1

Basic concepts, units, and laws of circuit theory

1.1 Properties of the electrical circuit

An electrical circuit comprises an arrangement of elements for the conversion, transmission and storage of energy. Energy enters a circuit via one or more *sources* and leaves via one or more *sinks*. In the sources energy is converted from mechanical, thermal, chemical or electromagnetic form into electrical form; in the sinks the reverse process takes place. Sources and sinks are linked by elements capable of transmitting and storing electrical energy. The familiar battery-operated flashlamp serves as a reminder of the energy flow processes in a circuit. In this device, energy is converted from chemical to electrical form in the battery and transmitted along wires to the lamp where most of the energy is converted into heat. A small but useful portion is emitted in the form of electromagnetic radiation in the visible part of the spectrum.

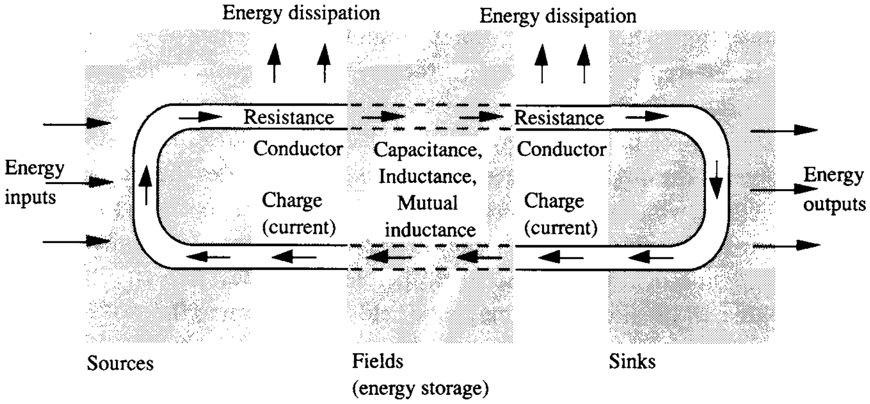
In an electrical circuit energy is conveyed through the agency of electrical *charge* and through the medium of *electric* and *magnetic fields*. An essential feature of any circuit, therefore, is the provision of conducting paths for the conveyance of charge. As indicated in fig. 1.1, sources and sinks are operative only when charge flows through them. The *rate* at which charge flows is referred to as the *current*; the greater the current the greater the energy transmitted between sources and sinks.

Charge is set in motion by the action of the electric field established throughout the circuit by the sources. This field provides the *electromotive force* (e.m.f.) which drives charge round the conducting paths in the circuit. Accompanying this flow of charge is the establishment of a magnetic field. Transmission of electrical energy is, therefore, manifest in a circuit by the presence of both electric and magnetic fields in addition to the movement of charge. The establishment of a field in a circuit is accompanied by an expenditure of energy, and this energy is stored within the region of space occupied by the field. On subsequent decay of the field, energy is released to the circuit and is eventually absorbed by the sinks. Thus energy can be both stored and conveyed through the medium of a field. However, for the latter process to occur the field must vary with time. Referring again to fig. 1.1, if the sources produce a constant e.m.f., the resulting currents and fields will all be constant and, in this case, there must be a continuous conducting path between sources and sinks along which charge can flow (indicated by the dashed lines in the figure). If, on the other hand, the sources produce a time-varying e.m.f., currents and fields will be time-varying and the conducting path need not be continuous.

This distinction leads to two of the major classes of circuits dealt with in this book: (1) *direct current* (d.c.) circuits in which fields are static and currents are constant and unidirectional: (2) *alternating current* (a.c.) circuits in which the directions of currents and fields alternate in a regular, periodic fashion.

It will be apparent from the above discussion that the electrical behaviour of a circuit is characterized by the strength and distribution of the currents and fields which arise when it is connected to an electrical energy source. The electrical characteristics of a circuit may, therefore, be described generally by means of three elemental properties: *resistance*, *capacitance* and *inductance* (including *mutual inductance*). Resistance is a property associated with the current-carrying paths in a circuit. Capacitance and inductance are properties associated respectively with the parts of a circuit in which electric and magnetic fields arise. Capacitive and inductive elements are often referred to as *storage elements* because of the energy storage properties of a field. A knowledge of the three elemental properties, for a particular circuit, allows us to specify, at least in principle,

Fig. 1.1. Elements of the electrical circuit.



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the magnitudes and directions of the currents which will flow as a result of the application of a given distribution of e.m.f.

Circuits containing only the three basic elements, resistance, capacitance and inductance, are termed *passive* circuits. (*Active* circuits contain also devices such as transistors which, unlike passive elements, are capable of energy amplification.)

If the elemental properties of a passive circuit depend only on its geometry and the materials of which it is made, the circuit is described as being *linear*. If, however, these properties depend additionally on the current or e.m.f. existing in the circuit at any instant, the circuit is described as being *non-linear*. Special techniques are required for the analysis of non-linear circuits; these are dealt with in chapter 7.

Finally, it should be noted that as an inevitable consequence of the movement of charge along a conductor, electrical energy is converted into heat (we are here excluding the superconducting type of circuit), thus the circuit itself acts inherently as an energy sink.

1.2 The lumped circuit model

Practical circuits consist of interconnected assemblies of components: *resistors*, *capacitors* and *inductors*, each designed to exhibit one elemental property to the exclusion of the others.* It is, however, impossible to manufacture a component exhibiting a single property in pure form. Furthermore, all of the interconnections between components will themselves possess each of the three elemental properties to some degree. Consequently, the way in which the elemental properties are distributed in a circuit is often ill defined and, in order to render the circuit amenable to analysis, it is usually necessary to make certain simplifying assumptions and approximations. The most basic of these consists in treating the circuit as if it were composed of pure, discrete elements connected together by conductors possessing no significant properties in themselves. This approach results in the so-called *lumped circuit model*.

Consider again the flashlamp the component parts of which are depicted in fig. 1.2(*a*). Each part, comprising battery, connecting wires, and lamp, possesses resistance which is distributed in some fashion round the closed path forming the circuit. The circuit also contains distributed capacitance and inductance, but only a cursory knowledge of the principles upon which this device operates tells us that these properties can be safely neglected. The circuit model, therefore, need include only resistance as shown in fig. 1.2(*b*). In this model the battery is represented by an energy source together

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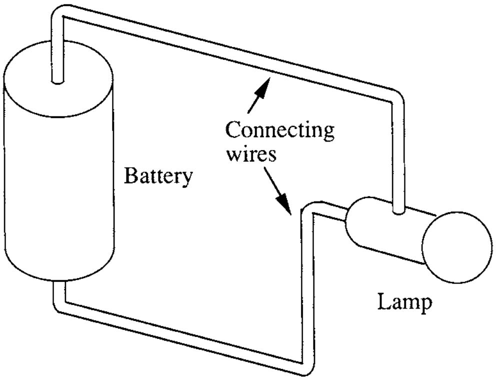
* Note that the circuit *component* is distinguished from the circuit *property* by the terminators *-or* and *-ance* respectively.

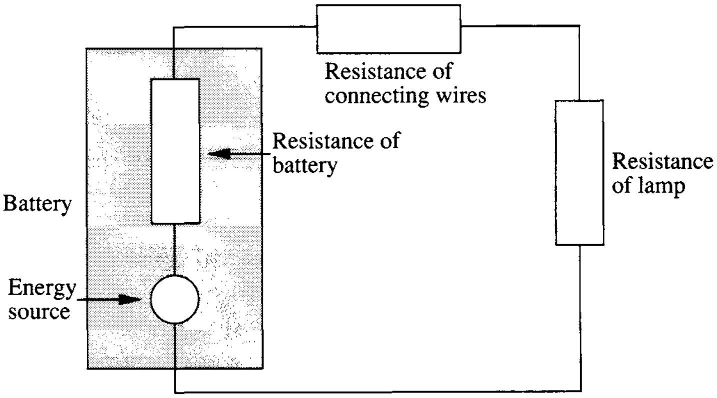
with a concentrated or lumped resistance which accounts for all distributed resistance within the battery. The distributed resistance of the connecting wires and the resistance of the lamp are similarly represented by separate lumped resistances. These lumped elements are joined by conductors which are assumed to be *perfect*, that is, by conductors having zero resistance.

The flashlamp exemplifies the simplest possible type of modelling in which there is a close correspondence between the component parts of the real circuit and the lumped elements of the model. Most of the circuits in this book fall into this category. It should be mentioned, however, that the process of devising suitable models for the type of circuit encountered in, for example, telecommunications systems which operate at high frequencies, is

Fig. 1.2. Circuit modelling.



(*a*) Flashlamp: physical components



(*b*) Lumped circuit model

often extremely difficult. Each component and interconnection may have to be represented by a combination of elemental properties and the designer may eventually have to select for analysis one among perhaps several possible lumped models, testing each against past experience or by means of actual circuit measurement.

The lumped circuit modelling technique is directly applicable only when the dimensions of the circuit under consideration are small compared with the wavelength corresponding to the frequency of the source excitation. Circuits not falling into this category, such as high-frequency transmission lines (characterized also by a continuous distribution of elemental properties), require special methods of analysis. The lumped modelling technique provides only a starting point for the development of the theory applicable to such circuits.

1.3 Charge and current

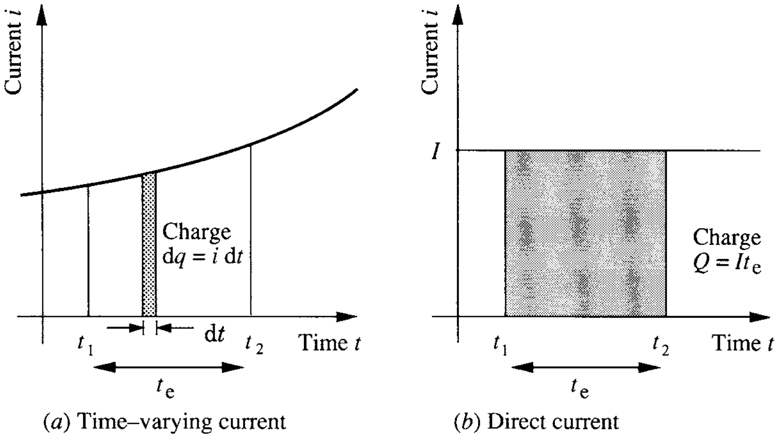
We have stated previously that current in a conductor is equal to the rate of flow of charge. If i is the instantaneous current, and a small quantity of charge $\mathrm{d}q$ flows in time $\mathrm{d}t$, then

$$i=\frac{\mathrm{d}q}{\mathrm{d}t} \tag{1.1}$$

The instantaneous current will in general vary with time (fig. 1.3(*a*)). We can calculate the total amount of charge q which flows during a time interval $t_1 \leq t \leq t_2$ by integrating (1.1).

$$\int_0^q \mathrm{d}q = q = \int_{t_1}^{t_2} i\,\mathrm{d}t$$

Fig. 1.3. Relationship between charge and current.



(*a*) Time–varying current (*b*) Direct current

The graphical interpretation of this integral is also shown in the figure.

If the time interval commences at the origin, $t_1=0$ and $t_2=t$, and the above integral becomes

$$q=\int_0^t i\,\mathrm{d}t \tag{1.2}$$

For a direct current of magnitude I (fig. 1.3(*b*)), the charge Q which flows in a time interval $t_1 \le t \le t_2$ is

$$Q=I\int_{t_1}^{t_2} \mathrm{d}t=I(t_2-t_1)=It_e \tag{1.3}$$

where $t_e=t_2-t_1$ is the elapsed time.

The units of charge and current are respectively the *coulomb* and the *ampere**.

Although the concept of charge is basic to our understanding of the way in which energy flows in an electrical circuit, the ampere is chosen as the fundamental electrical unit in the SI system rather than the coulomb. The reason for this is that it is easier to detect and measure charge in motion than at rest. The former gives rise to a magnetic field which in turn can be detected by utilizing forces resulting from interaction with other magnetic fields. (See definition of the ampere, appendix A.) This is discussed more fully in reference 6.

So far we have not considered the physical nature and origin of electrical charge and indeed for the purposes of the theory contained in this book it is unnecessary to do so. The established physical picture (according to the Rutherford–Bohr model of the atom) conceives of charge as being carried by atomic particles each bearing a discrete amount of charge. But, even in the smallest currents encountered in practice, the number of charge carriers involved in the transport process is very great and the discrete nature of the flow is not normally detectable. A concept of current as consisting of a smooth fluid-like flow is, therefore, adequate for nearly all practical purposes.

Detailed experimental observation reveals that charge carriers can possess two kinds of charge: positive and negative. Under the action of the same electric field, charges of different kind move in opposite directions. A given amount of positive charge moving along a conductor in one direction is indistinguishable, so far as any observable external effect is concerned,

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* Appendix A contains information on the International System of Units (SI), and an explanation of the symbols, abbreviations and nomenclature used throughout the text.

from the same quantity of negative charge moving in the opposite direction. By an internationally accepted convention, the direction of current flow is chosen to be that of the direction of motion of positive charge.

In metallic conductors the carriers are electrons which possess negative charge and move in a direction opposite to that of the defined direction of positive current. In semiconductors and electrolytes charge of both kinds exist (carried by electrons and positively charged holes, or ions) and the current is the net result of the movement of positive and negative charge in opposite directions. It must be emphasized, however, that in circuit analysis we are not normally concerned with the nature of charge flow from this microscopic point of view, and we, therefore, talk freely about positive charge moving in metallic conductors even though the charge is in reality carried by electrons.

The reference direction of positive charge flow or current in part of a circuit is indicated diagrammatically by means of an arrow placed on or alongside the conducting path in question. The direction of current between two points A and B in a circuit may also be indicated without ambiguity by means of a double subscript notation. Thus we may write I_{AB}, which is understood to mean a current of magnitude I amperes flowing in a conventional positive sense from A to B. A positive current flowing from B to A would be written I_{BA}; it follows therefore that $I_{BA} = -I_{AB}$. This notation will be valuable in our development of techniques for circuit analysis.

1.4 Potential difference, energy and power

Consider a current of constant magnitude flowing through a section of a metallic conductor AB as shown in fig. 1.4. It is observed experimentally that the passage of current through a conductor is accompanied by the release of energy in the form of heat. It follows that the potential energy of the charge entering the conductor at A must be greater than that of the charge leaving at B since the evolution of heat implies that work is done by the charge during its passage from A to B. A potential energy difference therefore exists between the points A and B. The SI unit of potential energy difference (or simply *potential difference* (p.d.)) is the *volt*, and we say that a voltage exists between A and B. The end of the conductor at the higher potential is indicated conventionally by a (+) sign and that at the lower potential by a (−) sign. A double subscript notation may also be used with advantage to express the magnitude and direction (or *polarity*) of a voltage existing between two points A and B in a circuit. We may write V_{AB} which is understood to mean a p.d. of constant magnitude V volts, A being *positive* with respect to B.

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Referring again to fig. 1.4, if a potential difference of one volt exists between A and B, then one coulomb of charge passing between A and B will produce one *joule* of heat energy. Generalizing this statement; if between two points on a metallic conductor there exists a constant potential difference of V volts, and a total of Q coulombs of charge passes between them, the heat output J, in joules, is given by

$$J = VQ \tag{1.4}$$

In terms of current this becomes, using (1.3),

$$J = VIt_e \tag{1.5}$$

where I is a current of constant magnitude and t_e is the elapsed time.

From (1.5) the power P (*watts*) is given by the energy dissipated in the conductor per unit time, that is,

$$P = \frac{J}{t_e} = VI \tag{1.6}$$

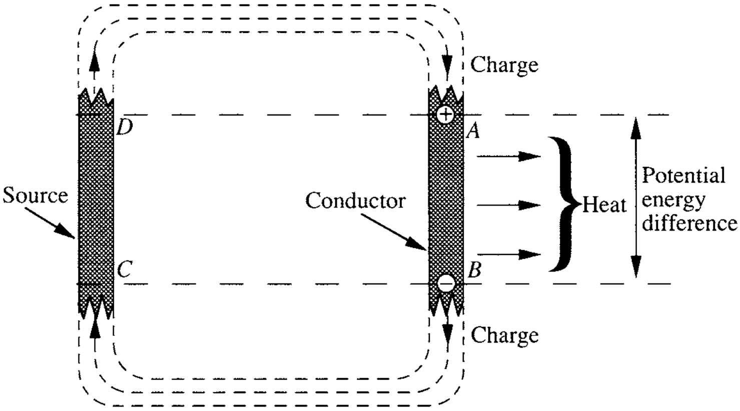
For the general case where both voltage and current vary with time, the energy is, at any instant of time t,

$$J = \int_0^t vi\,\mathrm{d}t \tag{1.7}$$

and the instantaneous power is

$$p = vi \tag{1.8}$$

Fig. 1.4. Potential difference. The potential energy lost by the charge as it flows from A to B is recovered as it flows from C to D.



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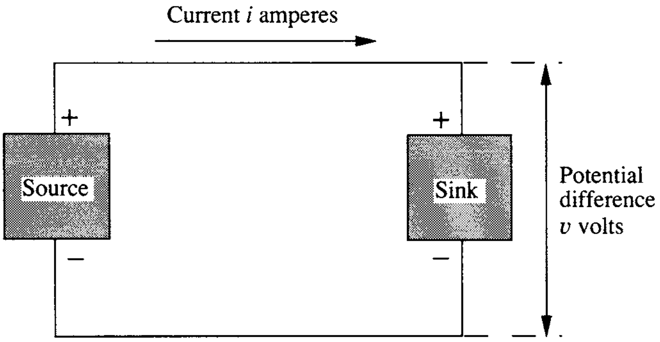
It follows from the principle of conservation of energy that if heat is to be dissipated continuously in the section AB, the potential energy lost by the charge in passing from A to B must be made up by a corresponding gain in potential energy elsewhere. In the system shown in fig. 1.4 this occurs as the charge passes through a section CD of a source. The magnitude of the potential difference across CD is, of course, identical to that across AB. For obvious reasons, the latter is often referred to as a voltage *drop* (or *fall*), and the former as a voltage *rise*.

Although the relationships shown in (1.7) and (1.8) have been established by considering the particular case of a metallic conductor, they apply generally to any sink in which electrical energy is converted to some other form. Consider the circuit shown in fig. 1.5. Source and sink are joined by perfect conductors so that the p.d. across both is the same and equal to v. The polarity of this voltage is, according to our convention, indicated by the (+) and (−) signs. Unit positive charge, on passing through the sink from the positive terminal to the negative terminal, loses a total potential energy of v volts, and on passing through the source from the negative terminal to the positive terminal this potential energy is completely regained. The instantaneous power flow from source to sink is given by the product vi.

For circuits containing a multiplicity of elements the magnitude and direction of power flow at any particular element or in any part of the circuit may be ascertained by considering the associated directions of the voltage and current at the terminals concerned. In fig. 1.6, P is any element or part of a circuit at which the instantaneous values of voltage and current are defined. If the product vi is positive, power is being delivered to P while if the product is negative, P is supplying power to the external circuit. In terms of our double subscript notation, power is delivered to P if the product $v_{AB}i_{AB}$ is positive. (Note carefully the order of the subscripts in this product.)

Fig. 1.5. Energy flow between source and sink. Instantaneous power $p = vi$ watts.



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If we apply this convention to fig. 1.5, we see that the direction of power flow is in accordance with the meaning which has so far been attached to the terms source and sink. That this is not always the case may be seen by comparing the two circuits shown in fig. 1.7.

In these circuits we assume that the voltage of source P is greater than that of Q and that, as a consequence, there will be a net e.m.f. acting in such a direction as to cause current to flow clockwise round the circuit as shown. Examination of the direction of this current in relation to the polarities of the two sources connected as in fig. 1.7(*a*), confirms that both sources are delivering power to the sink. However, if the polarity of Q is reversed, as in fig. 1.7(*b*), current enters its positive terminal, the product vi is positive, and we conclude that energy is being delivered to Q. In other words, what has hitherto been called a source is now effectively acting as a sink.

Many practical sources exhibit this property of reversibility. One common example is the battery which can be recharged by connecting it to

Fig. 1.6. Power in a circuit element: P receives power if product vi is positive; P delivers power if product vi is negative.

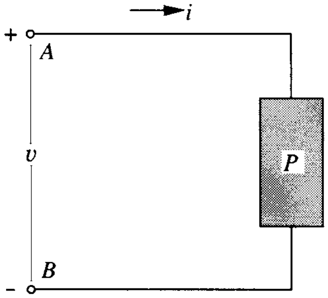
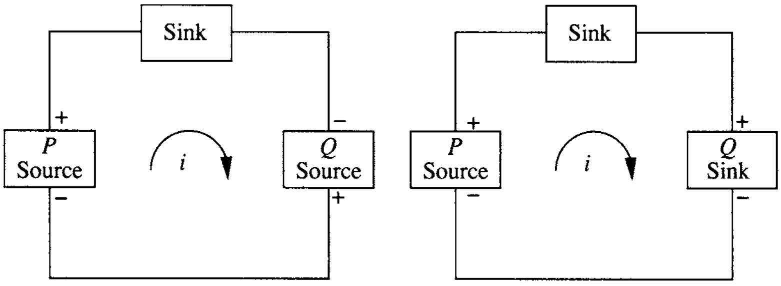


Fig. 1.7. (*a*) Sources P and Q deliver energy to sink. (*b*) Polarity of Q reversed: Q receives energy from P.



(*a*) (*b*)