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Coughlin is one of the principal contributors to the rapidly developing probabilistic voting literature, which is at the leading edge of public-choice theory. In this book, he both reviews the existing literature and presents new results that unify and extend his own contributions and other developments in the literature. The book will be important reading for scholars and students of public-choice theory in economics, political science, and other fields.

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# Probabilistic voting theory

PETER J. COUGHLIN  
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CAMBRIDGE UNIVERSITY PRESS  
 Cambridge, New York, Melbourne, Madrid, Cape Town, Singapore, São Paulo

Cambridge University Press  
 The Edinburgh Building, Cambridge CB2 2RU, UK

Published in the United States of America by Cambridge University Press, New York

[www.cambridge.org](http://www.cambridge.org)  
 Information on this title: [www.cambridge.org/9780521360524](http://www.cambridge.org/9780521360524)

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First published 1992

*A catalogue record for this publication is available from the British Library*

*Library of Congress Cataloguing in Publication data*

Coughlin, Peter J.  
 Probabilistic voting theory / Peter J. Coughlin.  
 p. cm.

Includes bibliographical references and index.

ISBN 0-521-36052-8

1. Social choice – Mathematical models. 2. Voting – Mathematical models. I. Title.

HB846.8.C68 1991

324'.01-dc20

91-14319

CIP

ISBN-13 978-0-521-36052-4 hardback

ISBN-10 0-521-36052-8 hardback

Transferred to digital printing 2006

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*to  
Kathy and Charlie*

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## Acknowledgments

The material in this book was developed over the last decade and has, during that period, been greatly improved by helpful comments and suggestions provided by Kenneth Arrow, Otto Davis, Arthur Denzau, James Enelow, Allan Feldman, Jerry Green, Melvin Hinich, Anne Krueger, John Ledyard, R. Duncan Luce, Steven Matthews, Martin McGuire, Dennis Mueller, Peter Murrell, Shmuel Nitzan, Wallace Oates, Mancur Olson, Peter Ordeshook, Thomas Palfrey, Thomas Romer, Howard Rosenthal, Amartya Sen, Steven Slutsky, and anonymous referees. This book has also been greatly improved by the editorial guidance provided by Colin Day and Emily Loose.

I gratefully acknowledge the financial assistance provided by National Science Foundation Grant No. SES-8409352 and the College of Behavioral and Social Sciences at the University of Maryland, which made it possible for me to prepare a draft version of this book. A second version was prepared during 1987-8 while I was a Fellow at the Center for Advanced Study in the Behavioral Sciences at Stanford University. I gratefully acknowledge the financial assistance provided by the Center (with support from National Science Foundation Grant No. BNS-8700864 and the Alfred P. Sloan Foundation) and the Graduate Research Board at the University of Maryland during that period. Last, but not least, I also gratefully acknowledge the financial assistance provided by a University of Maryland Sabbatical for the Fall 1989 semester, which made it possible for me to prepare the final version of this book.