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052141928X - Quantum Measurement
Vladimir B. Braginsky and Farid Ya. Khalili
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This book is an up-to-date introduction to the quantum theory of measurement, a fast developing field of intense current interest to scientists and engineers for its potential high-technology applications.

Although the main principles of the field were elaborated in the 1930s by Bohr, Schrödinger, Heisenberg, von Neumann and Mandelstam, it was not until the 1980s that technology became sufficiently advanced to allow its application in real experiments. Quantum measurement is now central to many ultra-high technology developments, such as “squeezed light,” single atom traps, and searches for gravitational radiation. It is also considered to have great promise for computer science and engineering, particularly for its applications in information processing and transfer. The book begins with a brief introduction to the relevant theory and goes on to discuss all aspects of the design of practical quantum measurement systems.

This book is essential reading for all scientists and engineers interested in the potential applications of technology near the quantum limit.

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Quantum Measurement

Vladimir B. Braginsky
and Farid Ya. Khalili
Moscow State University

Edited by
Kip S. Thorne
California Institute of Technology



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*A quantum phenomenon
is a phenomenon
Only if it is a recorded phenomenon*

John Archibald Wheeler

Editor's Foreword

The quantum theory of measurement has been widely regarded, since the 1940s, as an esoteric subject of little relevance to “real” physics. Quite the opposite was true in the 1920s and 1930s when Niels Bohr, John von Neumann, and others were struggling, by means of gedanken experiments, to develop an understanding of how measurements on single quantum objects were to be incorporated into quantum theory. However, the understanding they achieved turned out to be of little use in the quantum mechanical applications that commanded the attention of physicists between 1940 and 1980: the interaction of photons, atomic nuclei, and elementary particles, the theory of masers and lasers, the properties of matter (superfluidity, superconductivity, semiconductors), etc.

In each of these applications, although the phenomena studied were quantum mechanical at heart, the measurements used to probe them were so far removed from the quantum domain that the only feature of the quantum theory of measurement which entered into the experiments was the probability interpretation of squared amplitudes. There was no need to invoke what soon came to be regarded as an esoteric, problematic, dubious “collapse (reduction) of the wave function.”

Fundamentally, the reason for this irrelevance of the quantum theory of measurement was technological. The technology of 1940 - 1980 was not capable of making repetitive measurements on a single quantum mechanical system and thereby discovering in a second measurement how a first measurement had affected the system. Instead, the experiments of 1940 - 1980 typically entailed an ensemble of single measurements on a huge number of quantum systems (e.g. muons flying out of the interaction

region in a high-energy scattering experiment), with each measurement destroying the quantum system (muon) in the process of measuring its properties.

In the 1980s technology finally began to catch up with the measurement concepts of Bohr, von Neumann, and their 1930s colleagues, and forced modern physicists to elaborate and extend those concepts from the gedanken experiments of the 1930s to the actual experimental situations of the 1980s: The new technology entails repeated measurements on single quantum systems, measurements in which the more “esoteric” features of the quantum theory of measurement are essential.

One example is repeated measurements of the state of a mode of the electromagnetic field in an optical resonator, measurements that are central to the development and use of “squeezed light,” “frequency anticorrelated states,” “pure number eigenstates,” and other non-classical states of light. A second example is repeated measurements of a single atom that is held in an electromagnetic trap. These examples and others hold great promise for fundamental physics experiments (e.g. tests of quantum theory and of relativity theory, and searches for gravitational radiation), and also for practical applications (e.g. information processing and transfer, and high-stability clocks). Correspondingly, we can expect the quantum theory of measurement to become a central tool of scientists and engineers during the coming decades.

Almost all of the textbooks on quantum mechanics are written from the standard viewpoint of the 1940s - 1970s, the viewpoint that has little respect for or interest in the quantum theory of measurement. Correspondingly, it is essential that, until new textbooks take their place, they be supplemented by a small monograph that treats carefully and clearly the quantum theory of measurement and its applications to practical, high-technology experiments. This book is ideal for this purpose.

Despite being far removed from real experiment until recently, the quantum theory of measurement has long been an arena in which theorists carry out intense, emotional battles. Code words such as *many worlds interpretation*, *collapse of the wave function*, and *irreversibility of measurement* have generated enormous entropy among theorists over the past half century — and have also generated, in the end, some considerable understanding. The reader who wants a guide to the controversies (most of which are over issues of taste and viewpoint rather than over irreconcilable substance) will *not* find it in this small book.* Instead, this book

* For collections of papers that deal with the controversies and the insights they have brought, and with aspects of the quantum theory of measurement that are

Editor's Foreword

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focuses on aspects of the theory that by now, in the early 1990s, are rather well understood and fairly noncontroversial, and that, most especially, are central to the real experiments now being made possible by the rapid march of technology.

One can appreciate more fully this book's focus by knowing something about its authors. One of the authors, Vladimir B. Braginsky, is an experimental physicist who has made major contributions to the science of repetitive measurements in the quantum domain. His contributions include the invention of key new ideas for real quantum measurements (e.g. quantum nondemolition measurements and frequency anticorrelated quantum states), and the invention of key new experimental techniques (e.g. the use of "whispering gallery modes" of electromagnetic excitation of dielectric resonators). The second author, Farid Ya. Khalili, is a theorist who has contributed significantly to the modern extensions of the Bohr - von Neumann formal theory of quantum measurements. His contributions have helped extend the idealized Bohr - von Neumann theory into the domain of the practical experimental devices and techniques of real 1990s experiments.

Braginsky's experience with real quantum measurements and his deep physical intuition, when combined with Khalili's deep mathematical insights, gives this book a power that does not exist elsewhere in the pedagogical literature on measurement theory. This combination enables the book to serve simultaneously as an introduction to the formal theory of quantum measurements and as a guide to the physical concepts and the high-sensitivity measuring techniques that make the formal theory relevant to the 1990s and the 21st century.

Kip S. Thorne
California Institute of Technology

more esoteric and abstract (some might say more fundamental) than those dealt with in this book, see J. A. Wheeler and W. H. Zurek, editors, *Quantum Theory and Measurement* (Princeton University Press, Princeton, 1983); also H. S. Leff and A. F. Rex, editors, *Maxwell's Demon: Entropy, Information, Computing* (Adam Hilger, Bristol, 1990).

Notation

A	amplitude of oscillations
a, a^\dagger	creation and annihilation operators
B	correlation function or correlation matrix
C	capacity of a capacitor
c	speed of light
d	a geometric distance (e.g. distance between plates of a capacitor)
E	energy
E	electric field strength
e	charge of the electron
F	force
f	Fourier transform of force
H	magnetic field strength
H	Hamiltonian
k	mechanical rigidity
k_B	Boltzmann's constant
L	inductance
m	mass
N	number of quanta (chapter XI)
n	number of quanta (except in chapter XI)
	refractive index (chapter XI)
p, P	momentum
q, Q	generalized coordinate
Q	quality factor of an oscillator
q	electric charge
S	spectral density
s	Fourier transform

T	temperature
t	time
\mathbf{U}	evolution operator; electrical tension
V	volume
v	speed
w, W	probability; probability density
W	power
x, y, z	spatial Cartesian coordinates
X_1, X_2	quadrature amplitudes
δ	variation of a changing quantity
$\delta(\)$	Dirac delta function
Δ	uncertainty (standard deviation) of a random quantity
ξ	a dimensionless quantity of order unity
η	flux of the number of photons
λ	wavelength
Ω	reduction operator
ω	angular frequency
ρ	density operator; impedance for a traveling wave; energy density
τ	a time interval
τ^*	relaxation time
ϕ	phase of oscillations; angle
ψ	wave function
χ	generalized susceptibility
\circ	symmetrized product of operators: $\hat{Q} \circ \hat{P} \equiv \frac{1}{2}(\hat{Q}\hat{P} + \hat{P}\hat{Q})$

Quantities with hats (e.g. \hat{q}) are operators

Quantities with tildes (e.g. \tilde{q}) are the results of measurements

Spectral densities are normalized to the frequency range $-\infty$ to $+\infty$, so

$$\Delta^2 = \int_{-\infty}^{+\infty} S(\omega) \frac{d\omega}{2\pi}$$

is the squared standard deviation of a quantity with spectral density $S(\omega)$.