Addressing the need for new models for the analysis of social network data, Philippa Pattison presents a unified approach to the algebraic analysis of both complete and local networks. The rationale for an algebraic approach to describing structure in social networks is outlined, and algebras representing different types of networks are introduced. Procedures for comparing algebraic representations are described, and a method of analysing the representations into simpler components is introduced. This analytic method, factorisation, yields an efficient analysis of both complete and local social networks.

The first two chapters describe the algebraic representations of the types of networks, and the third chapter covers the ways in which representations of different networks can be compared. A general procedure for analysing the algebraic representations is then introduced, and a number of applications of the approach are presented in the final chapters.

The book should be of interest to all researchers interested in using social network methods.
Structural analysis in the social sciences 7

*Algebraic models for social networks*
Structural analysis in the social sciences

Mark Granovetter, editor

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The series Structural Analysis in the Social Sciences presents approaches that explain social behavior and institutions by reference to relations among such concrete social entities as persons and organizations. This contrasts with at least four other popular strategies: (a) reductionist attempts to explain by a focus on individuals alone; (b) explanations stressing the causal primacy of such abstract concepts as ideas, values, mental harmonies and cognitive maps (thus, "structurallism" or the Continent should be distinguished from structural analysis in the present sense); (c) technological and material determinism; (d) explanations using "variables" as the main analytic concepts (as in the "structural equation" models that dominated much of the sociology of the 1970s), where structure is that connecting variables rather than actual social entities.

The social network approach is an important example of the strategy of structural analysis; the series also draws on social science theory and research that is not framed explicitly in network terms, but stresses the importance of relations rather than the atomisation of reductionism or the determinism of ideas, technology or material conditions. Though the structural perspective has become extremely popular and influential in all the social sciences, it does not have a coherent identity, and no series yet pulls together such work under a single rubric. By bringing the achievements of structurally oriented scholars to a wider public, the Structural Analysis series hopes to encourage the use of this very fruitful approach.

Mark Granovetter
Algebraic models for social networks

Philippa Pattison

University of Melbourne
To my parents
Contents

List of figures and tables  page xi
Preface xix

1 Algebraic representations for complete social networks  1
Complete network data  5
Sources of network data  13
  The boundary of a network  13
  Relational content  14
  Network measurement  17
  Reliability and validity of network data  18
Structure in social networks  20
  Directed graphs  21
  Some analyses for social network data  22
Properties of a structural representation  32
An algebra for complete social networks  36
  Compound relations and network paths  37
  Comparing paths in networks and the Axiom of Quality  42
  The partially ordered semigroup of a network  44
  An algorithm for semigroup construction  49
Summary  54

2 Algebraic representations for local social networks  56
Types of local networks  58
  Representing local networks  61
An algebra for local social networks  62
  Paths in local networks  63
  Comparing paths in local networks  64
  The local role algebra of a local network  67
  An algorithm for constructing a local role algebra  68
  The local role algebra of a subset in a local network  70
## Contents

Role algebras 73
  Relations among role algebras: The nesting relation 75
  Presentation of role algebras 78
  Local role algebras and role-sets 79
Partial networks and partial role algebras 81
  The nesting relation for partial role algebras 86
  Analysis of local networks 86
Partially ordered semigroups and role algebras:
  A summary 88

3 Comparing algebraic representations 90
  Isomorphisms of network semigroups 91
  Some networks with isomorphic semigroups 93
Comparing networks: Isotone homomorphisms 96
  The π-relation of an isotone homomorphism 99
  Partial orderings among homomorphisms and π-relations 103
Lattices of semigroups and π-relations 104
  The joint homomorphism of two semigroups 110
  The common structure semigroup 113
  Lattices of semigroups: A summary 114
Local networks with isomorphic local role algebras 116
Comparing local role algebras: The nesting relation 119
Other classes of networks with identical algebras 123
  Trees 124
  Idempotent relations 128
  Monogenic semigroups 129
  Handbooks of small networks 133
  Summary 134

4 Decompositions of network algebras 135
Decompositions of finite semigroups 137
Direct representations 138
  Existence of direct representations 141
Subdirect representations 146
  Existence of subdirect representations 149
Factorisation 152
  Uniqueness of factorisations 155
  An algorithm for factorisation 156
  Using factorisation to analyse network semigroups 160
  The reduction diagram 161
Co-ordination of a partially ordered semigroup 162
Relationships between factors 163
Factorisation of finite abstract semigroups 165
Contents

A decomposition procedure for role algebras 166
Summary 171

5 An analysis for complete and local networks 172
An analysis for complete networks 173
Relational conditions of semigroup homomorphisms 173
Generalisations of structural equivalence 188
The correspondence definition 190
Searching for minimal derived set associations 199
Analysing entire networks 199
An example: Relational structure in a self-analytic group 201
Local networks 206
Derived local networks 207
A correspondence definition for local role algebras 208
Some applications 212
Local roles in the Breiger–Ennis blockmodel 212
A General Social Survey network 216
The snowball network L 220
Local role algebras for two-block two-generator models 222
Summary 223

6 Time-dependent social networks 224
A language for change 225
Some relational conditions for smooth change 226
An analysis of time-dependent blockmodels 228
The development of relational structure 230
A local role analysis of time-dependent blockmodels 234

7 Algebras for valued networks 238
The semigroup of a valued network 238
Binary network semigroups from valued networks 243
Local role algebras in valued local networks 247
Using valued network algebras 250

8 Issues in network analysis 251
Describing social context: Positions and roles 251
Positions and roles 252
The structure and content of relations 253
Some models for relational structure 256
Strong and weak ties 256
The balance model 258
The complete clustering model 258
The transitivity model 259
Contents

Other triad-based models 259
The First and Last Letter laws 260
Permutation models for kinship structures 260
Some other models 261
Describing common structure 261
Common relational forms in two self-analytic groups 262
Common relational forms in two community elites 266
Social structure 270
Analysing large networks 271

References 273
Appendix A Some basic mathematical terms 289
Appendix B Proofs of theorems 292
Author index 303
Subject index 307
Figures and tables

Figures

1.1 A directed graph representation of a friendship network among four members of a work group  page 5
1.2 Representations for symmetric network relations  7
1.3 Representations for a valued network relation  8
1.4 A multiple network W  9
1.5 Structural, automorphic and regular equivalence  26
1.6 The compound relation fH  38
1.7 Some compound relations for the network W  40
1.8 Hasse diagram for the partial order of S(W)  47
1.9 Hasse diagram for the partial order of S(N)  51
1.10 The Cayley graph of the semigroup S(N)  52
2.1 Some partial networks  57
2.2 Partial ordering for the local role algebra of the network L  67
3.1 The lattice Lₜ of isotone homomorphic images of S(N₄)  105
3.2 Hasse diagram for the partial order of S(N₄)  107
3.3 The lattice Aₜ for the abstract semigroup with multiplication table of S(N₄)  108
3.4 The lattice Lₙ(S(N₄)) of π-relations on S(N₄)  111
3.5 Extended automorphic equivalence  118
3.6 The lattice Lₙ of role algebras nested in Q  120
3.7 The lattice Lₙ(Q) of π-relations on the role algebra Q  122
3.8 Some relations in a small work group  125
3.9 Some directed out-trees  126
3.10 Some pseudo-order relations on four elements  129
3.11 Two transition graphs T and U  131
4.1 The π-relation lattice Lₚ(T) of the partially ordered semigroup T  146
4.2 The lattice Lₜ of isotone homomorphic images of T  146

xi
Figures and tables

4.3 The $\pi$-relation lattice $L_\pi(V)$ of the partially ordered semigroup $V$ 151
4.4 The $\pi$-relation lattice $L_\pi(U_3)$ of the semigroup $U_3$ 151
4.5 A $\pi$-relation lattice admitting two irredundant subdirect decompositions 153
4.6 A nondistributive, modular lattice 156
4.7 Reduction diagram for the factorisation of $V$ 162
5.1 Some network mappings 174
5.2 Automorphic, extended automorphic and regular equivalences in a network 178
5.3 Indegree and outdegree equivalences in a network 182
5.4 Some conditions for semigroup homomorphisms 184
5.5 The central representatives condition 186
5.6 Relations among equivalence conditions 189
5.7 The lattice $L_\pi(S(X))$ of $\pi$-relations of $S(X)$ 192
5.8 Searching for minimal derived set associations 200
5.9 Analysis of a complete network 200
5.10 Reduction diagram for the Breiger–Ennis semigroup BE1 204
5.11 Analysis of a local network 211
5.12 Reduction diagram for the local role algebra of the GSS network 217
5.13 Reduction diagram for the local role algebra of the network L 221
7.1 The decomposition theorem for valued network semigroups 246
8.1 Reduction diagram for the Ennis semigroup 265

Tables

1.1 The binary matrix of the friendship network in a small work group 6
1.2 Binary matrix representation of the multiple network $W$ 9
1.3 Types of complete network data 10
1.4 Relational content in a sample of network studies 15
1.5 Some approaches to network analysis 23
1.6 A blockmodel and multiple networks for which it is a fat fit, a lean fit and an $\alpha$-blockmodel ($\alpha = 0.5$) 30
1.7 Some compound relations for the network $W$ in binary matrix form 40
1.8 The blockmodel $N = \{L, A\}$ 41
<table>
<thead>
<tr>
<th>Figures and tables</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1.9 Primitive relations and compound relations of lengths 2 and 3 for the blockmodel N</td>
<td>42</td>
</tr>
<tr>
<td>1.10 The multiplication table and partial order for the partially ordered semigroup S(W)</td>
<td>46</td>
</tr>
<tr>
<td>1.11 Generating the semigroup of the blockmodel N</td>
<td>50</td>
</tr>
<tr>
<td>1.12 Multiplication table for the semigroup S(N)</td>
<td>51</td>
</tr>
<tr>
<td>1.13 Edge and word tables and partial order for the semigroup S(N)</td>
<td>53</td>
</tr>
<tr>
<td>2.1 Types of local network</td>
<td>60</td>
</tr>
<tr>
<td>2.2 The local network L in binary matrix form</td>
<td>62</td>
</tr>
<tr>
<td>2.3 Paths of length 3 or less in the network L having ego as source</td>
<td>64</td>
</tr>
<tr>
<td>2.4 Right multiplication table for the local role algebra of ego in the network L</td>
<td>66</td>
</tr>
<tr>
<td>2.5 Constructing the local role algebra of ego in the network L</td>
<td>69</td>
</tr>
<tr>
<td>2.6 The blockmodel network N</td>
<td>70</td>
</tr>
<tr>
<td>2.7 The local role algebra for block 1 in the network N</td>
<td>70</td>
</tr>
<tr>
<td>2.8 The local role algebra for the subset {1, 2} in the network N</td>
<td>71</td>
</tr>
<tr>
<td>2.9 Distinct submatrices in the local role algebra for the subset {1, 2} of the network N</td>
<td>72</td>
</tr>
<tr>
<td>2.10 Local role algebra for the subset {1, 2, 3, 4} of the network N</td>
<td>72</td>
</tr>
<tr>
<td>2.11 Quasi-orders on S(N) corresponding to the role algebras Q_1 and Q_{[1,2]}</td>
<td>77</td>
</tr>
<tr>
<td>2.12 Distinct relations in the semigroup S(N) of the network N</td>
<td>78</td>
</tr>
<tr>
<td>2.13 Relation plane for ego in the network L</td>
<td>80</td>
</tr>
<tr>
<td>2.14 Relation plane for block 1 in the network N</td>
<td>80</td>
</tr>
<tr>
<td>2.15 The role-set for block 1 in the network N</td>
<td>81</td>
</tr>
<tr>
<td>2.16 Truncated relation plane of order 2 for block 1 in the network N</td>
<td>82</td>
</tr>
<tr>
<td>2.17 Truncated relation plane of order 2 for ego in the network L</td>
<td>83</td>
</tr>
<tr>
<td>2.18 Partial local role algebra Q^{1}_{1} for block 1 in the network N</td>
<td>84</td>
</tr>
<tr>
<td>2.19 Partial local role algebra Q^{2}_{1} for ego in the network L</td>
<td>85</td>
</tr>
<tr>
<td>2.20 Partial local role algebra Q^{2}_{1} for ego in the network L</td>
<td>86</td>
</tr>
<tr>
<td>3.1 Two comparable networks (N_1 = {A, B}) and (N_2 = {A, B})</td>
<td>92</td>
</tr>
<tr>
<td>3.2 The partially ordered semigroups (S(N_1)) and (S(N_2)) of the networks (N_1) and (N_2)</td>
<td>93</td>
</tr>
<tr>
<td>3.3 The network B, which is an inflation of the network (N_1)</td>
<td>94</td>
</tr>
</tbody>
</table>
3.4 The network $N_3$, which is the disjoint union of the networks $N_1$ and $N_2$ of Table 3.1

3.5 Two comparable networks $N_1 = \{A, B\}$ and $N_4 = \{A, B\}$

3.6 The partially ordered semigroups $S(N_1)$ and $S(N_4)$

3.7 The partially ordered semigroups $S, T$ and $U$

3.8 The $\pi$-relation corresponding to the isotone homomorphism from $S$ onto $T$

3.9 The $\pi$-relation corresponding to the isotone homomorphism from $S(N_4)$ onto $S(N_4)$

3.10 Constructing a homomorphic image of the partially ordered semigroup $S$

3.11 Isotone homomorphic images of $S(N_4)$

3.12 Abstract homomorphic images of $S(N_4)$

3.13 Finding abstract homomorphic images of $S(N_4)$

3.14 $\pi$-relations corresponding to isotone homomorphisms of $S(N_4)$

3.15 The joint homomorphic image $J$ and the joint isotone homomorphic image $K$ of two semigroups $V$ and $W$

3.16 Common structure semigroups for the semigroups $V$ and $W$

3.17 Lattices of semigroups and $\pi$-relations

3.18 Some small local networks with identical role-sets

3.19 Role algebras nested in the role algebra $Q$

3.20 $\pi$-relations in $L_\pi(Q)$ for the role algebra $Q$

3.21 Two-element two-relation networks with identical partially ordered semigroups

4.1 A partially ordered semigroup $T$

4.2 Two partially ordered semigroups $S_1$ and $S_2$

4.3 The direct product $S_1 \times S_2$ of the semigroups $S_1$ and $S_2$

4.4 The $\pi$-relation corresponding to the isotone homomorphism from $T$ onto $S_1$

4.5 Isotone homomorphic images of the partially ordered semigroup $T$

4.6 $\pi$-relations in $L_\pi(T)$

4.7 Two partially ordered semigroups $U_1$ and $U_2$ and their direct product $U_1 \times U_2$

4.8 A subsemigroup $U$ of $U_1 \times U_2$ that defines a subdirect product of $U_1$ and $U_2$

4.9 A partially ordered semigroup $V$ isomorphic to the semigroup $U$

4.10 $\pi$-relations in $L_\pi(V)$

4.11 The $\pi$-relations $\pi_{16}$ generated by the ordering $1 > 6$ on the semigroup $V$
Figures and tables

4.12 $\pi$-relations generated by each possible additional ordering $i > j$ on the semigroup $V$ 159
4.13 Atoms in $L_{\pi}(S(N))$ and their unique maximal complements and corresponding factors 161
4.14 Co-ordinates for elements of the partially ordered semigroup $V$ in the subdirect representation corresponding to $\{\pi_1, \pi_2\}$ 163
4.15 Association indices for factors of the semigroups $T$ and $V$ 164
4.16 Multiplication table for a semigroup $S$ 165
4.17 $\pi$-relations in $A(S)$, presented as partitions on $S$ 167
4.18 The $\pi$-relations $\pi_s$ for each possible additional ordering $s > t$ on $Q$ 169
4.19 Atoms $z$ of the $\pi$-relation lattice of the local role algebra of the network $L$ and their unique maximal complements $\pi(z)$ 170
5.1 The network $X$ on $\{1, 2, 3\}$ and the derived network $Y$ on $\{a, b\}$ 191
5.2 The partially ordered semigroup $S(X)$ and the factors $A$ and $B$ of $S(X)$ 191
5.3 Distinct relations in $S(X)$ 192
5.4 The partial orderings $\leq_{\mu}$ and $\leq_{\phi}$ associated with the mapping $\mu$ on the node set of $X$ and the isotone homomorphism $\phi$ of $S(X)$ 194
5.5 A network $R = \{A\}$ on five elements 195
5.6 The partially ordered semigroup $S(R)$ of the network $R = \{A\}$ and factors of $S(R)$ 195
5.7 Distinct relations generated by the network $N$ 197
5.8 The partial order $\leq_{\phi}$ corresponding to the factor $S(N)/\pi_4$ of $S(N)$ 197
5.9 The partial orders corresponding to the derived sets $\{1, 2\}$ and $\{(134), (2)\}$ for the network $N$ 197
5.10 Derived sets associated with the factor $S(N)/\pi_4$ of the semigroup $S(N)$ 198
5.11 Derived networks corresponding to minimal derived set associations for the factor $S(N)/\pi_4$ of $S(N)$ 198
5.12 Minimal derived set associations and corresponding derived networks for other factors of $S(N)$ 199
5.13 The Breiger–Ennis blockmodel for a self-analytical group 202
5.14 The semigroup $BE1$ of the Breiger–Ennis blockmodel 203
5.15 Factors of $BE1$ 203
5.16 Other images of $BE1$ appearing in Figure 5.10 205
Figures and tables

5.17 Minimal derived set associations for some images of BE1 shown in Figure 5.10 205
5.18 Derived networks for associations with factors of BE1 206
5.19 The partial orders ≤ₚ and ≤ₚ for the local role algebra of block 1 of the network N 209
5.20 Derived set associations for the factors of the local role algebra of block 1 of N 210
5.21 Derived local networks corresponding to some minimal derived set associations for the factors of the local role algebra of block 1 210
5.22 Local role algebras for blocks in the Breiger-Ennis blockmodel 213
5.23 Factors of the local role algebras of Breiger-Ennis blocks 214
5.24 Minimal subsets for which factors of the Breiger-Ennis local role algebras are nested in the subset partial order 215
5.25 A local network from General Social Survey items 216
5.26 The local role algebra of the General Social Survey network 217
5.27 Factors for the role algebra of the GSS network 218
5.28 Other role algebras in the reduction diagram of Figure 5.12 219
5.29 Minimal subset associations for role algebras appearing in the reduction diagram of the GSS network 219
5.30 The local role algebra generated by the snowball network L 220
5.31 Role algebras identified in Figure 5.13 221
5.32 Some derived set associations for factors of the network L 222
5.33 Reducible role algebras from two-element two-relation local networks 223
6.1 Blockmodels for Newcomb Fraternity, Year 1, Weeks 1 to 15 229
6.2 Incidence of factors of Week 15 semigroup as images of semigroups for earlier weeks 231
6.3 Minimal partitions associated with identified images of S₁₅ and corresponding derived networks 232
6.4 Local role algebras for blocks in the Newcomb blockmodel at Week 15 234
6.5 Factors of the local role algebras for the Week 15 blockmodel 235
6.6 Incidence of Week 15 role algebra factors in earlier weeks 236
Figures and tables

7.1 A valued network $V = \{A, B\}$
7.2 Some max-min products for the valued relations $A$ and $B$ of the valued network $V$
7.3 The partially ordered semigroups $S(V)$ and $S(B)$ generated by the valued network $V$ and the blockmodel $B$
7.4 Components of the valued relations $A$ and $B$ of the valued network $V$
7.5 Components of the valued relations in $S(V)$ for the valued network $V$
7.6 Filtering relations for the semigroup $S(V)$
7.7 A valued local network
7.8 Distinct relation vectors from the valued local network of Table 7.7
7.9 The local role algebra of node 1 in the valued local network of Table 7.7
8.1 Some models for networks
8.2 The Ennis blockmodel
8.3 The semigroup $BE2$ of the Ennis blockmodel
8.4 The joint isotope homomorphic image $K$ of $BE1$ and $BE2$ and its factors $K1$ and $K2$
8.5 Derived set associations of $K1$ and $K2$ in the Breiger–Ennis and Ennis blockmodels
8.6 Images of $BE2$ appearing in Figure 8.1
8.7 The Alnlestadt blockmodel
8.8 The Towertown blockmodel
8.9 The Alnlestadt semigroup $A$
8.10 The Towertown semigroup $T$
8.11 The joint isotope homomorphic image $L$ of the semigroups $A$ and $T$
8.12 The $\pi$-relations $\pi_\alpha$ on $A$ and $\pi_\tau$ on $T$ corresponding to the joint isotope homomorphic image $L$
8.13 Minimal derived set associations with $L$ in the Alnlestadt and Towertown blockmodels and corresponding derived networks
Preface

A class of models for analysing social network data are described in this work. The models are offered in response to two related needs arising from current developments in social science research. Firstly, data on social networks are being gathered much more commonly, a fact that is reflected by the inclusion in 1985 of a set of network questions in the General Social Survey (Burt, 1984). As a result, there is a growing need for a variety of models that will enable the analysis of network data in a number of different forms. Secondly, the role of social networks in the development of social and psychological theory is increasingly prominent and calls for the development of data models attuned to a variety of theoretical claims about the nature of that role.

Arguments for the importance of social networks can be found in both the psychological and sociological domains. Social psychologists have documented their dissatisfaction with the “differential” view of social behaviour embodied in many psychological theories (e.g., Cantor & Kihlstrom, 1981; Fiske & Taylor, 1990; Harre & Secord, 1972; Magnusson & Endler, 1977; Moscovici, 1972) and have argued for an analysis of social behaviour that is more sensitive to the “meaningful” context in which it occurs. One aspect of that context is the network of social relations in which the behaviour in question is embedded, a contextual feature to which empirical studies of some kinds of behaviour have already given explicit recognition (e.g., Henderson, Byrne & Duncan-Jones, 1981).

On the sociological side, the case for the importance of social networks was initiated much earlier, and those studies that demonstrated the salience of social and personal networks have become classics (e.g., Barnes, 1954; Bort, 1957; Coleman, Katz & Menzel, 1957). Indeed, a considerable amount of attention has been devoted to the problems of obtaining information about social networks and representing it in some explicit form (e.g., Fischer, 1982; Harary, Norman & Cartwright, 1965; Henderson et al., 1981; Laumann, 1973; White, Boorman & Breiger, 1976). Moreover, in addition to their role in making social context explicit, social networks have played a significant part in the “aggregation”
problem, a role that Granovetter (1973), in particular, has made clear. The aggregation problem is the process of inferring the global, structural implications of local, personal interactions (White, 1970). Granovetter has demonstrated that the problem is not straightforward and has shown in several instances how an understanding of the local social network assists the task of inferring macro level social behaviour (Granovetter, 1973; also, Skog, 1986).

The models for which an analysis is developed in this book have therefore been chosen to be sensitive to these two main themes for the role of social networks in social theory: as an operational form of some aspects of social context and as a vehicle for the aggregation of local interactions into global social effects. The claim is not made that the models selected are unique in filling this role, although it will be argued that their properties are closely aligned with a number of theoretical mechanisms proposed for them.

The starting point for the models is the characterisation of social networks in terms of blockmodels by White et al. (1976) and the subsequent construction of semigroup models for role structure (Boorman & White, 1976; also, Lorrain & White, 1971). In presenting the construct of a blockmodel as a representation for positions and roles from multiple network data, White et al. argued that it was necessary to develop a view of concrete social structure that did not depend on the traditional a priori categories or individual attributes in the sociologists’ battery but rather on the networks of relations among individuals. They claimed that blockmodels provide a means for representing and ordering the diversity of concrete social structures, and they showed how the semigroup of a blockmodel provides a representation of its relational structure at a more abstract, algebraic level.

Later, Winship and Mandel (1983) and Mandel (1983) extended the blockmodel framework to include a representation for what they termed “local” roles. In so doing, they decoupled the notion of local role from the global role structure approach of Boorman and White, thus pointing the way to an algebraic characterisation of role in incomplete or ego-centred networks.

In this book, I have attempted to develop an integrated method of analysis for these and some related algebraic characterisations of role structure in social networks. I argue that the algebraic description of structure is natural from the perspective of social theory and extremely useful from the perspective of data analysis. In particular, it allows for a general means of analysing network representations into simple components, a property that greatly enhances the descriptive power of the representations. A major theme of the work is that the provision of such a means of analysis is a necessary precursor to adequate practical evaluation.
Preface

of the representations. Moreover, an eventual by-product of this form of analysis should be a catalogue of commonly occurring structural forms and the conditions under which they occur and, hence, a more systematic development of projects initiated by Lorrain and by Boorman and White in their accounts of simple structural models.

The first two chapters describe the algebraic representations adopted for complete and local networks, respectively. The question of which networks possess identical algebraic representations is addressed in chapter 3, together with the more general question of how to compare the algebraic representations of different networks. In chapter 4, a general procedure for analysing the algebraic representations of complete and local networks is described. The task of relating this analysis to aspects of network structure is taken up in chapter 5, where a number of illustrative applications of the overall analytic scheme are also presented. Chapter 6 contains an application of the scheme to complete and local networks measured over time, while chapter 7 presents the algebraic representations that can be constructed for valued network data. Finally, chapter 8 discusses the contribution of the analysis to some important issues for network analysis, including the description of positions and roles, structural models for networks and the comparison of network structures.

The work has benefited from the assistance of many people. Warren Bartlett lent a great deal of encouragement and support in the early stages of the work, and Harrison White and Ronald Breiger have given help in many different ways over a number of years. Many of the ideas developed here have their source in earlier work by Harrison White and François Lorrain and also by John Boyd; the work also owes much to many insightful commentaries by Ron Breiger. I am grateful, too, to Stanley Wasserman for his helpful remarks on two drafts; and I am especially indebted to my family – Ian, Matt and Alexander, my parents and my parents-in-law – for their help and patience.