

Index

- absolute value of a c -number, exercise 1.6
- action functional, 232
 - for Bose oscillator, 273
 - for Fermi oscillator, 261
 - for linear supersymmetric model, 286
 - for nonlinear supersymmetric model, 293
 - for simple Fermi system, 250
 - for supersymmetric system in curved space, 370
- adjoint representation, 150–1
- antisupercommutator, 65
- antisupersymmetric matrix, 28
- a -numbers, 3
 - functions of real a -numbers, 9
 - superanalytic functions of, 4
- arc length, exercise 2.9
- atlas, 52
 - locally finite atlases, 110–11
- auxiliary functions, 132ff.
 - left auxiliary function, 133
 - right auxiliary function, 133
- bases (of supervector spaces), 16ff.
 - canonical bases, 99–100
 - change of basis, 24–5
 - coordinate bases, 62
 - dual bases, 30
 - for forms, 85
 - orthosymplectic bases, 99–100
 - pure bases, 17ff.
 - pure real bases, 20ff.
 - standard bases, 24
- Betti numbers, 324, 342, 385
- body
 - nonvanishing body of a supervector, 25
 - of a matrix, 2
 - of a super Lie group, 123
 - of a supermanifold, 55
 - of a supernumber, 1
 - vanishing body of a supervector, 25
- Bose–Fermi supersymmetry, 285ff.
- Bose–Fermi supersymmetry group, 289–92, 295–6, 374
- Bose oscillator, 273ff.
 - action functional for, 273
 - coherent states for, 277ff.
- Feynman propagator for, 281–3
- Hamiltonian for, 276
- mode functions for, 275
- positive and negative frequency functions for, 275
- supercommutator function for, 274
- C_a , 3
- C_c , 3
- canonical bases, 99–100
- canonical coordinates, 129–30
- canonical diffeomorphisms of super Lie groups, 124
- canonical form of metric tensor, 98
- canonical systems, 307ff.
- canonical transformation group, 217–18
- Cartan–Killing matrix, 169, 178
 - of $D(2.1, a)$, 197
 - of $F(4)$, 200
 - of $G(3)$, 206
 - of $GL(m, n)$, 182
 - of $O\text{Sp}(m, n)$, 189
 - of $P(m)$, 192
 - of $Q(m)$, 194
 - of $SL(m, n)/GL(1, 0)$, 186
 - of $SL(m, n)$, 184
- Cartan super Lie groups, 216ff.
 - canonical transformation group $\text{Can}(M, \omega)$, 217–18
 - contact transformation group, 219–20
 - diffeomorphism group $\text{Diff}(M)$, 216
 - $H(n)$, 222–3
 - $\tilde{H}(n)$, 222–3
 - $\text{SDiff}(M, \mu)$, 216–17
 - $S(n)$, 221–2
 - $\tilde{S}(n)$, 221–2
 - $W(n)$, 221
- characteristic functions, 314
- charts, 52
- Chern–Gauss–Bonnet formula, 388–9
- chronological ordering, 246, 247
- classical super Lie groups, 180ff.
 - $A(m, m)$, 190
 - $A(m, n)$, 190
 - $B(m, n)$, 190
 - $C(n)$, 190

- $D(m, n)$, 190
- $GL(m, n)$, 180ff.
- orthosymplectic group $OSp(m, n)$, 187ff.
- $P(m)$, 191–2
- $Q(m)$, 193–4
- $\tilde{Q}(m)$, 193
- $SL(m, m)/GL(1, 0)$, 185–6
- $SL(m, n)$, 183ff.
- classification of transitive realizations of super Lie groups, 147–8
- c -numbers, 3
 - absolute value of, exercise 1.6
 - functions of real c -numbers, 6–7
 - superanalytic functions of, 4
- coadjoint representation, 151
- coherent states
 - for Bose oscillator, 277ff.
 - for Fermi oscillator, 266ff.
 - for supersymmetric system in curved space, 381
 - stationary path between
 - for Bose oscillator, 283
 - for Fermi oscillator, 270
- cohomology class, 324
- coincidence limits, 365, 368
- commutator subgroup, 341
- compact supermanifolds, 56
- completeness relations, 268, 279, 322, 377, 382
- complex conjugation
 - of supernumbers, 5–6
 - of supervectors, 16
 - of tensors, 73
- components
 - of connections, 89
 - of contravariant vector fields, 61
 - of covariant vector fields, 69–70
 - of dual supervectors, 30
 - of supervectors, 16
 - of tensors and tensor fields, 72ff.
- configuration spaces, 227ff.
 - curved, 327ff.
 - nontrivial, 307ff.
- conformal group
 - global conformal group, 107ff.
 - local conformal group, 104
- conformal Killing vector fields, 106–7
- conformally flat Riemannian supermanifolds, 104–5
- conformally related Riemannian supermanifolds, 103–4
- congruences, 82
 - left-invariant congruences, 126–7
 - right-invariant congruences, 126–7
- connection 1-forms, exercise 2.7
- connections, 88ff.
 - components of, 89
 - invariant connections on coset spaces, 153ff.
 - invariant connections on group supermanifolds, 163ff.
 - Riemannian connections, 100–1
 - transformations laws for, 89
- conservation laws for supersymmetric systems, 287, 293, 373–4
- contact transformation group, 219–20
- contractions of tensor fields, 77
- contragradiant representations, 149–50
- contravariant vector fields, 58ff.
 - components of, 61
- conventional super Lie groups, 173
 - structure of, 176
- coordinate functions, 54
- coordinate systems, 52
 - for coset spaces, 146–7
 - super-Cartesian coordinates, 103
- coset spaces, 140–1
 - coordinates for, 146–7
 - geometry of, 152ff.
 - invariant connections on, 153ff.
 - invariant measure functions on, 153ff.
 - invariant metric tensor fields on, 154ff
 - invariant tensor fields on, 153
 - left coset spaces, 140
 - right coset spaces, 140
 - super Poincaré group/ $\overline{SO}(1, 3)$, 210ff.
- cotangent spaces, 69
- covariant derivative, 88
 - with respect to time, 329
- covariant functional differentiation, 331–2
- covariant variation, 328–9
- covariant vector fields, 69ff.
 - components, of, 69–70
- covering translation, 344
- curvature tensor field, 101
- curvature 2-forms, exercise 2.7
- defining representation, 179
- delta function
 - on \mathbf{R}_a^n , 10
 - on \mathbf{R}_a^n , 12–13
 - on \mathbf{R}_c , 8
- derivations
 - exterior derivative, 86
 - Lie derivations, 78ff.
 - of forms, 85
- derivative mapping, 81
 - of connections, exercise 2.3
 - of measure functions, exercise 2.4
- diffeomorphism group, 216
- diffeomorphisms, 53
- differentiable mappings, 50–3

- differential forms, 71, 83ff.
- differentials, 70
- dimension
 - of supermanifold, 52
 - of supervector spaces, 17, 19
- distant parallelism, 96
 - distant parallelism in group supermanifolds, 165–6
- draggings
 - dragging of tensor fields, 83
 - left draggings, 124
 - right draggings, 124
- dual bases, 30
- dual supervector spaces, 28ff.
- dynamical equations, 232

- embeddings, 54
- Euler–Poincaré characteristic, 385ff.
 - Chern–Gauss–Bonnet formula for, 388–9
 - functional integral for, 388ff.
 - supertrace representation of, 385
 - topological invariance of, 385–6
- exceptional super Lie groups, 194ff.
 - $D(2, 1, \alpha)$, 194ff.
 - $F(4)$, 197ff.
 - $G(3)$, 201ff.
- exponential mapping, 129–30
- extending representation, 176
 - for $D(2, 1, \alpha)$, 194
 - for $F(4)$, 197
 - for $G(3)$, 201
 - for $GL(m, n)$, 183
 - for $OSp(m, n)$, 189
 - for $P(m)$, 192
 - for $Q(m)$, 194
 - for $\tilde{Q}(m)$, 193
 - for $SL(m, m)/GL(1, 0)$, 186
 - for $SL(m, n)$, 184
- exterior derivative, 86, 380
- exterior product, 84
- external sources, 245

- Fermi oscillator, 261ff.
 - action functional for, 261
 - coherent states for, 266ff.
 - Feynman propagator for, 272–3
 - Hamiltonian for, 263
 - mode functions for, 262
 - positive and negative frequency functions for, 262
 - supercommutator function for, 262
- Feynman functional integral, 248–9
 - ambiguity in, 336ff.
 - for Bose oscillator, 281–3
 - for Fermi oscillator, 268–9
 - evaluation of, 269ff.
 - for simple Fermi system, 254–5
 - evaluation of, 256ff.
 - for system in curved space, 332ff.
- Feynman propagator
 - for Bose oscillator, 283–4
 - for Fermi oscillator, 272–3
- field of local frames, 68
- flat Riemannian supermanifolds, 103
- forms, 83ff.
 - closed, 319
 - derivations of, 85
 - exterior derivative of, 86
 - exterior product of, 84
 - operator-valued, 312
- Fourier transforms
 - over \mathbf{R}_a^n , 10–11
 - over \mathbf{R}_a^n , 13
 - over \mathbf{R}_c , 8
- fully reducible representations, 160
- functional derivative, 228
- functionals, 228
 - over configuration space, 228
 - over the space of histories, 231
- functions
 - over \mathbf{R}_a^n , 9
 - over \mathbf{R}_c , 6–7
 - superanalytic functions, 3
- fundamental domain, 344
- fundamental group, 339
- fundamental representation, 179

- Gaussian integrals (over $\mathbf{R}_c^m \times \mathbf{R}_a^n$), 44ff.
- geometry of coset spaces, 152ff.
 - geometry of super Poincaré group/ $SO(1, 3)$, 212–13
- global conformal group, 107ff.
- Grassmann algebras, 1
- Green's functions
 - advanced Green's function, 233
 - for Bose oscillator, 274, 284
 - for Fermi oscillator, 261–2, 272–3
 - for simple Fermi system, 250, 260
 - for standard canonical systems, 309, 356
 - for supersymmetric system in curved space, 371–2
 - reciprocity relations for, 234
 - retarded Green's function, 233
 - role of G^* in the functional integral, 335, 361–2, exercise 6.11
 - superdeterminants of, 333–4, 358–9, exercise 6.14
- Hamiltonian operator
 - chronologically ordered Hamiltonian, 349ff.
 - for Bose oscillator, 276
 - for Fermi oscillator, 263

- for simple Fermi system, 251
 - for standard canonical systems, 325ff.
 - for supersymmetric systems, 289, 293, 375–6
 - for system in a curved space, 347ff.
- Hamilton–Jacobi theory, 352ff.
 - for Bose oscillator, 279ff.
- heat-kernel expansion, 363ff.
- Hodge operator, 380
- homology, 342ff.
- homotopy, 337ff.
- homotopy equivalence classes, 338
- homotopy group, 338–9
- homotopy mesh, 339

- infinitesimal disturbances, 232–3
- inner automorphisms, 150
- inner product, 28, 87
- integral supercurves, 82
- integration
 - over \mathbf{R}_a^n (Berezin integration), 9–10
 - over \mathbf{R}_a^n , 12–13
 - over \mathbf{R}_c , 37ff.
 - over $\mathbf{R}_c^m \times \mathbf{R}_a^n$, 37ff.
 - over super Lie groups, 166ff.
 - over supermanifolds, 110ff.
- invariant connections on coset spaces, 153ff.
 - on super Poincaré group/ $\overline{\text{SO}}(1, 3)$, 212
- invariant measure functions on coset spaces, 153ff.
 - on super Poincaré/ $\overline{\text{SO}}(1, 3)$, 212
- invariant tensor fields on coset spaces, 153
 - invariant metric tensor fields on coset spaces, 154ff.
 - on super Poincaré group/ $\overline{\text{SO}}(1, 3)$, 212
- irreducible sets of matrices, exercise 1.8
- isotropy subgroups, 139, 140

- Jacobi fields, 356ff.

- Killing flows, 141ff.
 - on super Poincaré group/ $\text{SO}(1, 3)$, 211
- Killing vector fields, 106

- Lagrangian, 251, 263, 307, 327
- Laplacian operator, 380
- left draggings, 124
- left-invariant congruences, 126–7
- left-invariant vector fields, 125
 - left-invariant local frame fields, 125–6
- left translations, 124
- Lie derivative, 78ff.
 - of connections, exercise 2.3
 - of contravariant vector fields, 78
 - of covariant vector fields, 79
 - of forms, 86
 - of measure functions, exercise 2.4
 - of metric tensor field, 106
 - of scalar fields, 78
- Lie superalgebras, 174
- linearly independent supervectors, 16
- linear operators, 238–9
 - physical observables, 239–40
 - self-adjoint linear operators, 239
- local conformal group, 104
- local frames, 67–8
 - field of, 68
- locally finite atlases, 110–11
- loop expansion, 360ff.

- matrices,
 - antisymmetric matrix, 28
 - a -type matrix, exercise 1.9
 - body of 2,
 - c -type matrix, exercise 1.9
 - irreducible sets of, exercise 1.8
 - soul of, 2
 - superdeterminant of, 34–7, exercise 1.10
 - symmetric matrix, 28
 - supertrace of, 33, exercise 1.9
 - supertranspose of, 25–8, exercise 1.9
- matrix representations of super Lie algebras, 151–2
 - defining representation, 179
 - extending representation, 176
 - fully reducible representations, 160
 - fundamental representation, 179
- matrix representations of super Lie groups 149ff.
 - adjoint representation, 150–1
 - coadjoint representation, 151
 - contragrading representations, 149–50
 - defining representation, 179
 - extending representation, 176
 - fundamental representation, 179
 - generators of, 152
- measure functions, 110
 - invariant measure functions on coset spaces, 153ff.
 - invariant measure functions on group supermanifolds, 166–7
- metric tensor field, 97
 - canonical form of, 98
 - invariant metric tensor fields on coset spaces, 154ff.
 - invariant metric tensor fields on group supermanifolds, 168–9
- mode functions
 - for Bose oscillator, 275
 - for Fermi oscillator, 262
- momentum operator, 316ff.
 - lack of uniqueness of, 318
 - position representation of, 322ff.

- Morse index, 362
 multitenor fields, 76
- one-parameter Abelian subgroups, 127ff.
 orbits, 137–8
 orientable supermanifolds, 56–7
 orthosymplectic bases, 99–100
 orthosymplectic group, 187ff.
- paracompact supermanifolds, 56
 parallelism at a distance, 96
 parallelism at a distance in group
 supermanifolds, 165–6
 parallel transport, 93–4
 partitions of unity, 110–11
 path integral, 332ff.
 (see Feynman functional integral)
- Peierls bracket, 235
 equivalence to Poisson bracket, 310
 for standard canonical systems, 309ff.,
 325
 identities satisfied by, 236
 phase space, integration over, 347–8
 physical observables, 239–40
 physical state vectors, 238
 position operator, 313–4
 position representation, 321ff.
 projection m -form, 312–3
 position representation of, 327
 projectively Hausdorff spaces, 51
 projectively Hausdorff supermanifolds,
 56
- pseudorepresentations
 of $F(4)$, 200–1
 of $G(3)$, 206–7
 of $Q(m)$, 194
 of $SL(m, m)/GL(1, 0)$, 185
- quantization on nontrivial manifolds,
 311ff.
- quantum systems, 240ff.
 Bose oscillator, 273ff.
 Fermi oscillator, 261ff.
 linear supersymmetric model, 285ff.
 nonlinear supersymmetric model, 292ff.
 simple Fermi system, 250ff.
- \mathbb{R}_a^n , 6
 \mathbb{R}_a^n , 12
 \mathbb{R}_c^n , 6
 $\mathbb{R}_c^m \times \mathbb{R}_a^n$, 13
 integration over, 37ff.
 topology of, 50–2
- realizations
 of super Lie groups, 137ff.
 faithful realizations, 137
 transitive realizations, 138
 of the body of a supermanifold, 57
 reciprocity relations, 234
 representations of super Lie algebras,
 151–2
 defining representation, 179
 extending representation, 176
 fully reducible representations, 160
 fundamental representation, 179
 representations of super Lie groups, 149ff.
 adjoint representation, 150–1
 coadjoint representation, 151
 contragradient representations, 149–50
 defining representation, 179
 extending representation, 176
 fundamental representation, 179
 generators of, 152
 Ricci tensor field, 102
 Riemannian connections, 100–1
 Riemannian supermanifolds, 97ff.
 flat Riemannian supermanifolds, 103
 Riemann tensor field, 92
 right draggings, 124
 right-invariant congruences, 126–7
 right-invariant vector fields, 125
 right-invariant local frame fields, 125–6
 right translations, 124
- scalar fields, 52
 Schrödinger equation, 325ff., 348, 380
 Schwinger variational principle, 242ff.
 simple Fermi system, 250ff.
 action functional for, 250
 advanced and retarded superclassical
 trajectories for, 257
 average superclassical trajectory for,
 259–60
 Hamiltonian for, 251
 supercommutator function for, 250
- skeleton
 of a super Lie group, 123
 of a supermanifold, 54
- soul
 of a matrix, 2
 of a supernumber, 1
 soul subspace, 51, 55
- space of histories, 231
 spontaneously broken supersymmetry,
 299ff.
- standard canonical systems, 307ff.
 Green's function for, 309
- sub-supermanifold, 53
 superalgebras, 239
 the superalgebra $\mathcal{A}(C)$, 312ff.
 superanalytic functions, 3
 integration of, 5
 of a -numbers, 3
 of c -numbers, 4

- super Bianchi identity, 93
- super-Cartesian coordinates, 103
- superclassical dynamical systems, 227ff.
 - Bose oscillator, 273ff.
 - Fermi oscillator, 261ff.
 - linear supersymmetric model, 285ff.
 - nonlinear supersymmetric model, 292ff.
 - simple Fermi system, 250ff.
 - supersymmetric model in curved space, 370ff.
- supercommutator, 65
 - for derivations, 87
- supercommutator function, 235
 - for Bose oscillator, 274
 - for Fermi oscillator, 262
 - for simple Fermi system, 250
 - for supersymmetric system in curved space, 372
- supercurves, 53
 - a*-type supercurves, 53
 - congruences of, 82
 - c*-type supercurves, 53
 - integral supercurves, 81
 - supergeodesics, 94–5
 - tangents to, 63
- super de Sitter group, 207ff.
 - contraction to super Poincaré group, 209–10
- superdeterminant, 34ff., exercise 1.10
 - computation of for Feynman functional integral, 332–4, 358–9, exercise 6.14
- supergeodesics, 94–5
- super Hilbert spaces, 237ff.
 - linear operators in, 238–9
 - physical elements of, 238
 - the superalgebra $\mathcal{A}(C)$ for, 312ff.
- super-Jacobian, 41ff.
- super Jacobi identity, 65
 - for the Peierls bracket, 236
- super Killing group, 108, 109
- super Lie Algebras, 130–1
- super Lie bracket, 66
- super Lie groups, 123ff.
 - body of, 123, 124
 - Cartan super Lie groups, 216ff.
 - classical super Lie groups, 180ff.
 - conventional super Lie groups, 173
 - exceptional super Lie groups, 194ff.
 - geometry of, 163ff.
 - invariant connections on, 163ff.
 - invariant measure functions on, 166–7
 - invariant metric tensor fields on, 168–9
 - matrix representations of, 149ff.
 - parallelism at a distance in, 165–6
 - realizations of, 137ff.
 - skeleton of, 123
 - unconventional super Lie groups, 174–5
- super Lorentz group, 213ff.
 - representations of, 214ff.
- supermanifolds, 50ff.
 - body of, 55
 - compact supermanifolds, 56
 - conformally flat Riemannian supermanifolds, 104–5
 - conformally related Riemannian supermanifolds, 103–4
 - orientable supermanifolds, 56–7
 - paracompact supermanifolds, 56
 - projectively Hausdorff supermanifold, 56
 - Riemannian supermanifolds, 97ff.
 - supermanifolds as configuration spaces, 230
 - topology of, 53
- supernumbers, 1
 - a*-numbers, 3
 - body of a supernumber, 1
 - c*-numbers, 3
 - imaginary supernumbers, 6
 - pure supernumbers, 15
 - real supernumbers, 5–6
 - soul of a supernumber, 1
- super Poincaré group, 209ff.
 - coset space of 210ff.
- supersymmetric matrix, 28
- supersymmetric models, 285ff.
 - in curved space, 370
 - action functional for, 370
 - conserved quantities for, 373–4
 - Green's functions for, 372
 - Hamiltonian for, 373, 376
 - supercommutator function for, 372
 - linear supersymmetric model, 285ff.
 - action functional for, 286
 - conserved quantities for, 287
 - Hamiltonian for, 186
 - supercommutators for, 286
 - nonlinear supersymmetric model, 292ff.
 - action functional for, 293
 - conserved quantities for, 295
 - energy spectrum for, 197ff.
 - Green's functions for, 294
 - Hamiltonian for, 293
 - supercommutator function for, 294
- supersymmetry group, 289, 296, 374
- supertrace, 33, exercise 1.9
 - representation of, 382–3, exercise 5.10
- supertranspose, 25ff., exercise 1.9
- supervectors, 14ff.
 - addition of, 14
 - complex conjugation of, 16
 - complex supervectors, 16
 - imaginary supervectors, 16
 - linearly independent supervectors, 16

- multiplication of, by supernumbers, 14
 - nonvanishing body of, 25
 - pure supervectors, 15
 - real supervectors, 16
 - type-*a* supervectors, 15
 - type-*c* supervectors, 15
 - vanishing body of, 25
 - zero supervector, 14
- supervector spaces, 14ff.
 - bases for, 16
 - dimensionality of, 17, 19
 - dual supervector spaces, 28ff.
 - structure constants, 131–2
 - symmetric supermanifolds, exercise 3.13
- tangent spaces, 62
- tangent bundle, 307
- tensor products, 74–5
- tensors and tensor fields, 72ff.
 - components of, 72
 - contractions of, 77
 - curvature tensor field, 101
 - metric tensor field, 97
 - Ricci tensor field, 102
 - Riemann tensor field, 92
 - torsion tensor field, 90–1
 - unit tensor field, 77–8
 - Weyl tensor field, 103–4
- topology
 - of $\mathbb{R}_c^m \times \mathbb{R}_a^n$, 50–2
 - of supermanifolds, 53
- torsion subgroup, 342
- torsion tensor field, 90–1
- torsion 2-form, exercise 2.7
- transition amplitudes, 244ff., 254ff. 268ff., 278ff., 340ff., 345ff., 386
 - partial amplitudes, 339ff.
- two-loop contributions, 368ff., 387
- unconventional super Lie groups, 174–5
- unit tensor field, 77–8
- universal covering space, 343ff.
- Van Vleck–Morette determinant, 281, 354ff., 359
- variational derivative, 228
- vector operators, 314–5
- Weyl tensor field, 103–4
- WKB approximation, 361ff.
- Z_2 -graded algebras, 174