Nuclear Superfluidity
Pairing in Finite Systems

Nuclear Superfluidity is the first modern text devoted exclusively to pair-correlations in nuclei. It begins by exploring pair-correlations in a variety of systems including superconductivity in metals at low temperatures and superfluidity in liquid $^4$He and in neutron stars. The book goes on to introduce basic theoretical methods, symmetry breaking and symmetry restoration in finite many-body systems. The last four chapters are devoted to introducing new results on the role of polarization effects in the structure of both normal and exotic nuclei. Central to this discussion is the fact that while bare nucleon–nucleon interactions are essential for the production of pair-correlations in nuclei, the coupling of pairs of nucleons to low-energy nuclear collective excitations also plays an important role.

This book will be essential reading for researchers and students in both experimental and theoretical nuclear physics, and related research fields such as metal clusters, fullerenes and quantum dots.

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Nuclear Superfluidity
Pairing in Finite Systems

DAVID M. BRINK and RICARDO A. BROGLIA
For my family – DMB
For Angela, Donatella, Gianandrea and Bettina – RAB
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Preface

The present monograph aims to provide an account of the basic results obtained in the exploration of the subject of nuclear superfluidity, placing special emphasis on recent developments coming out from ongoing research, in particular medium polarization and pairing in exotic nuclei.

The marked mass dependence of the abundance of nuclear species testifies to the fact that nucleons in nuclei move essentially independently of each other in an average potential produced by the effect of all the other nucleons. Special stability is ascribed to the closing of shells in correspondence with magic numbers. Adding nucleons to a closed shell system polarizes the shells, leading eventually to deformations. The best studied of these phase transitions are associated with deformations in: (a) three-dimensional space (violating angular momentum conservation, i.e. rotational invariance), (b) gauge space (violating particle number conservation, i.e. gauge invariance).

The phenomenon of spontaneous symmetry breaking in gauge space (i.e. the fact that the Hamiltonian describing a system is symmetric with respect to gauge transformations while the state is not) is intimately connected with nuclear superfluidity. This is the subject of Chapters 1, 2 and 3.

While potential energy always prefers special arrangements and thus leads to the spontaneous symmetry-breaking phenomena, fluctuations favour symmetry, leading to collective modes (intimately connected with symmetry restoration): pairing rotations and pairing vibrations, analogues in gauge space to rotations in three-dimensional space of the system as a whole, as well as to (multipole) surface vibrations respectively. This is the subject of Chapters 4, 5, and 6. Pairing also plays a central role in phenomena like exotic decay, alpha decay and fission where a nucleus divides into two smaller subsystems. Some aspects of this important subject are taken up in Chapter 7.

Associated with spontaneous symmetry breaking the system acquires emergent properties not contained in the Hamiltonian describing it, nor in the particles
composing it, in particular, an order parameter and a generalized rigidity. The order parameter measures the magnitude of the distortion: e.g. the quadrupole moment (which measures the number of aligned single-particle orbitals) in the case of deformed nuclei in normal space, the pairing gap (which measures the ‘binding energy’ of pairs of nucleons moving in time-reversal states and forming Cooper pairs, the building blocks of fermion superfluidity) in the case of gauge deformations. Generalized rigidity indicates the fact that pushing a deformed nucleus at one of the tips (through, e.g. a time-dependent Coulomb field induced by the passage of a heavy ion), the push is propagated over the whole system and one can set the nucleus into rotation. Two-particle transfer reactions provide the push to set a superfluid nucleus into rotation in gauge space, as was first found by Josephson in connection with superconductivity in condensed matter. This brings us to the subtitle of the present monograph: pairing in finite systems.

Although one can draw many analogies between the behaviour of infinite and of finite many-body systems (FMBS), there are also major differences. In particular, while in transition between the normal and superfluid phase taking place in infinite systems, all particles moving close to the Fermi surface play a similar role, in the case of FMBS very few orbitals control the phenomenon, providing also most of the stability of the new phase. This fact provides the possibility, among other things, of studying superfluidity in terms of individual orbitals, both in the case of the (standard) s-wave pairing as well as in the case of d-wave pairing. At variance to the case of infinite systems where pairing vibrations are hard to observe, FMBS provide the framework to study the spectrum of pairing vibrations both in superfluid as well as in normal nuclei, their interweaving and resulting anharmonicity phenomena which are at the basis of the condensation of Cooper pairs and thus of superfluidity. The ubiquitous role played by pairing vibrations (Chapter 5) in nuclear structure (pairing phase transition, dealignment, nuclear masses, etc.) is discussed in detail in Chapters 6 and 8.

A microscopic calculation of superfluidity in nuclei starts from a mean-field calculation which provides the single-particle levels of the bare nucleons. Bouncing inelastically off the nuclear surface, nucleons can excite a collective vibration at a given instant of time and reabsorb it at a later time. Through this process the bare nucleons become dressed. Physically, this means that what one measures (the experimental single-particle energies, closely related to the effective mass of a nucleon inside the nucleus) is not the bare mass but something else which includes the effect of the virtual processes mentioned above.

Collective surface vibrations excited by a nucleon at a given time may also be reabsorbed by another nucleon at a later time. Such a process turns out to be of importance in renormalizing the bare nucleon–nucleon pairing interaction arising from the exchange of mesons, and is the subject of Chapters 8, 9 and 10.

Because halo nuclei (i.e. nuclei where the excess of one type of nucleons forces the least bound particles into very extended orbits) are highly polarizable,
they provide a particularly testing ground and novel framework to assess the role polarization effects play in pairing correlations in nuclei. This is the subject of Chapter 11.

A number of important themes are not covered in any detail by the present monograph: in particular, two-neutron transfer reactions and proton–neutron pairing. Concerning the first subject an extensive literature is available, including review papers and chapters of books. Concerning the second subject, although much theoretical work has been published on $T = 0$ p-n pairing, the experimental evidence remains, to date, circumstantial at best. There is an extensive literature on $T = 1$ n-p pairing, which includes several detailed review papers.

It could be remarked that the present treatise also does not cover all the work of large shell model studies of pairing in nuclei. This is true. One has to remember, however, that nuclear field theory (NFT) used in connection with the Bloch–Horowitz (perturbation theory) formalism, or with the Dyson equation, leads also to an (essentially) exact diagonalization of pairing as well as of mean field. What is accomplished in one approach (shell model) using very large configuration spaces, is obtained in the second approach (NFT) by accurately dressing the elementary modes of nuclear excitation (single-particle motion and collective vibrations) and the vertex controlling their mutual interweaving.

We feel uncomfortable about not including a chapter on pairing at finite temperature and on pairing in other FMBS, such as atomic clusters or quantum dots. However, this feeling is partially mitigated by the fact that the first subject is standard in any book on condensed matter physics, and the results can essentially be taken over to the case of atomic nuclei (see Brink (1994)). The second subject can be found among the topics covered by recently published books, written also by practitioners of nuclear physics (see e.g. Lipparini (2003), Broglia et al. (2004)).

Over the years we have received much illumination on the subject dealt with in the present monograph from a number of people. DMB would like to acknowledge stimulating discussions with physicists at the European Center for Theoretical Studies in Nuclear Physics and Related Areas (ECT*) in Trento, Italy and especially with Ben Mottelson, Renzo Leonardi, Sandro Stringari and Aage Winther. He also achieved a deeper understanding of nuclear structure as a result of discussions with George Bertsch in Seattle, Brian Buck in Oxford and members of the theory groups in many laboratories including Catania, Kyushu, Kyoto, Lund, MSU, Orsay, Milan. Padova, Pisa, Saclay, Sapporo, Surrey and Tokyo. RAB wishes to acknowledge the debt he owes to George Bertsch, Ole Hansen and Claus Riedel. The interaction and collaboration with Aage Bohr and Ben Mottelson, started in 1965 and continued through the years, has been a major source of inspiration. At that time, when he arrived at the Niels Bohr Institute of Copenhagen, the main foundations of nuclear superfluidity were already solidly established. Nonetheless pairing was again a major subject of research. In fact,
the exploration of the collective degrees of freedom was under way, triggered also, as has always happened in connection with major developments in nuclear physics, by the availability of new experimental data. This time, two-neutron transfer data came from the group of Ole Hansen and Ove Nathan. RAB was fortunate to participate closely in this exploration with Daniel Bes (teacher, collaborator and friend from whom he learned so much through the years) as well as with Ole Hansen. RAB wishes also to thank Ben Bayman, Gerry Brown, Bob Schrieffer, Peter Schuck, Vladimir Zelevinsky and Witek Nazarewicz for much illumination. His debt to Francisco Barranco is very large, and increases as each day goes by, as a consequence of our common striving to understand pairing in a variety of FMBS setups. Thanks are also due to Pier Francesco Bortignon and to Gianiuca Colò for advice and collaboration on particular subjects discussed in this book, and to Enrico Vigezzi for useful criticism. RAB also acknowledges a most fruitful, lifelong collaboration with Aage Winther. During the many versions of the book Francesco Marini has taken care of all the technical aspects of the manuscript. His help has been invaluable to us. Last but not least, thanks are also due to all the students of the fourth year course of ‘Nuclear Structure Theory’, which RAB has delivered since 1986 at the Department of Physics of the University of Milan, building on the wealth of experience gathered in previous years at the Niels Bohr Institute and at Stony Brook (State University of New York) in the presentation of particular subjects covered in the present book.

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Milan, May 2004