underlying unstable dynamics.

Having stated our view of standard economics and our offered alternative, we now survey the historic viewpoint of physics. In particular, Galileo did not merely discover a mathematical model of nature, he discovered two inviolable local laws of nature: the law of inertia and the local law of gravity. Both of those local laws survived the Einsteinian and quantum revolutions. Following the lessons of Galileo, Kepler, and Newton, scientists have amassed indisputable evidence that mindless nature behaves mathematically lawfully. But “motion” guided by minds is an entirely different notion. Social behavior is generally complicated, it may be artificially regulated by the enforcement of human law, or it may be completely lawless. Neo-classical economists try to model human preferences using *a priori* models of behavior (utility maximization) that have been falsified. More recent work in both econophysics and economics uses agent-based modeling, which is like trying to replace thinking, hopeful, and fearful agents with fixed rules obeyed by spins on a lattice. In this text we will instead adopt an inherently macroeconomic, or phenomenological, viewpoint. We will not try to model what agents prefer or do, but instead will simply ask real markets what the observed statistics can teach us. In particular, we will try to discover regularities in the form of equations of motion for log returns of prices. The discovery of a correct class of dynamic models is far beyond the reach of regression analysis in econometrics.

The history of physics shows that mathematical law cannot be discovered from empirical data unless *something* is repeated systematically. Wigner has explained the basis for the discovery of mathematical laws of motion in local invariance principles. But the method of the natural sciences cannot be found in standard economic theorizing and data analysis. In financial economics, where no correct dynamical model has been discovered, the term “stylized

facts” appears. “Stylized facts” are supposed to be certain statistical features of the data. But even there, certain hidden assumptions in statistical analysis have implicitly and unquestionably been taken for granted without checking for their validity. We’ll show (Chapter 7) how a common method of data analysis leads to spurious stylized facts, to features “deduced statistically” that are really not present in the empirical data. We avoid generating spurious statistical results by constructing an approximate statistical ensemble for the analysis of a single, historic nonstationary time series.

Karl Popper only put into words what physicists since Galileo, Kepler, and Newton have done. Science consists of falsifiable propositions and theories. Falsifiable models have no free parameters to tweak that would make a wrong model fit adequate data (data with enough points for “good statistics”). A falsifiable model is specified completely by empirically measurable parameters so that, if the model is wrong, then it can be proven wrong via measurement. Examples of falsifiable models in economics and finance are neo-classical economics and the original Black–Scholes Gaussian returns model. Both models have been falsified. In science the skeptics, not the believers, must be convinced via systematic, repeatable measurements. The application of the idea of “systematic repeated observations,” the notion of a statistical ensemble, is applied to the analysis of a single, historic time series in Chapter 7. The basis for the statistical ensemble is an observed repetitiveness in traders’ behavior on a daily time scale. We predict a new class of falsifiable dynamical model.

In Chapter 3 we will emphasize the distinction between local and global predictions. “Local” means in a small region near a given point (x,t) , whereas “global” means over large displacements $x(t,T) = x(t + T) - x(t)$ for different initial times t and large time lags T . The limitations on global predictability in perfectly well-defined deterministic dynamical systems are well defined, and inform the way that I understand and present stochastic dynamics and market models. We will distinguish local from global solutions of stochastic processes. In particular, we see no good reason to expect universality of market dynamics, and find no statistical evidence for that notion. Our analysis shows that finance markets vary in detail from one financial center to another (e.g. New York to Tokyo), and may not obey exactly the same dynamics.

The reader is encouraged to study Wigner’s (1960) essay on the unreasonable effectiveness of mathematics in nature and his book *Symmetries and Reflections* (1967), and Velupillai’s corresponding essay on the unreasonable ineffectiveness of mathematics in economics (2005). We turn next to Wigner’s explanation of the basis for discovering laws of motion: local invariance principles.

1.2 Invariance principles and laws of nature

It's important to have a clear picture of just how and why standard economic theorizing differs from theoretical physics. To see the difference, the reader may compare any micro- or macroeconomics text with any elementary physics or astronomy text. The former describes only mental constructs like equilibrium of supply and demand that are not observed in real markets; the latter present the accurate mathematical descriptions of the historic experiments and observations on which physics and astronomy are based. In particular, where equilibrium is discussed, real examples are presented (a flower pot hanging from a ceiling, for example). Physics and astronomy are about the known mathematical laws of nature. Economics texts are about stable equilibria that do not exist in any known market. Why, in contrast, has mathematics worked so precisely in the description of nature?

Eugene Wigner, one of the greatest physicists of the twentieth century and the acknowledged expert in symmetry principles, wrote most clearly about the question: why are we able to discover mathematical laws of nature? (Wigner, 1967) An historic example points to the answer. In order to combat the prevailing Aristotelian ideas, Galileo proposed an experiment to show that relative motion doesn't matter. Motivated by the Copernican idea, his aim was to explain why, if the earth moves, we don't feel the motion. His proposed experiment: drop a ball from the mast of a uniformly moving ship on a smooth sea. It will, he asserted, fall parallel to the mast just as if the ship were at rest. Galileo's starting point for discovering physics was therefore the principle of relativity. Galileo's famous thought experiment would have made no sense were the earth not a local inertial frame for times on the order of seconds or minutes.¹ Nor would it have made sense if initial conditions like absolute position and absolute time mattered.

The known mathematical laws of nature, the laws of physics, do not change on any observable time scale. Physicists and chemists were able to discover that nature obeys inviolable mathematical laws only because those laws are grounded in local invariance principles, local invariance with respect to frames moving at constant velocity (principle of relativity), local translational invariance, local rotational invariance and local time-translational invariance. These local invariances are the same whether we discuss Newtonian mechanics, general relativity, or quantum mechanics. Were it not for these underlying invariance principles it would have been impossible to discover

¹ There exist in the universe only local inertial frames, those locally in free fall in the net gravitational field of other bodies; there are no global inertial frames as Mach and Newton assumed. See Barbour (1998) for a fascinating and detailed account of the history of mechanics.

1.3 Humanly invented law can always be violated

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mathematical laws of nature in the first place. Why is this? Because the local invariances form the theoretical basis for *repeatable identical experiments/observations* whose results can be reproduced by different observers independently of where and at what time the observations are made, and independently of the state of relative motion of the observational machinery. This leads us to the idea of a statistical ensemble based on repetition, a main topic of Chapter 7.

In physics, astronomy, and chemistry, we do not have merely *models* of the behavior of matter. Instead, we know mathematical laws of nature that cannot be violated intentionally. They are beyond the possibility of human invention, intervention, or convention, as Alan Turing, the father of modern computability theory, said of arithmetic in his famous paper defining computability. Our discussion above informs us that *something* must be systematically repeated if we're to have any chance to discover equations of motion. The motion of the ball is trivial periodic; it has a cycle of period zero. A simple pendulum has a cycle of period one. Finance data don't generate deterministic cycles, but instead, as we'll show, exhibit a certain statistical periodicity.

Mathematical laws of nature have been established by repeatable identical (to within some decimal precision) experiments or observations. Our aim is to try to mimic this so far as is possible in finance. To qualify as science, a model must be falsifiable. A falsifiable theory or model is one with few enough parameters and definite enough predictions, preferably of some new phenomenon, that it can be tested observationally and, if wrong, can be proven wrong. A theory is not established because its promoters believe it. To gain wide acceptance, a theory must convince the skeptics, who should perform their own experiments or observations. In economics this has not been the method of choice. As various books and articles have correctly observed, textbook economic theory is not empirically based but rather is an example of socially constructed modeling. Rational expectations (Chapter 10) provides the latest example.

1.3 Humanly invented law can always be violated

Physics and economics are completely different in nature. In economics, in contrast with physics, there exist no known inviolable mathematical laws of "motion"/behavior. Instead, economic law is either legislated law, dictatorial edict, contract, or in tribal societies the rule of tradition. Economic "law," like any legislated law or social contract, can always be violated by willful people and groups. The idea of falsification via observation has not yet taken

root in adequately thick topsoil. Instead, an internal logic system called neo-classical economic theory was invented via postulation and still dominates academic economics, the last contributor being Robert Lucas, who's given credit for the "rational expectations revolution" in economic theory. Neo-classical economics is not derived from empirical data. The good news is that the general predictions of the theory are specific and have been falsified. The bad news is that this is still the standard theory taught in economics textbooks, where there are many "graphs" but few if any that can be obtained from or justified by unmassaged, real market data.

In his very readable book *Intermediate Microeconomics*, Hal Varian (1999), who was a dynamical systems theorist before he was an economist, writes that much of (neo-classical) economics (theory) is based on two principles:

The optimization principle. People try to choose the best patterns of consumption they can afford.

The equilibrium principle. Prices adjust until the amount that people demand of something is equal to the amount that is supplied.

Both of these principles sound like common sense, and we will see that they turn out to be more akin to common sense than to science. They have been postulated as describing markets, but lack the required empirical underpinning.

Because the laws of physics, or better said the known laws of nature, are based on local invariance principles, they are independent of initial conditions like absolute time, absolute position in the universe, and absolute orientation. We cannot say the same about markets: socio-economic behavior is not necessarily universal but may vary from country to country. Mexico is not necessarily like China, which is certainly not like the USA or Germany. Many econophysicists, in agreement with economists, would like to ignore the details and hope that a single universal "law of motion" governs markets, but that idea remains only a hope. We will see in Chapter 4 that there is but a single known law of socio-economic invariance, and that is not enough for universally valid market dynamics.

The best we can reasonably hope for in economic theory is a model that captures and reproduces the essentials of historical data for specific markets during some epoch, like finance markets since c. 1990. We can try to describe mathematically what has happened in the past, but there is no guarantee that the future will be the same. Insurance companies provide an example. There, historic statistics are used with success in making money under normally expected circumstances, but occasionally there comes a "surprise" whose risk was not estimated correctly based on past statistics, and the companies consequently lose a lot of money through paying unexpected claims.

Some people may fail to see that there is a difference between economics and the hardest unsolved problems in physics. One might object: we can't solve the Navier-Stokes equations for turbulence because of the butterfly effect or the computational complexity of the solutions of those equations, so what's the difference with economics? Economics cannot be fairly compared with turbulence. In fluid mechanics we know the equations of motion based on Galilean invariance principles. In turbulence theory we cannot predict the weather. However, we *understand* the weather physically and can describe it qualitatively and reliably based on the equations of thermo-hydrodynamics. We understand very well the physics of formation and motion of hurricanes and tornadoes, even if we cannot predict when and where they will hit. No comparable basis for qualitative understanding exists in economic theory.

1.4 Origins of econophysics

Clearly, econophysics should not try to imitate academic economic theory, nor should econophysics rely on standard econometric methods. We are not trying to make incremental improvements in theory, as Yi-Cheng Zhang has so poetically put it, we're trying instead to replace the standard models and methods with entirely new results. Econophysics began in this spirit in 1958 with M. F. M. Osborne's discovery of Gaussian stock market returns (the lognormal pricing model), Mandelbrot's emphasis on Martingales for describing hard-to-beat markets, and then Osborne's falsification in 1977 of the supply–demand curves. From the practical side, a supply–demand mismatch of physics PhDs to academic jobs, and new research opportunities in practical finance, drew many physicists to “Wall Street.” Physics funding had exploded in America after Sputnik was launched by the USSR in October, 1957, but had tapered off by 1971, when academic jobs in physics began to dry up (see Derman's informative autobiography (2004), which is a history of that era). In 1973 the Black–Scholes theory of option pricing was finally published after a struggle of several years against editors who insisted that finance wasn't economics, and large-scale options trading was legalized at the same time. The advent of deregulation as a dominant government philosophy in the 1980s (along with the opening of China to investment c. 1980, following the Nixon-Kissinger visit to Chairman Mao and Chou En-Lai in 1973), the collapse of the USSR in 1989–1991, and the explosion of computing technology in the 1980s all played determining roles in the globalization of capital. With computerization, finance data became more accurate and more reliable than fluid turbulence data, inviting physicists to build falsifiable finance

models. All of these developments opened the door to the globalization of trade and capital and led to a demand on modeling and data analysis in finance that many physicists have found to be either interesting or lucrative.

1.5 A new direction in econophysics

One can ask why physicists believe that they're more qualified than economists to explain economic phenomena, and *if* physicists *then* why not also mathematicians, chemists, and biologists? Mathematicians dominate both theoretical economics and financial engineering, and by training and culture they are a strongly postulatory tribe that at worst ignores real market data, and at best (financial engineering) proves powerful theorems about Gaussian models while introducing no new empirically based models to solve the fundamental problem of market dynamics (see for example the closing words in Steele's (2000) book!). Chemists and biologists are certainly empirically oriented, but are trained to focus on details that physicists usually find boring. Physicists are trained to see the connections between seemingly different phenomena, to try to get a glimpse of the big picture, and to present the simplest possible mathematical description of a phenomenon that includes no more factors than are necessary to describe the empirical data. Physicists are trained to isolate cause and effect. A good physicist like Feynman has more in common with a radio or car repairman than with a mathematician. A few highlights of a debate between econophysicists and economists can be found in Gallegati *et al.* (2006), Ball (2006), and McCauley (2006). An interesting discussion of an entirely different nature can be found in Solomon and Levy (2003).

Since the word was coined by Gene Stanley in 1995 (Mantegna and Stanley, 1999), the term econophysics has been characterized largely by three main directions, not necessarily mutually exclusive. First, there was the thorough mathematical solution of the Minority Game inspired by the Fribourg school of econophysics (Challet *et al.*, 2005), and related models of agent-based trading (Maslov, 2000). That work partly evolved later into studies of networks (Caldarelli, 2007) and "reputation systems" (Masum and Zhang, 2004). The foray into finance is illustrated by Dacorogna *et al.* (2001), Farmer (1999), and Bouchaud and Potters (2000). Models of market crashes have been constructed by A. Johansen and Sornette (2000). Most popular, however, has been the reliance on econophysics as the attempt to explain economic and finance data by scaling laws (the Hurst exponent) and fat-tailed probability distributions. The work on fat tails was initiated historically by Pareto and was revived by Mandelbrot around 1960. Since 1995, fat tails and scaling studies have been inspired by the Boston School led by Gene Stanley,

who also opened Physica A to econophysics. Econophysics is still unrecognized as science by the American Physical Society, but fortunately the European Physical Society has had a Finance and Physics section since 1999 or earlier. Without Gene Stanley and Physica A, econophysics would never have gotten off the ground. Hurst exponent scaling was also emphasized in the earlier era by Mandelbrot, with his papers on rescaled range (R/S) analysis and fractional Brownian motion. If we would judge what econophysics is by the number of papers in the field, we would say that the main ideas of econophysics are agent-based models, fat tails, and scaling. But this is not enough to determine the underlying market dynamics.

Blazing a new trail, we offer an alternative approach to econophysics. We follow Osborne's lead (and validate Mandelbrot's Martingale efficient market hypothesis) and focus on the discovery of falsifiable classes of market dynamics models deduced directly from empirical data. In particular, we will present evidence for diffusive models that don't scale in log returns, nor do we find evidence for fat tails in log returns. We offer a view of finance market dynamics that contradicts the standard so-called stylized facts. Our method of analysis, unlike the other approaches, is based on statistical ensembles. In particular, we do not use time averages ("sliding windows") on nonstationary time series.

Econophysics does not mean lifting tools and models from statistical physics and then applying them directly to economics. Economics is not like chemistry, where all results follow at least in principle from physics. Neither is economics a trivial science that can be formulated and solved by transferring methods and ideas directly from physics, mathematics, or from any other field. We use the theory of stochastic processes both in data analysis and modeling, but we've had to invent new classes of stochastic models, and have found it necessary to clarify some older mathematical ideas, in order to understand finance markets. As Lars Onsager once asserted, a theoretical physicist should not start with a mathematical tool and then look around for data to explain. Instead, a "real theorist" should study the data and invent the required mathematical tools. That's what Galileo, Kepler, and Newton did. That's also what Lars did when he solved the 2D Ising model, and also earlier when he produced an exact solution to the pdes describing the dissociation and recombination of ions of a weak electrolyte in an electric field. Both were amazing mathematical feats, and the latter was directly applicable to experimental data. Econophysics, simply stated, means following the example of physics in observing and modeling markets.

2

Neo-classical economic theory

2.1 Why study “optimizing behavior”?

Globalization via deregulation and privatization is supported by the implicit and widespread belief in an economic model that teaches the avoidance of government intervention in socio-economic life. The old *laissez faire* belief was revived in the Reagan-Thatcher era, and then gained ground explosively after the collapse of central planning in communist countries. The old fight through the 1970s was between the idea of regulated markets in the west and strong central planning under communism. The question for our era is whether markets should be regulated for social purposes, as they were in western Europe prior to the fall of the wall,¹ or whether the current *laissez faire* binge will continue in spite of its inherent financial instabilities and the irreversible loss of jobs in previously well-off western nations. In particular, *laissez faire* teaches that regulations should have been avoided, and this has led to the peculiar problem that financial derivatives are a highly leveraged and unregulated form of credit creation. In contrast, the standard economic theory to be described in this chapter does not admit “money” in any form, shape, or fashion.

The “losing side” in the Cold War has adopted capitalism with a vengeance, and is now beating its former enemies: China and Russia, as of 2007, sit on the largest Dollar reserves in the world. With imports outrunning exports in the west, the problems that follow from deregulation and privatization are now felt in the so-called “First World” countries: degradation of the currency, so far mainly the Dollar, and unemployment due to the systematic loss of manufacturing capacity to cheap labor. The financial pressure to deregulate everything

¹ The vast middle ground represented by the regulation of free markets, along with the idea that markets do not necessarily provide the best solution to all social problems, is not taught by “Pareto efficiency” in the standard neo-classical model.