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Preface

Our aim in editing *Twistors in Mathematics and Physics* has been to collect together review articles which reflect the wide diversity of ideas and techniques which constitute modern twistor theory. Whilst the origins and much continuing work in twistor theory are in the area of fundamental physics, there is an ever-growing body of ‘twistor mathematics’ which has now taken on a life of its own. This is reflected by articles on representation theory and differential geometry, among other subjects.

The main objective in the ‘twistor programme’ for fundamental physics is a theory which unites Einstein’s general relativity and the world of quantum physics—a theory in which the rôle of complex holomorphic geometry is fundamental; Penrose’s article in this volume reviews its current status. Other contributors have covered the advances which have occurred since the major successes of Penrose’s *non-linear graviton* and Ward’s construction of the *Yang–Mills instantons*—the most notable of these is probably Penrose’s definition of *quasi-local mass* in general relativity, and topics such as the twistor description of vacuum space-times with symmetries and twistor particle theory are also covered.

Twistor mathematics is now a wide-ranging subject in itself, and articles in this volume cover differential geometry, integrable systems and several topics related to representation theory. The process by which Penrose originally encoded solutions of field equations on Minkowski space in terms of holomorphic functions on regions in twistor space, namely the *Penrose transform*, has been generalized to a complex homogeneous spaces, and so has applications in the theory of invariant operators (Verma modules) and the construction of unitary representations. The reader will find all these topics, and others, covered here.

Our hope is that this volume will be of use to workers in all areas of twistor theory and the many areas connected with it. We particularly hope that it will encourage the continued cross-fertilisation of ideas, particularly between pure mathematics and mathematical physics, which has always been one of the subject’s particular strengths.
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