1

Introduction

A plasma is an ionized gas consisting of positively and negatively charged particles with approximately equal charge densities. Plasmas can be produced by heating an ordinary gas to such a high temperature that the random kinetic energy of the molecules exceeds the ionization energy. Collisions then strip some of the electrons from the atoms, forming a mixture of electrons and ions. Because the ionization process starts at a fairly well-defined temperature, usually a few thousand K, a plasma is often referred to as the "fourth" state of matter. Plasmas can also be produced by exposing an ordinary gas to energetic photons, such as ultraviolet light or X-rays. The steady-state ionization density depends on a balance between ionization and recombination. In order to maintain a high degree of ionization, either the ionization source must be very strong, or the plasma must be very tenuous so that the recombination rate is low.

The definition of a plasma requires that any deviation from charge neutrality must be very small. For simplicity, unless stated otherwise, we will assume that the ions are singly charged. The charge neutrality condition is then equivalent to requiring that the electron and ion number densities be approximately the same. In the absence of a loss mechanism, the overall charge neutrality assumption is usually satisfied because all ionization processes produce equal amounts of positive and negative charge. However, deviations from *local* charge neutrality can occur. Usually these deviations are small, since as soon as a charge imbalance develops, large electric fields are produced that act to restore charge neutrality. Systems that display large deviations from charge neutrality, such as vacuum tubes and various electronic devices, are not plasmas, even though some aspects of their physics are similar.

To be a plasma the charged particles must be in an unbound gaseous state. This requirement can be made more specific by requiring that the random kinetic energy be much greater than the average electrostatic energy, and is imposed to provide a distinction between a plasma, in which the particles move relatively freely, and condensed matter, such as metals, where electrostatic forces play a dominant role. In a plasma long-range electrical forces are much more important than short-range forces. Because many particles "feel" the same long-range forces, a plasma is dominated by "collective" motions involving correlated movements of large numbers of particles rather than uncorrelated interactions between neighboring particles. Long-range forces lead to many complex effects that do not occur in an ordinary gas.

Plasmas can be divided into two broad categories: natural and man-made. It is an interesting fact that most of the material in the visible universe, as much as 99% according to some estimates, is in the plasma state. This includes the Sun, most stars, and a significant fraction of the interstellar medium. Thus, plasmas play a major role in the universe. Plasma physics is relevant to the formation of planetary radiation belts, the development of sunspots and solar flares, the acceleration of high velocity winds that flow outward from the Sun and other stars, the generation of radio emissions from the Sun and other astrophysical objects, and the acceleration of cosmic rays.

In the Earth's atmosphere, the low temperatures and high pressures that are commonly present are not favorable for the formation of plasmas except under unusual conditions. Probably the most common plasma phenomenon encountered in the Earth's atmosphere is lightning. In a lightning discharge the atmospheric gas is ionized and heated to a very high temperature by the electrical currents that are present in the discharge. Because of the high recombination rate the resulting plasma exists for only a small fraction of a second. Less common is ball lightning, which consists of a small ball of hot luminous plasma that lasts for up to tens of seconds. Another terrestrial plasma phenomenon, readily observable at high latitudes, is the aurora, which is produced by energetic electrons and ions striking the atmosphere at altitudes of 80 to 100 km. At higher altitudes, from one hundred to several hundred km, the Earth is surrounded by a dense plasma called the ionosphere. The ionospheric plasma is produced by ultraviolet radiation from the Sun, and also exists on the nightside of the Earth because the recombination rate is very low at high altitudes. The ionosphere plays an important role in radio communication by acting as a reflector for low-frequency radio waves. At even higher altitudes, the Earth is surrounded by a region of magnetized plasma called the magnetosphere. Planetary magnetospheres have now been observed at all the magnetized planets and exhibit many of the plasma processes that are believed to occur at magnetized astronomical objects such as neutron stars.

Numerous applications of basic plasma physics can be found in man-made devices. One of the most important of these is the attempt to achieve controlled thermonuclear fusion. Because fusion requires temperatures of 10^7 K, or more, to overcome the Coulomb repulsion between nuclei, controlled fusion necessarily involves very high temperatures. Since a fusion plasma would be quickly cooled by the walls of any ordinary container, considerable effort has gone into attempts to contain plasmas by magnetic fields, using a so-called "magnetic bottle." Although the principles of such magnetic confinement may appear at first glance to be straightforward, attempts to achieve controlled fusion using magnetic confinement have been complicated by collective effects that develop when large numbers of particles are introduced into the machine. The effort to find a technologically and economically attractive configuration for confining a dense, hot plasma remains one of the main challenges of fusion research. Besides fusion, numerous other devices involving plasmas also exist. Fluorescent lights and various other devices involving plasma discharges, such as electric arc welders and plasma etching machines, are in common daily use. More advanced devices include magnetohydrodynamic generators for producing electricity from high-temperature gas jets, ion engines for spacecraft propulsion, various surface treatment processes that involve the injection of ions into metal surfaces, and high-frequency electronic devices such as traveling wave tubes and magnetrons.

The purpose of this book is to provide the basic principles needed to analyze a broad range of plasma phenomena. Since both natural and man-made plasmas are of potential interest, a special effort has been made in this book to provide examples from both space and laboratory applications.

2

Characteristic parameters of a plasma

Before starting with a detailed discussion of the processes that occur in a plasma, it is useful to identify certain fundamental parameters that are relevant to the description of essentially all plasma phenomena.

2.1 Number density and temperature

In an ordinary material there are usually three parameters, pressure, density and temperature, that must be specified to determine the state of the material, any two of which can be selected as the independent variables. A plasma almost always involves considerably more parameters. For a plasma consisting of electrons and various types of ions, it is necessary to define a number density for each species, denoted by n_s where the subscript s stands for the sth species. Since the electrons and ions respond differently to electromagnetic forces, the number densities of the various species must be regarded as independent variables. In general, a plasma cannot be characterized by a single density.

The temperature of particles of type s is directly proportional to their average random kinetic energy. In thermal equilibrium the distribution of velocities of particles of type s is given by the Maxwellian distribution

$$f_s(\mathbf{v}) = n_s \left(\frac{m_s}{2\pi\kappa T_s} \right)^{3/2} \mathrm{e}^{-\frac{m_s v^2}{2\kappa T_s}}, \tag{2.1.1}$$

where $f_s(\mathbf{v})$ is the distribution function, $\mathbf{v}$ is the velocity, m_s is the mass of the particles, κ is Boltzmann's constant, and T_s is the temperature. The distribution function is normalized such that $f_s(\mathbf{v})$ integrated over all velocities gives the number density of particles of type s,

$$\int_{-\infty}^{\infty} f_s(\mathbf{v})\, \mathrm{d}v_x\, \mathrm{d}v_y\, \mathrm{d}v_z = n_s. \tag{2.1.2}$$

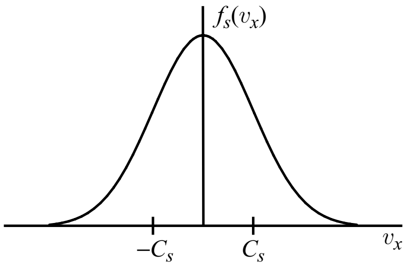

Figure 2.1 The Maxwellian velocity distribution.

A plot of the Maxwellian distribution as a function of v_x is shown in Figure 2.1 (for $v_y = v_z = 0$). It is a relatively simple matter to show that the root-mean-square velocity is given by $\sqrt{3}C_s$, where

$$C_s = \sqrt{\frac{\kappa T_s}{m_s}}. \tag{2.1.3}$$

Hereafter, C_s will be referred to as the thermal speed. The average kinetic energy is given by

$$\left\langle \frac{1}{2} m_s v^2 \right\rangle = \frac{3}{2} \kappa T_s, \tag{2.1.4}$$

where the angle brackets indicate an average. The above equation shows that the temperature is directly proportional to the average kinetic energy of the particles.

According to a general principle of statistical mechanics called the H-theorem, the Maxwellian distribution is the unique distribution function that arises when a gas is in thermal equilibrium [Huang, 1963]. For a plasma in thermal equilibrium, not only should the distribution function for each species be a Maxwellian but the temperature of all species must be equal. However, because collisions occur very infrequently in a tenuous plasma, the approach to thermal equilibrium is often very slow. Therefore, non-equilibrium effects are quite common in plasmas. Since the electron and ion masses are very different, the rate of energy transfer between electrons and ions is much slower than between electrons or between ions. Therefore, when a plasma is heated, substantial temperature differences often develop between the electrons and ions. Non-equilibrium distributions also occur when an electron beam or an ion beam is injected into a plasma. Under these circumstances the velocity distribution function of the beam usually cannot be represented by a Maxwellian distribution. Such non-thermal distributions produce many interesting effects that will be discussed later.

2.2 Debye length

All plasmas are characterized by a fundamental length scale determined by the temperature and number density of the charged particles. To demonstrate the existence of this length scale consider what happens when a negative test charge Q is placed in an otherwise homogeneous plasma. Immediately after the charge is introduced, the electrons are repelled and the ions are attracted. Very quickly, the resulting displacement of the electrons and ions produces a polarization charge that acts to shield the plasma from the test charge. This shielding effect is called Debye shielding, after Debye and Hückel [1923] who first studied the effect in dielectric fluids. The characteristic length over which shielding occurs is called the Debye length.

To obtain an expression for the Debye length, it is useful to consider a homogeneous plasma of electrons of number density n_e and temperature T_e, and a fixed background of positive ions of number density n_0. After the negative test charge Q has been inserted and equilibrium has been established, the electrostatic potential Φ is given by Poisson's equation

$$\nabla^2\Phi = -\frac{\rho_q}{\epsilon_0} = -\frac{e}{\epsilon_0}(n_0 - n_e), \tag{2.2.1}$$

where ρ_q is the charge density, ϵ_0 is the permittivity of free space and e is the electronic charge. To obtain a solution for the electrostatic potential, it is necessary to specify the electron density as a function of the electrostatic potential. We assume that at infinity, where $\Phi = 0$, the electrons have a Maxwellian velocity distribution with a number density n_0. From general principles of kinetic theory it can be shown that the velocity distribution function for the electrons is given by

$$f_e(v) = n_0\left(\frac{m_e}{2\pi\kappa T_e}\right)^{3/2} e^{-\frac{(\frac{1}{2}m_e v^2 + q\Phi)}{\kappa T_e}}, \tag{2.2.2}$$

where $q = -e$. This equation is like the Maxwellian distribution discussed previously, but has an additional factor $\exp[-q\Phi/\kappa T]$. This factor comes from a principle of statistical mechanics that states the number of particles with velocity $\mathbf{v}$ is proportional to $\exp[-W/\kappa T]$, where W is the total energy [Huang, 1963]. The total energy is given by the sum of the kinetic energy and the potential energy, $W = (1/2)m_e v^2 + q\Phi$. By integrating the distribution function over velocity space, it is easy to show that the electron density is given by

$$n_e = n_0\, e^{\frac{e\Phi}{\kappa T_e}}. \tag{2.2.3}$$

Substituting the above expression into Poisson's equation (2.2.1), one obtains the non-linear differential equation

$$\nabla^2\Phi = -\frac{n_0 e}{\epsilon_0}\left(1 - \mathrm{e}^{\frac{e\Phi}{\kappa T_e}}\right). \tag{2.2.4}$$

This differential equation can be solved analytically if we assume that $e\Phi/\kappa T_e \ll 1$. Expanding the exponential in a Taylor series and keeping only the first-order term, one obtains the linear differential equation

$$\nabla^2\Phi = \frac{n_0 e^2}{\epsilon_0 \kappa T_e}\,\Phi. \tag{2.2.5}$$

Since the plasma is isotropic, the electrostatic potential can be assumed to be spherically symmetric. The above equation then simplifies to

$$\frac{\partial^2}{\partial r^2}(r\Phi) - \frac{n_0 e^2}{\epsilon_0 \kappa T_e}(r\Phi) = 0, \tag{2.2.6}$$

which has the general solution

$$\Phi = \frac{A}{r}\,\mathrm{e}^{-r/\lambda_D}, \tag{2.2.7}$$

where r is the radius and A is a constant. The factor λ_D is the Debye length and is easily shown to be given by

$$\lambda_D^2 = \frac{\epsilon_0 \kappa T_e}{n_0 e^2}. \tag{2.2.8}$$

The constant A is determined by requiring that the solution reduce to the Coulomb potential as the radius goes to zero. The complete solution is then given by

$$\Phi = \frac{1}{4\pi\epsilon_0}\,\frac{Q}{r}\,\mathrm{e}^{-r/\lambda_D}, \tag{2.2.9}$$

and is called the Debye–Hückel potential. A plot of the Debye–Hückel potential (for negative Q) is shown in Figure 2.2. As can be seen, the potential decays exponentially, with a length scale given by the Debye length, λ_D. A simple, practical formula for the Debye length is

$$\lambda_D = 6.9\sqrt{T_e/n_0}\ \mathrm{cm}, \tag{2.2.10}$$

where T_e is in K and n_0 is in cm^{-3}.

The derivation of the Debye length given above is deceptively simple and hides some subtleties inherent in the concept, especially in collisionless plasmas for which the assumption of a Maxwellian distribution (2.2.2) is open to question. For instance, consider the following interesting paradox involving the role of ions. If the

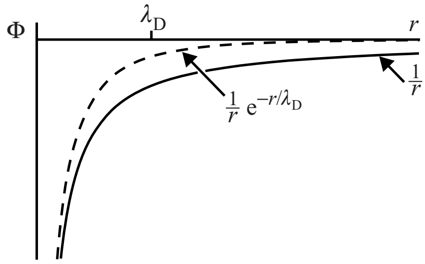

Figure 2.2 A comparison of the Debye–Hückel potential (dashed line) with the Coulomb potential (solid line) for a negative test charge.

ions were mobile, then by a simple extension of the treatment used for the electrons it would appear that the ion number density should be given by

$$n_i = n_0 e^{-\frac{e\Phi}{\kappa T_i}}. \tag{2.2.11}$$

However, this equation does not provide a correct representation of the ion density. This is because the ions are accelerated by the negative charge Q, and particle flux conservation dictates that as the ion velocity increases, the ion density should decrease. This suggests that the mobility of the ions actually decreases the positive charge density, in contrast with equation (2.2.11) which implies that the ion density should increase as r decreases. This effect is called *anti-shielding*, since it decreases the charge density in the shielding region. The resolution of this paradox requires a more sophisticated understanding of distribution functions than we have at the moment, and is postponed to Chapter 10.

2.2.1 Plasma sheaths

When an object of finite size is placed in a plasma with approximately equal electron and ion temperatures, it acquires a net negative charge because the electron thermal speed, $C_e = \sqrt{\kappa T_e/m_e}$, is much greater than the ion thermal speed, $C_i = \sqrt{\kappa T_i/m_i}$, which causes more electrons to hit the object than ions. As the object charges negatively, the electrons start to be repelled, just as when a negative test charge is introduced into the plasma. Equilibrium occurs when the electron current collected by the object balances the incident ion current. An electrically polarized region is thereby formed around the object. This polarized region is called a plasma sheath, or sometimes a positive ion sheath, because the electrons are largely excluded from the sheath. The exact form of the electrostatic potential distribution is a complicated boundary value problem and can only be solved analytically for certain simple geometries such as a sphere, a cylinder, or a planar surface. If the radius of curvature is much larger than the Debye length, so that the surface can be regarded as locally planar, then the potential decays exponentially with a characteristic length scale given by the Debye length. In these simple cases it is easy to show, by equating the incident electron and ion currents, that the equilibrium potential of the surface is given to a good approximation by

$$V = -\frac{\kappa T_{\rm e}}{2e}\left[\ln\left(\frac{m_{\rm i}}{m_{\rm e}}\right) + \ln\left(\frac{T_{\rm e}}{T_{\rm i}}\right)\right]. \tag{2.2.12}$$

Note that for a given ion-to-electron mass ratio the equilibrium potential is controlled entirely by the electron temperature. Because of the weak logarithmic dependence, this potential is typically only a few times the electron thermal energy. For a proton–electron plasma with equal electron and ion temperatures $V = -3.75\kappa T/e$.

If the object is exposed to ultraviolet radiation, as in the case of a spacecraft exposed to sunlight, then the emitted photoelectron current must be added to the equilibrium current balance condition. Under these conditions the object can charge to a positive potential if the photoelectron flux exceeds the incident electron flux. Modifications to the equilibrium potential can also occur if secondary electrons are produced by energetic particles striking the surface.

2.3 Plasma frequency

If the electrons in a uniform, homogeneous plasma are displaced from their equilibrium position, an electric field arises because of charge separation. This electric field produces a restoring force on the displaced electrons. Since the magnitude of the charge imbalance is directly proportional to the displacement, the restoring force is given by Hooke's law, $F = -k\Delta x$, where Δx is the displacement and k is the effective "spring constant." Since the electrons have inertia, the system behaves as a harmonic oscillator. The resulting oscillations are called electron plasma oscillations or Langmuir oscillations, after Tonks and Langmuir [1929] who first discovered these oscillations.

To compute the oscillation frequency, let us assume that the plasma consists of a uniform slab of electrons of number density n_0 and a fixed background of positive ions of the same density. Suppose we now displace the slab of electrons to the right by a small distance Δx, as shown in Figure 2.3. The slab can be divided into three regions. Region 1 has a net positive charge, region 2 has no net charge, and region 3 has a net negative charge. The electric field in region 2 can be computed using Gauss' law and is given by

$$E = \frac{n_0 e \Delta x}{\epsilon_0}. \tag{2.3.1}$$

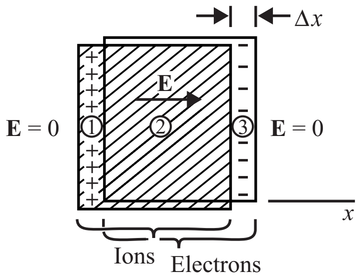

Figure 2.3 A simple slab model that illustrates electron plasma oscillations.

If the slab of electrons is then released, the equation of motion for the electrons is given by

$$m_{\mathrm{e}} \frac{\mathrm{d}^2 \Delta x}{\mathrm{d}t^2} = (-e)E = -\frac{n_0 e^2}{\epsilon_0} \Delta x, \tag{2.3.2}$$

which simplifies to

$$\frac{\mathrm{d}^2 \Delta x}{\mathrm{d}t^2} + \left(\frac{n_0 e^2}{\epsilon_0 m_{\mathrm{e}}} \right) \Delta x = 0. \tag{2.3.3}$$

The above equation is just the harmonic oscillator equation. The oscillation frequency ω_{pe} is determined by the term in parentheses

$$\omega_{\mathrm{pe}}^2 = \frac{n_0 e^2}{\epsilon_0 m_{\mathrm{e}}}, \tag{2.3.4}$$

and is called the electron plasma frequency. A simple formula for the electron plasma frequency (in hertz) is given by

$$f_{\mathrm{pe}} = 8980\sqrt{n_0}\ \mathrm{Hz}, \tag{2.3.5}$$

where the number density n_0 is in electrons cm^{-3}. Note that the electron plasma frequency is determined solely by the number density of the electrons.

If a plasma contains several species, it is customary to define a plasma frequency for each species according to the equation

$$\omega_{\mathrm{p}s}^2 = \frac{n_s e_s^2}{\epsilon_0 m_s}, \tag{2.3.6}$$

where e_s, m_s, and n_s are the charge, mass, and number density of that species. It is easily verified that the plasma frequency, the Debye length, and the thermal speed