This is an updated and expanded second edition of a successful and well-reviewed text presenting a detailed exposition of the modern theory of supermanifolds, including a rigorous account of the super-analogs of all the basic structures of ordinary manifold theory.

The exposition opens with the theory of analysis over supernumbers (Grassman variables), Berezin integration, supervector spaces and the superdeterminant. This basic material is then applied to the theory of supermanifolds, with an account of super-analogs of Lie derivatives, connections, metric, curvature, geodesics, Killing flows, conformal groups, etc. The book goes on to discuss the theory of super Lie groups, super Lie algebras, and invariant geometrical structures on coset spaces. Complete descriptions are given of all the simple super Lie groups. The book then turns to applications. Chapter 5 contains an account of the Peierls bracket for superclassical dynamical systems, super Hilbert spaces, path integration for fermionic quantum systems, and simple models of Bose–Fermi supersymmetry. The sixth and final chapter, which is new in this revised edition, examines dynamical systems for which the topology of the configuration supermanifold is important. A concise but complete account is given of the path-integral derivation of the Chern–Gauss–Bonnet formula for the Euler–Poincaré characteristic of an ordinary manifold, which is based on a simple extension of a point particle moving freely in this manifold to a supersymmetric dynamical system moving in an associated supermanifold. Many exercises are included to complement the text.

Review comment on the first edition

'*Supermanifolds* is destined to become the standard work for all serious study of super-symmetric theories of physics.' *Nature*

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Contents

Preface to first edition

This book is an outgrowth of a book on quantum gravity that the author started to write nine years ago in collaboration with Christopher Isham. It began as an Appendix to the quantum gravity book, but subsequent developments modified the original plan. Firstly, new results in quantum gravity, particularly in supergravity and in the applications of topology to quantum field theory, appeared so rapidly that the timing of the collaborative volume became inopportune. Secondly, the theory of supermanifolds had so many loose ends which needed to be dealt with that the original Appendix grew beyond reasonable size limits and turned into a book in its own right.

A previous generation of theoretical physicists could function adequately with a knowledge of the theory of ordinary manifolds and ordinary Lie groups. With the discovery of Bose–Fermi supersymmetry all this changed. Nowadays the theorist must know about supermanifolds and super Lie groups. The purpose of the present volume is to provide him with an easily accessible account of these mathematical structures. Mathematicians will find much of this book incomplete and expressed in language that they have nowadays passed beyond, but it is probably pitched about right for the average physicist. It still has something of the character of an Appendix in its lack of any account of how it relates to supergravity and other locally supersymmetric theories. For a time it was to have appeared as Volume I of a two-volume work on supermanifolds and supersymmetry written in collaboration with Peter van Nieuwenhuizen and Peter West. However, delays caused by new developments in supergravity theory, particularly in higher-dimensional Kaluza–Klein versions of supergravity theory, rendered this linkage impractical. The second volume will ultimately appear (in the same Cambridge University Press series), but rather than delay the first volume further, the decision was made to publish the two as separate books. While waiting for the second book to appear, the reader of the present volume who wishes to establish linkages to physics will have to content himself with studying the elementary applications of supermanifold theory selected in chapter 5 of this volume and with reading the already vast literature on supersymmetric theories.

The author wishes to express his gratitude to the following for their support at various times during the writing of this book: The Warden and Fellows of All Souls College, Oxford, where the book was begun, the John Simon Guggenheim Foundation, the United States National Science Foundation, the North Atlantic Treaty Organization and the University of Texas.

Preface to the second edition

At the end of the fifth and last chapter of the first edition the author wrote that if the book were ever to be revised it would include an account of the beautiful work of E. Witten and of L. Alvarez-Gaumé on supersymmetry, Morse theory and the Atiyah–Singer index theorem. Chapter 6 of this revised edition is a partial fulfilment of the promise. The aim of chapter 6, like that of chapter 5, is almost exclusively pedagogical. Unlike chapter 5, however, chapter 6 deals with nontrivial supermanifolds, and the author discovered that there are numerous fine points in the theory of the Feynman functional integral for such supermanifolds that are not adequately covered in the literature, even on a formal level. To be pedagogically helpful the book *has* to deal with these issues, given the fact that, despite the essential role it plays in chapter 6, the functional integral is used in a formal way rather than as a rigorous tool. This has meant that, in order to keep the reader's confidence, the author has had to expend a large part of his effort on the functional integral itself and hence could include only a little of the flavor of the index theorem, as it touches the Euler–Poincaré characteristic. The effort to display the internal consistency of the functional integral formalism has nevertheless been useful in that it presents a challenge to the student to attempt what must surely be possible, namely, to establish the functional integral at last on a fully rigorous basis for both bosonic and fermionic systems.