CAMBRIDGE

Cambridge University Press
978-0-521-42908-5 - Relativity on Curved Manifolds
F. de Felice and C. J. S. Clarke
Table of Contents
More information

CONTENTS

Preface *xi*

Geometry and Physics: An Overview 1
1 Geometry 1
2 Special relativity 4
3 General relativity 7
4 Local, global and infinitesimal 11

1 The Background Manifold Structure 14
1.1 Topological spaces 14
1.2 Maps 15
1.3 Coordinate neighbourhoods 17
1.4 Differentiable manifolds 19
1.5 Maps of manifolds 20
1.6 The tangent space 21
1.7 Bases in the tangent space 24
1.8 Transformation properties of vector components 25
1.9 The tangent map 27
1.10 The cotangent space 28
1.11 Bases in the cotangent space 29
1.12 The dual tangent map 31
1.13 Tensors 32
1.14 Symmetry operations on tensors 37
1.15 The metric tensor 39
1.16 Raising and lowering of tensor indices 47
1.17 Alternating tensors 48
1.18 Exterior algebra 51

© Cambridge University Press www.cambridge.org

CAMBRIDGE

Cambridge University Press
978-0-521-42908-5 - Relativity on Curved Manifolds
F. de Felice and C. J. S. Clarke
Table of Contents
More information

1.19 Measure of lengths and the world-function 55

2 Differentiation 57
2.1 Tensor fields and congruences 57
2.2 The Lie derivative 62
2.3 The connector 68
2.4 Parallel propagation and geodesics 72
2.5 Transformation properties of the connector 77
2.6 The covariant derivative 78
2.7 Torsion and normal coordinates 82
2.8 Compatibility of the metric with the connection 84
2.9 Parallelism 85
2.10 Applications of the covariant derivative 86
2.11 The exterior derivative 88
2.12 Frobenius theorems 90
2.13 Isometries on M 99

3 The Curvature 102
3.1 The Riemann tensor 102
3.2 Symmetry properties of the Riemann tensor and the Gaussian curvature 106
3.3 Significance of a curvature tensor vanishing everywhere 109
3.4 The Ricci tensor, the curvature scalar, the Weyl tensor 111
3.5 The Bianchi identities 114
3.6 The equation of geodesic deviation 117
3.7 The covariant derivative of the world-function 120
3.8 Maximally symmetric spaces 125

4 Space-time and Tetrad Formalism 129
4.1 The space-time manifold and the physical observer 129
4.2 Construction of a tetrad 133

© Cambridge University Press www.cambridge.org

CAMBRIDGE

Cambridge University Press
978-0-521-42908-5 - Relativity on Curved Manifolds
F. de Felice and C. J. S. Clarke
Table of Contents
More information

4.3 Relations among tetrads and the Lorentz group 135
4.4 The propagation laws for tetrads 138
4.5 The Ricci rotation coefficients 139
4.6 Differential operators related to a tetrad frame 141

5 Spinors and the Classification of the Weyl Tensor 146
5.1 Outline 146
5.2 The group $SL(2, \mathbb{C})$ 146
5.3 Lie algebras 151
5.4 Bivector algebra 155
5.5 Spinors 159
5.6 The spinor connection 166
5.7 The spinor curvature 168
5.8 The torsion case 172
5.9 Conformal spinors 174
5.10 The Weyl spinor and the Petrov classification 177

6 Coupling Between Fields and Geometry 185
6.1 Newtonian fluids 185
6.2 Generalization to special relativity 187
6.3 Coupling between fields and geometry: the field action 188
6.4 The gravitational action and the Einstein equations 191
6.5 The energy-momentum tensor of a perfect fluid 195
6.6 The energy-momentum tensor of a single particle 198
6.7 The energy-momentum tensor of the electromagnetic field 200
6.8 The energy-momentum pseudotensor 201

7 Dynamics on Curved Manifolds 206
7.1 Conservation laws 206

© Cambridge University Press www.cambridge.org

CAMBRIDGE

Cambridge University Press
978-0-521-42908-5 - Relativity on Curved Manifolds
F. de Felice and C. J. S. Clarke
Table of Contents
More information

7.2 The equations of motion of an extended body 211
7.3 The centre-of-mass description 216
7.4 Motion of a point particle 221
7.5 Constants of motion 225
7.6 Maxwell's equations for a free electromagnetic field 227
7.7 Maxwell's equations in the presence of charges and currents 228
7.8 The radiation field 231
7.9 The light cone 236
7.10 Stationary space-times 238
7.11 The geometry of stationary null surfaces 244

8 Geometry of Congruences 250
8.1 Tetrad decomposition of the Riemann tensor 250
8.2 The expansion equation 255
8.3 The vorticity equation 258
8.4 The Einstein equations in tetrad form 259
8.5 The geometry of null rays 261
8.6 Singularities 267

9 Physical Measurements in Space-time 274
9.1 The concept of measurement 274
9.2 The measurement of time intervals and space distances 275
9.3 Measurements of angles 281
9.4 Curvature effects in the measurement of angles 282
9.5 Measurement of frequency 286
9.6 Measurement of relative velocities 287
9.7 The velocity composition law 295
9.8 Energy and momentum of a particle 297
9.9 Measurement of electric and magnetic fields 298
9.10 The properties of a fluid 300
9.11 The equations of motion of a fluid 304
9.12 The small curvature limit 306

© Cambridge University Press www.cambridge.org

CAMBRIDGE

Cambridge University Press
978-0-521-42908-5 - Relativity on Curved Manifolds
F. de Felice and C. J. S. Clarke
Table of Contents
More information

9.13 Gravitational radiation 314

10 Spherically Symmetric Solutions 320
10.1 The spherically symmetric line element 320
10.2 The external Schwarzschild solution 322
10.3 The internal Schwarzschild solution 326
10.4 The global structure of spherically symmetric space-times 329
10.5 The extended external Schwarzschild solution 332
10.6 Penrose diagrams 338
10.7 Time-like geodesics in the external Schwarzschild solution 342
10.8 The precession of the apsidal points 347
10.9 The plunging-in observer 349
10.10 Null geodesics in the external Schwarzschild solution 351
10.11 The bending of light rays 354
10.12 The Reissner-Nordström solution 358
10.13 The extended Reissner-Nordström solution 361
10.14 Particle behaviour near the Reissner-Nordström singularity 369
10.15 Homogeneous and isotropic cosmology 372
10.16 The Friedmann solutions 378
10.17 Cosmological effects 384

11 Axially Symmetric Solutions 389
11.1 The axially symmetric line element: the canonical form 389
11.2 The Kerr solution 392
11.3 Physical interpretation of the Kerr metric 397
11.4 The space-time structure 400
11.5 Time-like geodesics 406
11.6 Rotationally induced effects 408
11.7 The angular geodesic equation 413
11.8 The equatorial circular geodesics 414
11.9 Null geodesics 418

© Cambridge University Press www.cambridge.org

CAMBRIDGE

Cambridge University Press
978-0-521-42908-5 - Relativity on Curved Manifolds
F. de Felice and C. J. S. Clarke
Table of Contents
More information

11.10 The Kerr-Newman solution 421
11.11 The Weyl and the T-S solutions 424
11.12 Hawking radiation: an overview 426

Notation *431*

References *434*

Index *441*

© Cambridge University Press www.cambridge.org