INTRODUCTION TO PLASMA PHYSICS

With Space and Laboratory Applications

The emphasis of this text is on basic plasma theory, with applications to both space and laboratory plasmas. All mathematical concepts beyond those normally covered in an advanced calculus course are fully explained.

Topics covered include single particle motions, kinetic theory, magnetohydrodynamics, small amplitude waves in both cold and hot plasmas, non-linear phenomena and collisional effects. Applications include planetary magnetospheres and radiation belts, the confinement and stability of plasmas in fusion devices, the propagation of discontinuities and shock waves in the solar wind, and the analysis of various types of plasma waves and instabilities that can occur in planetary magnetospheres and laboratory plasma devices. This book is structured as a text for a one- or two-semester introductory course in plasma physics at the advanced undergraduate or first-year graduate level. It can also serve as a resource book on the basic principles of plasma physics.

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Preface

This textbook is intended for a full year introductory course in plasma physics at the senior undergraduate or first-year graduate level. It is based on lecture notes from courses taught by the authors for more than three decades in the Department of Physics and Astronomy at the University of Iowa and the Department of Applied Physics at Columbia University. During these years, plasma physics has grown increasingly interdisciplinary, and there is a growing realization that diverse applications in laboratory, space, and astrophysical plasmas can be viewed from a common perspective. Since the students who take a course in plasma physics often have a wide range of interests, typically involving some combination of laboratory, space, and astrophysical plasmas, a special effort has been made to discuss applications from these areas of research. The emphasis of the book is on physical principles, less so on mathematical sophistication. An effort has been made to show all relevant steps in the derivations, and to match the level of presentation to the knowledge of students at the advanced undergraduate and early graduate level. The main requirements for students taking this course are that they have taken an advanced undergraduate course in electricity and magnetism and that they are knowledgeable about using the basic principles of vector calculus, i.e., gradient, divergence and curl, and the various identities involving these vector operators. Although extensive use is made of complex variables, no special background is required in this subject beyond what is covered in an advanced calculus course. Relatively advanced mathematical concepts that are not typically covered in an undergraduate sequence, such as Fourier transforms, Laplace transforms, the Cauchy integral theorem, and the residue theorem, are discussed in sufficient detail that no additional preparation is required. Although this approach has undoubtedly added to the length of the book, we believe that the material covered provides an effective and self-contained textbook for teaching plasma physics. MKS units are used throughout.
Preface

For the preparation of this text we would especially like to thank Kathy Kurth who did the typing and steadfastly stuck with us through the many revisions and additions that occurred over the years. We would also like to thank Joyce Chrisinger and Ann Persoon for their outstanding work preparing the illustrations and proofreading, and Dr. C.-S. Ng and Dr. Z. Ma for checking the accuracy of the equations. Don Gurnett would like to acknowledge the salary support provided by the Carver Foundation during the preparation of this manuscript, and Amitava Bhattacharjee would like to acknowledge the generous support of the Faculty Scholar Program at the University of Iowa and the Peter Paul Chair at the University of New Hampshire.