Symplectic techniques in physics

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Preface

This book is based on lectures on symplectic geometry that we have given over the past few years at MIT, Harvard University, and the University of Tel Aviv. Symplectic geometry – especially under its old name, "the theory of canonical transformations" – is a venerable topic in mathematical physics. It has recently experienced a great rejuvenation and is currently an active area of research. Our purpose in this book is twofold: to provide an introduction to the subject and to present the central results of the subject from a modern point of view. There is, accordingly, a difference in style and tone between Chapter I and the rest of the book.

Chapter I is directed at the general reader interested in mathematics or physics. The mathematical prerequisites are quite modest. A knowledge of calculus and the rudiments of linear algebra suffice for most of the chapter. Although some mention of more sophisticated topics is made from time to time, the passages containing such material are inessential and can be glossed over without loss of continuity. The mix of mathematics and physics is homogeneous. We have tried the genetic approach. Using the various theories of light as our paradigm, we have attempted to explain the development of the mathematical and physical ideas involved in symplectic geometry. Despite its elementary character, most of the key ideas of the book are to be found, albeit in embryonic form, in Chapter I.

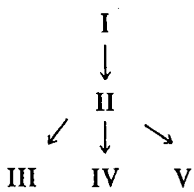
Chapter II presents the key mathematical results of the book and describes several of the important physical applications. The style is more formal and the mathematical demands are greater than those of Chapter I. The reader is expected to have some familiarity with the basics of differential geometry and a degree of mathematical sophistication. The mix between the mathematics and the physics is less homogeneous here. We have indicated those sections that can be skipped by a reader more interested in the physical applications than in the mathematical proofs.

Chapters III, IV, and V are practically independent of one another. Chapter III is mainly concerned with the use of symplectic geometry as a tool for formulating (possible) laws of physics. The main theme is the principle of general covariance due to Einstein, Infeld, and Hoffman in a form recently explained and recast by Souriau. Section 41 contains one of the main mathematical results of the book.

Chapter IV is devoted to the use of the interaction between group theory and symplectic geometry for the solution of various mechanical systems. We have tried to explain how a broad variety of integrable mechanical systems can be explained using overt or hidden symmetries. We have, in the main, restricted attention to finite-dimensional systems, although we do discuss some evolution equations of the Korteweg–de Vries type. We have not included the very interesting work involving the Kac–Moody algebras.

Chapter V serves two purposes. The first part presents some of the key results in the theory of Lie algebras that are needed and used throughout the book. Although these results, and our treatment of them, are completely standard, we provide them here for the convenience of the reader. We also illustrate these results in computations of physical interest. The second part is a discussion of the deformation theory of symplectic group actions. The results are all due to Don Coppersmith, and much of the last few sections is taken directly from his unpublished Harvard thesis. We thank him for his kind permission to reproduce these results here. Much work remains to be done on this highly interesting and important topic.

The diagram of logical dependence is thus roughly as follows:



We should pay tribute to some of the people who originated the ideas and techniques that are the subject matter of a large part of the last three chapters of this book. The coadjoint representation and coadjoint orbits appear for the first time in Kirillov's work on classification of the irreducible representations of the nilpotent Lie groups. The "orbit method" in its symplectic setting was extensively developed by Kostant in the late sixties and early seventies. The moment mapping also plays an important role in Kostant's work at this time, and its physical implications were emphasized by Souriau in his beautiful monograph (1970). The idea of reduction occurs for the first time in the short paper of Marsden and Weinstein (1974) and was subsequently used by them and their students and collaborators for a wide range of applications in mathematics and physics. The recent work, described in Chapter IV, on complete integrability owes much to the work of Arnold and Moser in the late sixties and early seventies. In particular, it was from their work that the idea emerged that the complete integrability of most of the classical examples of completely integrable systems could be accounted for by "hidden symmetries." The material in the first part of Chapter IV, on Lagrangian fibrations and the geometric structure of completely integrable systems, is largely adapted from Duistermaat's beautiful paper (1980); and the material in Section 33, on stationary phase, is based partly on work of Duistermaat and Heckman and partly on work (some of it unpublished) of Bott and Berline and Vergne.

We wish to thank Professor G. Emch for a careful reading and many constructive comments on Chapter I. We also wish to thank Bert Kostant for long and fruitful collaboration on many of the topics covered in this book and for his permission to reproduce many joint results, and Jerry Marsden and Alan Weinstein for intense but friendly competition. We would also like to express our appreciation to many of our friends who provided illuminating conversations, unexpected insights, and moral support during the three or four years that this book was in gestation (as course notes for graduate courses at Harvard and MIT). Among them are Michael Atiyah, Bob Blattner, Raoul Bott, David Kahzdan, Richard Melrose, John Rawnsley, J. M. Souriau, Joe Wolf, and Alejandro Uribe.

V. G.
S. S.

December 1982