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Excerpt
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1 Historical introduction: photons and measurements using photons

In the standard, textbook treatment of quantum mechanics, the contact with experiment is described in terms of probabilities for obtaining this, that, or another result, when identical experiments are performed on a huge number (*ensemble*) of identical objects. Such an ensemble description was adequate during the first half century of quantum theory, because the technology of that era was incapable of making a measurement without destroying or severely changing the measured object (photon, atom, ...). In recent years, however, advances in technology and technique have made possible repetitive quantum measurements on a single quantum object, with each measurement influencing the object only minimally. Such measurements cannot be analyzed using solely the ensemble theory. Additional theoretical concepts are needed. The purpose of this book is to describe the methods and theory of such measurements (as well as the more elementary theory of ensembles of measurements).

This book is written for two types of readers: those who have had only a little previous contact with quantum mechanics, and those who have had much.

Readers of the first type are presumed to understand atomic physics at a qualitative level. The unstarred sections of this book offer such readers an overview of the present state-of-the-art of quantum measurements on single objects and prospects for the future, as well as an elementary overview of the theory of such measurements.

Readers of the second type are presumed to understand quantum mechanics at the level of an advanced undergraduate or first-year graduate course. This book can serve as a supplement to standard textbooks for such a course. The unstarred sections may repeat, in part, material learned from the standard textbooks, but will also extend that material into the domain of measurements on single objects. The starred sections offer the second type of reader an advanced, theoretical understanding of such measurements.

The principal questions addressed by this book are described at the end of this chapter. As background for their description, this chapter presents a short historical excursion into the origin and development of the principal ideas and methods of quantum measurements.

1.1 The discovery of photons

As is well known, quantum physics began with Max Planck's postulate of the discreteness of the energy in a mode of an electromagnetic resonator:[1] the energy comes in discrete quanta, each of which has an energy $\boldsymbol{E}_{\text{quantum}}$ proportional to the mode's angular frequency of oscillation ω:

$$\boldsymbol{E}_{\text{quantum}} = \hbar\omega \ . \tag{1.1}$$

The constant $\hbar$ was later named after Planck. This postulate enabled Planck to develop a formal theory that describes accurately the observed spectral distribution of the energy of thermal radiation.

Historically, the next idea was Einstein's:[2] the flux of electromagnetic radiation with frequency ω, like the energy of an electromagnetic oscillator, also consists of discrete quanta, each equal to $\hbar\omega$. This quantum of energy can interact with an object as a whole. Einstein combined this idea with the conservation of energy to derive the following simple formula for the photoelectric effect:

$$\hbar\omega = \frac{1}{2} m_e v^2 + \boldsymbol{E}_{\text{bind}} \ . \tag{1.2}$$

Here ω is the frequency of the incoming electromagnetic radiation, $\frac{1}{2} m_e v^2$ is the kinetic energy of an electron ejected from a metal by the incoming radiation, and $\boldsymbol{E}_{\text{bind}}$ is the binding energy of the electron to the metal—a quantity characteristic of the specific metal being used. Einstein's photoelectric formula was confirmed by experiments.

Both of these seminal ideas, Planck's and Einstein's, had a character that was clearly nonclassical; i.e., they could not be derived from Maxwell's equations. These ideas led unavoidably to the conclusion that electromagnetic radiation has not only wavelike properties, but also, simultaneously, particle-like properties. They forced physicists to adapt themselves to the fact that a stream of electromagnetic radiation with frequency ω consists of "seed" portions of energy equal to $\hbar\omega$. Many years later these portions were given the name "photons." Initially this term was used only in the optical region of the electromagnetic spectrum, but now it typically is used throughout the spectrum.

P.N. Lebedev[3] measured the pressure of light (a purely classical effect) several years before the experimental confirmation of Einstein's photoelectric formula (1.2). Lebedev's experiments verified with high confidence the following statement: Any portion $\mathbf{E}$ of energy in electromagnetic radiation carries a mechanical momentum P given by

$$P = \mathbf{E}/c \ , \tag{1.3}$$

where c is the speed of propagation of electromagnetic radiation. By comparing the results (1.2) and (1.3) of the photoelectric and light pressure experiments, one is forced to conclude that each photon carries a mechanical momentum

$$P_{\text{photon}} = \frac{\hbar\omega}{c} \ . \tag{1.4}$$

Experiments on the interference and diffraction of light, when performed with very low light intensities,[4] revealed further that an interference pattern (a classical, pure-wave effect) shows up on a photographic plate only when the number of photons falling on the plate is very large. Each photon in such an experiment is *completely destroyed* (ceases to exist) by interacting with the plate's silver chloride molecules. When the photon is destroyed there appears somewhere on the photographic plate an atom of free silver, which will act as an embryo from which, by photographic developing, a small seed of silver will grow. The silver embryo is much smaller than an electromagnetic wavelength.

This is remarkable. In the interference process (e.g. in the two-slit experiment of Fig. 1.1), the photon must have been influenced by the locations of both slits, since the interference pattern depends on the distance between them. This means that the photon must have occupied a volume larger than the slit separation. On the other hand, when it fell on the photographic plate, the photon must have become localized into the tiny volume of the silver embryo. Later the terms "collapse of the wave function" and "reduction of the wave packet" were used to describe such localization. This collapse or reduction process became one of the key concepts in the quantum theory of measurement.

The wave properties of the photon show up in the fact that the *probability* of collapse at a certain place on the photographic plate (and the accompanying birth of a silver atom there) is proportional to the light

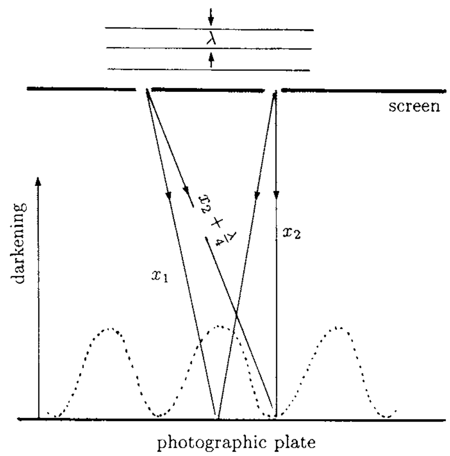

Fig. 1.1 Two-slit interference experiment.

intensity, which is calculated classically using the wave theory. The particle properties show up in the birth of silver atoms, one by one. And the interference pattern appears only after a very large number of photons (an *ensemble* of photons) have fallen on the plate.

Thus, the photon turns out to be neither a particle nor a wave (in the classical sense of these words), but a qualitatively more complicated object; and it behaves like a wave or like a particle depending on the details of the experiment performed.

1.2 The wave and particle properties of photons

In the process of its "death" on a photographic plate a photon behaves like a particle, while in the interference process it behaves like a wave. The question naturally arises, can particle properties show up in experiments where the photon does not die? Yes, as we shall see clearly in many examples later in this book. A somewhat fuzzy example, which is useful for delineating the connection between a photon's particle and wave properties, is *localization* of a freely propagating photon. (Localization is a property common to particles and to short wave packets of classical waves.)

The techniques of modern nonlinear optics permit one to prepare very short pulses of light in dielectric waveguides (optical fibers). These pulses can have durations shorter than $\tau \simeq 1\times10^{-14}$ sec.[5] Such a pulse in the fiber is localized into a spatial length $\Delta x = c\tau \simeq 3\times10^{-4}$ cm, i.e. several optical wavelengths. If the total energy of such an optical pulse is known, then by passing the pulse through several wide-bandwidth absorbers, one can reduce the wave packet's energy to approximately $\hbar\omega$.

Now, the classical theory of waves tells us that the duration τ and bandwidth $\Delta\omega$ of a wave packet are connected by the simple relation

$$\Delta\omega\cdot\tau \gtrsim 1 \ . \tag{1.5}$$

Thus, localization into the spatial interval $\Delta x = c\tau$ must produce an *uncertainty* of the energy. Using equations (1.1) and (1.5), we obtain

$$\Delta \boldsymbol{E}_{\text{photon}}\cdot\tau \gtrsim \hbar \ . \tag{1.6}$$

In other words, whenever an experimenter *prepares* "short" photons (photons well localized in space), there inevitably will be a substantial, random unpredictability of the photon energy:

$$\boldsymbol{E}_{\text{photon}} = \hbar\bar{\omega} \pm \frac{\hbar}{2\tau} \ , \tag{1.7}$$

where $\bar{\omega}$ is the mean frequency. However, so long as $\hbar/2\tau$ is small compared to $\hbar\bar{\omega}$, we can be sure that the wave packet contains only one photon.

Instead of the term "photon localized in space," many publications use the phrase "monophotonic state," or "particle-like wave packet." Recently L. Mandel and S. Hong[6] have succeeded in preparing a photon in a monophotonic state, using a scheme proposed by D. Klyshko[7]. The details of their experiment are sketched in Fig. 1.2

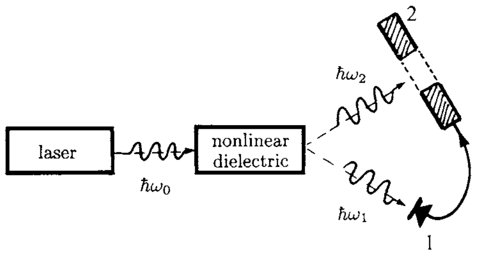

Fig. 1.2 Experimental preparation of a monophotonic state, as carried out by S. Hong and L. Mandel[6] according to a scheme first proposed by D. Klyshko.[7] The photon $\hbar\omega_0$ is split into two photons $\hbar\omega_1$ and $\hbar\omega_2$ in the nonlinear dielectric. The photon $\hbar\omega_1$ is registered by the photodetector 1, which opens the optical gate 2 for a short time τ, permitting the photon $\hbar\omega_2$ to pass. The result is a single photon beyond the gate, localized in a wavepacket of duration τ. This scheme works with a nonideal photodetector (one with quantum efficiency less than unity) just as well as with an ideal one.

Below we shall use the term "photon" in the traditional sense—to denote a quantum of electromagnetic energy of any type: one that is spread out over the volume of a resonator, or one that is localized in a monophotonic state. We shall emphasize localization only when appropriate.

For a monophotonic state the energy uncertainty $\Delta \boldsymbol{E}_{\text{photon}} \gtrsim \hbar/\tau$ leads, according to formula (1.4), to the following momentum uncertainty:

$$\Delta P_{\text{photon}} \gtrsim \frac{\hbar \Delta\omega}{c} . \tag{1.8}$$

The uncertainty of the photon's position (relative to the center of the wave packet) is

$$\Delta x = \frac{c\tau}{2} = \frac{c}{2\Delta\omega} . \tag{1.9}$$

By multiplying equations (1.8) and (1.9), we obtain the fundamental position-momentum uncertainty relation for the monophotonic state:

$$\Delta x \cdot \Delta P \gtrsim \frac{\hbar}{2} . \tag{1.10}$$

There is no conflict between the above discussion and the fact that Max Planck's original formula $\boldsymbol{E}_{\text{photon}} = \hbar\omega$ [Eq. (1.1)] is a precise one. To illustrate this, imagine sending a monophotonic state through a prism (or any other device that decomposes electromagnetic radiation into its spectral components), and onto a photographic plate. The state will produce just one seed of silver on the plate, since it contains just one photon. From the position of this seed relative to the prism, the experimenter can infer the *frequency* of the photon with very high accuracy. In principle, the plate can be replaced by a net of microcalorimeters (devices that measure very accurately the energy of electromagnetic radiation by converting it into some other form of energy). Thereby the experimenter can infer with very high accuracy the photon's *energy* (from the microcalorimeter) and its *frequency* (from the location of the microcalorimeter). The laws of quantum mechanics guarantee that this energy and frequency will be related precisely by Planck's law (1.1). Because this experiment determines, with high precision, the photon's energy and correspondingly its momentum, it must be that in passing through the prism the photon's wave packet gets lengthened by enough to preserve the uncertainty relations $\Delta \boldsymbol{E}_{\text{photon}}\tau \gtrsim \hbar$ and $\Delta x \Delta P \gtrsim \hbar/2$. Indeed, one can show that, whenever a monophotonic state is sent through a spectral analyzer, its spatial length increases. This is not peculiar to quantum mechanics; it is true, also, for any classical wave packet.

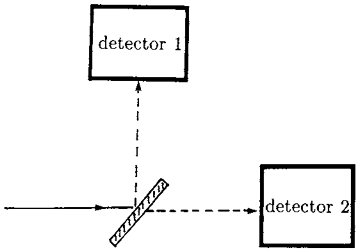

Fig. 1.3 After passing through the beam splitter, the monophotonic state is registered either by detector 1 or by detector 2, and not by both.

To assure ourselves that the photon, despite its wave properties, is also a single, integral particle, we can carry out the following experiment. In the path of a monophotonic state we place a half-silvered mirror (a beam splitter), followed by two photodetectors; cf. Fig. 1.3. Only one of the two detectors will register the photon: detector 1 if the photon passes through the mirror; detector 2 if it is reflected by the mirror. Never, when dealing with one photon, will an experimenter see the monophotonic state's energy get split into two parts and be registered in both detectors (as would be the case for a classical wave packet).

In conclusion, it is worth noting that the particle-wave duality of electromagnetic radiation produces a large number of very interesting effects, which show up in interactions of photons with materials, of photons with phonons, of photons with photons, etc. These effects are now attracting considerable attention among experimenters and theorists, as a result of rapid developments in the state of the art of nonlinear electrodynamics. Despite the popularity of this subject, however, a number of features of photonic interactions remain to be discovered and studied. This book may help to lay foundations for such studies by analyzing and describing various features of the interactions of groups of photons during the measurement process.

1.3 The Heisenberg uncertainty relations

In our historical excursion, we have made a leap of approximately 70 years from the experiments that verified Einstein's photoelectric relation (1.2) to experiments with monophotonic states. Between these experiments, and after Niels Bohr's formulation of his "old quantum theory" postulates, Louis de Broglie hypothesized that ordinary particles have wave properties, and his postulate was confirmed by experiments on the diffraction of electrons (the experiments of Davisson and Germer[8]). In the mid 1920s Werner Heisenberg[9] and Erwin Schrödinger[10] began to formulate mathematically the foundations of the "new" (present-day) quantum mechanics; and in the late 1920s those foundations were completed by Paul Dirac[11] and John von Neumann.[12]

The mathematical formulation of quantum mechanics was far quicker and easier to establish than a full understanding of the *physical essence* of quantum phenomena. Indeed, a full physical understanding is not yet in hand, even now. The most important contribution to our physical understanding was that of Niels Bohr. In his famous discussions with Albert Einstein, Bohr developed the foundations for the physical interpretation of quantum theory—foundations that are now accepted by the majority of physicists.[13,14]

The principal interests of the creators of quantum theory and the quantum theory of measurement were in the microworld. Experimenters during those years focused attention on interactions of *ensembles* of photons with ensembles of nuclei, atoms, and electrons, and on interactions between ensembles of elementary particles. Correspondingly, the creators of quantum theory paid little attention to features of quantum mechanics that lie outside the domain of microworld ensembles; most especially, they largely ignored phenomena associated with measurements of single objects.

As an introduction to such phenomena, we shall recall two thought experiments about measurements of single objects that *were* developed by the creators of quantum theory, as part of their effort to clarify the methodology of quantum measurements: Heisenberg's microscope (end of 1920s), and von Neumann's Doppler speed meter (beginning of 1930s).

The Heisenberg microscope

We shall describe a version of the Heisenberg microscope that is closer to the main contents of this book than Heisenberg's original version.

Suppose that one wishes to measure the position x_1 of a macroscopic body with mass m. To do so, one can attach to the body a stick with diameter less than or of order the wavelength of light. (This can actually be done with modern technology.) If we know in advance the *approximate* position of the mass m, then we can arrange a lens and a photographic plate as shown in Fig. 1.4. The stick must be close to the focal plane of the lens, and the optical amplification factor will be approximately L_2/L_1, where L_1 is the focal length. We can then send in a stream of photons from the side and wait for an *individual* photon to be scattered by the stick, pass through the lens's aperture a, impinge on the photographic plate, and there collapse and produce a small seed of silver.

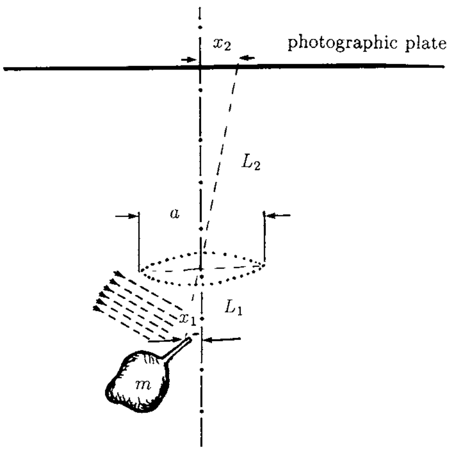

Fig. 1.4 The Heisenberg microscope.

The transverse position x_2 of the silver seed, relative to the lens's optical axis, can be determined to an accuracy much better than an optical wavelength. From x_2 we infer the transverse position $x_1 = -x_2 L_1/L_2$ of the stick, relative to the optical axis. However, we cannot claim that the photon was scattered by the stick at a position precisely equal to this x_1. Because of the photon's wave properties, the scattering may have occurred, with roughly equal probabilities, anywhere within a distance

$$\Delta x_{\text{measure}} \simeq \frac{1}{6}\lambda\frac{L_1}{a} \tag{1.11}$$

of the location $x_1 = -x_2 L_1/L_2$. Here λ is the photon's wavelength. (This fuzziness in the location of the scattering is well known in optics, for light propagating in the opposite direction to that of Fig. 1.4: A plane wave, when focused by a lens, forms a spot whose size is $\simeq (\lambda/3)(L_1/a)$ — the "Airy spot.") The $\Delta x_{\text{measure}}$ of equation (1.11) is the error in the inferred position x_1 when one observes the scattering of a single photon.

Now, because the photon has a momentum [equation (1.4)], and because it must have passed through the lens's aperture a, it must have given to the stick (and its attached mass m) a random momentum in the x direction, with unknown sign and with magnitude of order

$$\Delta P_{\text{perturb}} \gtrsim \frac{\hbar\omega}{c}\frac{a}{2L_1}, \tag{1.12}$$

for $a/L_1 \ll 1$. From the product of equations (1.11) and (1.12) we obtain the following form of the Heisenberg uncertainty relation

$$\Delta x_{\text{measure}} \cdot \Delta P_{\text{perturb}} \geq \frac{\hbar}{2} . \tag{1.13}$$

This thought experiment illustrates the main elements of a quantum measurement:

a) The extraction of information about the measured quantity x_1 (usually called the *observable*), to within a definite error.

b) The inevitable perturbation of another quantity P_1 (or quantities) by the measurement process.

c) An inevitable irreversible process (in our case the death of the photon and birth of the silver seed)—a process that, in fact, is macroscopic in a sense that we shall explore later.

Despite the similar forms of the formulae $\Delta x \cdot \Delta P \geq \hbar/2$ [equation (1.10) for a monophotonic state] and $\Delta x_{\text{measure}} \cdot \Delta P_{\text{perturb}} \geq \hbar/2$ [equation (1.13) for the Heisenberg microscope], their essences are fundamentally different. Equation (1.10) is a fundamental property of the physical state of a quantum object; in it the object's position and momentum have equal footing. According to it, a quantum object, independently of its prehistory (how its state was prepared) cannot have precisely defined values of its position and momentum simultaneously—and this is true in the same sense as a classical wave packet's inability to have simultaneous, precise values of its position and wave number (or frequency). By contrast, equation (1.13) is a fundamental property of the process of measurement; and the uncertainties in the object's position and momentum appear in it in different ways: the position uncertainty is an error in the measurement, the momentum uncertainty is a perturbation given to the object by the measuring process. Nevertheless, both equation (1.10) and equation (1.13) carry the name "Heisenberg uncertainty relation."

In fact, the physical roots of the two relations (1.10) and (1.13) are the same (see chapters II and III and also the beginning of chapter V): From the fact that the state of the *measuring device* has unavoidable uncertainties described by (1.10), it follows that a measurement of the mass's position inevitably perturbs its momentum in the manner of (1.13) (and, similarly, a measurement of the mass's momentum inevitably perturbs its position in a manner analogous to (1.13); see the Doppler speed meter, below). Conversely, the perturbation that inevitably accompanies any measurement prevents one from ever preparing a quantum object in a state with simultaneous, precise values of its position and momentum.