GROUP STRUCTURE OF GAUGE THEORIES

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Preface

It has been known for many years that the gravitational and electromagnetic interactions of matter can be formulated as gauge theories – based on the Lorentz group $SO(3,1)$ and the compact ‘internal’ phase group $U(1)$, respectively. But over the past two decades it has gradually come to be accepted that the remaining two (known) fundamental interactions of matter, namely the strong and weak nuclear interactions, are also gauge interactions, a property that had been hidden by confinement for the strong interactions and by spontaneous symmetry breaking for the weak ones.

To be more precise, it has now been established beyond reasonable doubt that the weak nuclear interactions combine with electromagnetism to form a gauge interaction based on the compact internal non-abelian group $U(2)$, and, although the evidence is less direct, it is accepted that the strong interactions are gauge interactions based on the compact simple internal (colour) group $SU(3)$. The upshot of these results is that the (known) non-gravitational interactions are now described by a gauge theory based on a compact internal group with Lie algebra $SU(3) \times SU(2) \times U(1)$. (The global group is actually $SU(3) \times SU(2)$ because of certain discrete correlations in the particle classification, chapter 9.)

If the $SU(3) \times SU(2)$ theory of the non-gravitational interactions is correct, it represents an immense advance because gauge theories, by their nature, determine the form of the interactions, leaving only a finite number of constants as free parameters. In fact it means that the form of all the fundamental interactions is now known. Furthermore, since gauge theories have a geometrical interpretation in terms of fibre bundles, it means that even the non-gravitational interactions have a geometrical significance and are thus brought a step nearer to gravitation.

On the other hand the fact that all the interactions have a common gauge structure does not mean that they are fully unified, because the gravitational interaction has special properties not shared by the others (the existence of the metric and the equivalence principle, for example) and the three other interactions remain separate in the sense that the $SU(3) \times SU(2) \times U(1)$ algebra consists of three irreducible pieces, with a
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separate coupling constant for each piece. For this reason it has been suggested that the gauge group $SU(3) \times U(2)$ is actually only a subgroup of a larger, simple, compact gauge group $G$, which has only one coupling constant and which truly unifies the three non-gravitational interactions. Theories based on such groups $G$ are called grand unified theories (GUTs) and have been extensively studied in recent years. Although the most spectacular prediction of GUTs, namely proton decay, has not yet been (and may even never be) observed, there is a certain amount of indirect evidence for GUTs (chapter 10), notably from the particle classification, from renormalization group considerations and from cosmology.

Compact gauge theories are, in principle, generalizations of electromagnetism from $U(1)$ to non-abelian groups, but the generalization is not trivial for two reasons. First, the intrinsic group structure (Lie algebras, representations, invariants, etc.) is much more complicated than in the abelian case. Second, spontaneous symmetry breaking, which enters only in the special case of superconductivity for electromagnetism, plays a central role for the non-abelian theories.

The aim of the present monograph is to provide a review of the group structure both of the non-abelian gauge theories themselves and of their spontaneous symmetry breaking. The presentation is pitched at about the graduate student level and, as so not to overlap with the many excellent treatments of other aspects of gauge theories (renormalization, phenomenology, confinement, topology, etc.), it concentrates on two aspects. These are the group theoretical background, particularly the global group theory (Part I) and the algebraic structure of the gauge interactions and of their symmetry breakdown patterns (Part II). The spontaneous symmetry breaking is treated in some detail (at the classical level) because many results in this area have not previously been available in book form. It should be stated, however, that the investigation of symmetry breaking patterns is still at an early stage of development and so the results presented should be regarded as pioneering ones.

The general plan of the monograph may be seen from the list of contents, but a few remarks may be in order. In chapters 1–5, where the group-theoretical background is given, some of the more technical equipment (tables of branching rules and Clebsch–Gordan coefficients for example) has been omitted because it is available elsewhere and space did not permit a reasonable résumé. In the chapters on spontaneous symmetry breaking (8, 11, 12) it is assumed, for definiteness, that the symmetry breakdown is caused by a local scalar potential, but it is fairly evident that because of the group-theoretical nature of the results most of them would survive
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in a much broader context, e.g. if the scalar field were composite. Indeed this is one of the justifications for the group-theoretical approach. With regard to the references, and the suggestions for further reading, the literature on both Lie groups and gauge theory is so vast there was no hope of providing a comprehensive bibliography, and accordingly these sections have been limited to those references which are strictly relevant, to recent reviews (many of which, notably Langacker (1981), contain further lists of references) and to books.

Finally, I should like to take this opportunity to thank Professors Nikolas Kuiper (director) and Louis Michel for their kind hospitality at the Institut des Hautes Etudes Scientifiques, Bures-sur-Yvette, for most of the academic year 1983–4, when much of the monograph was written. I should also thank Louis Michel, whose influence pervades not only the book but the whole literature on symmetry and symmetry breaking, for many invaluable discussions and comments.

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