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This book provides a comprehensive introduction to the operation and theory of electrical circuits for students in engineering and the physical sciences. The methods of circuit analysis are clearly explained and illustrated with the aid of numerous worked examples. Applications of the theory relevant to the fields of electronics, telecommunication and power systems are treated throughout.

The text is suitable for first- and second-year undergraduate courses in UK universities and polytechnics. It is also appropriate for a first course in circuits for undergraduate Electrical Engineering majors in US colleges and universities.

Selected topics are treated at a more advanced level to cater for students specializing in electrical subjects in the final years of their degree course.

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# Electrical circuits

*AN INTRODUCTION*

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The program listings in Appendix C are available on IBM-PC compatible diskette from the authors. For diskette prices and ordering procedure write, enclosing a self-addressed envelope, to Dr K.C.A. Smith, 50 Selwyn Road, Cambridge, CB3 9EB.

## Preface

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This book provides an introduction to electrical circuits that will serve as a foundation for courses in electronics, communications and power systems at first degree level. The first three chapters will be found particularly suitable as prerequisite reading for the companion volume in this series; *Analogue and digital electronics for engineers* by H. Ahmed and P.J. Spreadbury. Engineering and science students not intending to specialise in electrical subjects will find in this book most of the circuit theory required for a first degree.

The level of presentation presupposes that students will have encountered the basic ideas of electromagnetism and electrical circuits, including the laws of Faraday, Ohm and Kirchhoff. These ideas are reviewed in chapter 1. Mathematical skills are assumed to extend to the solution of first-order differential equations, and to the elements of complex algebra. Courses in mathematics taken concurrently with those in electrical subjects during the earlier part of a degree course would be expected to fill in progressively the additional mathematical background required; the subject matter has been arranged with this in mind. Sections which may give rise to mathematical difficulties on a first reading, or which may be too specialised for the general student's requirements, are indicated by an obelus (†).

A traditional approach to the development of electrical circuit theory is adopted: the concept of linearity, and the circuit theorems and analytical techniques which stem from this concept, are all presented in chapter 2 within the context of d.c. circuits. The methods and techniques of linear circuit analysis thus established are then extended to a.c. circuits in chapter 3. Familiarity with the basic material contained in chapters 1–2, together with sections 3.1–3.8 and 4.1–4.7 of chapters 3 and 4, will allow the remainder of the book to be read on a selective basis appropriate for the particular courses being followed by the student.

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Chapters 4 and 5 deal mainly with various aspects of power transmission in electrical circuits; in chapter 6 methods for the general transient and steady-state analysis of circuits are described, with emphasis on Laplace transform techniques, and chapter 7 deals with the analysis of circuits incorporating non-linear elements. Chapter 8 covers the theory of two-port networks, including the modelling of non-linear devices such as the transistor; later sections of this chapter will be of interest primarily to electrical engineering students.

A suite of simple computer programs, written in BASIC, is included as an appendix, which is designed to assist the student in working through the numerous illustrative examples and problems contained in the text.

We are indebted to Professor K.F. Sander for major contributions to chapter 8; to Mr J. Barron for supplying the basis for the formal proofs of the linear network theorems contained in Appendix B; and to Dr D.E. Roberts, Dr D.M. Holburn and Mr K.L. Chau for advice on programming, and for checking and testing the programs listed in Appendix C. We wish also to acknowledge the skill and patience of Mrs Pat Silk in the preparation and typing of the greater part of the manuscript. Questions from the examination papers of several universities have been included; their permission is gratefully acknowledged.