CAMBRIDGE

Cambridge University Press
978-0-521-42725-8 - Arithmetical Functions
Wolfgang Schwarz and Jurgen Spilker
Table of Contents
More information

Contents

Preface xi

Acknowledgments xv

Notation xvii

Chapter I Tools from Number Theory 1

I.1. Partial Summation 2

I.2. Arithmetical Functions, Convolution, Möbius Inversion Formula 4

I.3. Periodic Functions, Even Functions, Ramanujan Sums 15

I.4. The Turán-Kubilius Inequality 19

I.5. Generating Functions, Dirichlet Series 25

I.6. Some Results on Prime Numbers 31

I.7. Characters, L-Functions, Primes in Arithmetic Progressions 35

I.8. Exercises 39

Photographs 43

Chapter II Mean-Value Theorems and Multiplicative Functions, I 45

II.1. Motivation 46

II.2. Elementary Mean-Value Theorems (Wintner, Axer) . . 49

II.3. Estimates for Sums over Multiplicative Functions (Rankin's Trick) 56

II.4. Wirsing's Mean-Value Theorem for Sums over Non-Negative Multiplicative Functions 65

II.5. The Theorem of G. Halász on Mean-Values of Complex-Valued Multiplicative Functions 76

II.6. The Theorem of Daboussi and Delange on the Fourier-Coefficients of Multiplicative Functions 78

II.7. Application of the Daboussi-Delange Theorem to a Problem of Uniform Distribution 81

II.8. The Theorem of Saffari and Daboussi, I. 82

II.9. Daboussi's Elementary Proof of the Prime Number Theorem 85

II.10. Mohan Nair's Elementary Method in Prime Number Theory 91

© Cambridge University Press www.cambridge.org

CAMBRIDGE

Cambridge University Press
978-0-521-42725-8 - Arithmetical Functions
Wolfgang Schwarz and Jurgen Spilker
Table of Contents
More information

II.11. Exercises 93

Chapter III Related Arithmetical Functions 97
III.1. Introduction, Motivation 98
III.2. Main Results 101
III.3. Lemmata, Proof of Theorem 2.3 104
III.4. Applications 110
III.5. On a Theorem of L. Lucht 115
III.6. The Theorem of Saffari and Daboussi, II 117
III.7. Application to Almost-Periodic Functions 118
III.8. Exercises 121

Chapter IV Uniformly Almost-Periodic Arithmetical Functions 123
IV.1. Even and Periodic Arithmetical Functions 124
IV.2. Simple Properties 133
IV.3. Limit Distributions 139
IV.4. Gelfand's Theory: Maximal Ideal Spaces 142
IV.4.A. The maximal ideal space $\Delta_{\mathcal{B}}$ of $\mathcal{B}^u$ . . 142
IV.4.B. The maximal ideal space $\Delta_{\mathcal{D}}$ of $\mathcal{D}^u$ . . 147
IV.5. Application of Tietze's Extension Theorem 155
IV.6. Integration of Uniformly Almost-Even Functions . . 156
IV.7. Exercises 162

Chapter V Ramanujan Expansions of Functions in $\mathcal{B}^u$ 165
V.1. Introduction 166
V.2. Equivalence of Theorems 1.2, 1.3, 1.4, 1.5 168
V.3. Some Lemmata 171
V.4. Proof of Theorem 1.5 175
V.5. Proof of Lemmas 3.4 and 3.5 178
V.6. Exercises 184

Chapter VI Almost-Periodic and Almost-Even Arithmetical Functions 185
VI.1. Besicovich Norm, Spaces of Almost Periodic Functions 186
VI.2. Some Properties of Spaces of q-Almost-Periodic Functions 197

© Cambridge University Press www.cambridge.org

CAMBRIDGE

Cambridge University Press
978-0-521-42725-8 - Arithmetical Functions
Wolfgang Schwarz and Jurgen Spilker
Table of Contents
More information

Contents

VI.3. Parseval's Equation 206
VI.4. A Second Proof for Parseval's Formula 208
VI.5. An Approximation for Functions in $\mathcal{B}^1$ 210
VI.6. Limit Distributions of Arithmetical Functions . . . 212
VI.7. Arithmetical Applications 215
VI.7.A. Mean-Values, Limit Distributions 215
VI.7.B. Applications to Power-Series with Multiplicative Coefficients 218
VI.7.C. Power Series Bounded on the Negative Real Axis 221
VI.8. A $\mathcal{B}^q$-Criterion 224
VI.9. Exercises 229
Photographs 231

Chapter VII The Theorems of Elliott and Daboussi 233
VII.1. Introduction 234
VII.2. Multiplicative Functions with Mean-Value $M(f) \neq 0$, Satisfying $\|f\|_2 < \infty$ 239
VII.3. Criteria for Multiplicative Functions to Belong to $\mathcal{B}^1$ 243
VII.4. Criteria for Multiplicative Functions to Belong to $\mathcal{B}^q$ 251
VII.5. Multiplicative Functions in $\mathcal{A}^q$ with Mean-Value $M(f) \neq 0$ 257
VII.6. Multiplicative Functions in $\mathcal{A}^q$ with Non-Void Spectrum 261
VII.7. Exercises 266

Chapter VIII Ramanujan Expansions 269
VIII.1. Introduction 270
VIII.2. Wintner's Criterion 271
VIII.3. Mean-Value Formulae for Multiplicative Functions 276
VIII.4. Formulae for Ramanujan Coefficients 280
VIII.5. Pointwise Convergence of Ramanujan Expansions 284
VIII.6. Still Another Proof for Parseval's Equation 289
VIII.7. Additive Functions 291
VIII.8. Exercises 291

© Cambridge University Press www.cambridge.org

CAMBRIDGE

Cambridge University Press
978-0-521-42725-8 - Arithmetical Functions
Wolfgang Schwarz and Jurgen Spilker
Table of Contents
More information

Chapter IX Mean-Value Theorems and Multiplicative Functions, II 293
IX.1. On Wirsing's Mean-Value Theorem 294
IX.2. Proof of Theorem 1.4 298
IX.3. The Mean-Value Theorem of Gabor Halász 303
IX.4. Proof of Proposition 3.3 309
IX.5. Exercises 311
Photographs 313

Appendix 315
A.1. The Stone-Weierstrass Theorem, Tietze's Theorem . . 315
A.2. Elementary Theory of Hilbert Space 316
A.3. Integration 319
A.4. Tauberian Theorems (Hardy-Littlewood-Karamata, Landau-Ikehara) 321
A.5. The Continuity Theorem for Characteristic Functions 323
A.6. Gelfand's Theory of Commutative Banach Algebras 325
A.7. Infinite Products 327
A.8. The Large Sieve 329
A.9. Dirichlet Series 331

Bibliography 333
Author Index 353
Subject Index 357
Photographs 365
Acknowledgements 367

© Cambridge University Press www.cambridge.org