

1 Equations of motion

1.1 Introduction

This is a book about the fluid motions which set the performance of devices such as propulsion systems and their components, fluid machinery, ducts, and channels. The flows addressed can be broadly characterized as follows:

- (1) There is often work or heat transfer. Further, this energy addition can vary between streamlines, with the result that there is no “uniform free stream”. Stagnation conditions therefore have a spatial (and sometimes a temporal) variation which must be captured in descriptions of the component behavior.
- (2) There are often large changes in direction and in velocity. For example, deflections of over 90° are common in fluid machinery, with no one obvious reference direction or velocity. Concepts of lift and drag, which are central to external aerodynamics, are thus much less useful than ideas of loss and flow deflection in describing internal flow component performance. Deflection of the non-uniform flows mentioned in (1) also creates (three-dimensional) motions normal to the mean flow direction which transport mass, momentum, and energy across ducts and channels.
- (3) There is often strong swirl, with consequent phenomena that are different than for flow without swirl. For example, static pressure rise can be associated almost entirely with the circumferential (swirl) velocity component and thus essentially independent of whether the flow is forward (radially outward) or separated (radially inward). In addition the upstream influence of a fluid component, and hence the interaction between fluid components in a given system, can be qualitatively different than that in a flow with no swirl.
- (4) The motions are often unsteady. Unsteadiness is necessary for work exchange in turbomachines. Waves, oscillations, and self-excited unsteadiness (instability) not only affect system behavior, but can sometimes be a limiting factor on operational regimes.
- (5) A rotating reference frame is a natural vantage point from which to examine flow in rotating machinery. Such a reference frame, however, is a non-inertial coordinate system in which effects of Coriolis and centrifugal accelerations have a major role in determining the fluid motions.
- (6) Perhaps the most important features of internal flows, however, are the constraints imposed because the flow is bounded within a duct or channel. This influence is felt in all flow regimes, but it is especially marked when compressibility is involved, as in many practical applications. If the effects of wall friction, losses in the duct, or energy addition or extraction are not assessed correctly, serious adverse effects on mass flow capacity and performance can result.

2 Equations of motion

In the succeeding chapters we will see *when* these different effects are important, *why* they are important, and *how* to define and analyze the magnitude of their influence on a given fluid motion.

In this chapter we present a summary of the basic equations and boundary conditions needed to describe the motion of a fluid. The discussion given is self-contained, although it is deliberately brief because there are many excellent sources, with extended discussions of the topics covered; these are referred to where appropriate.

1.2 Properties of a fluid and the continuum assumption

For the applications in this book, we define a fluid as an isotropic substance which continues to deform in any way which leaves the volume unchanged as long as stresses are applied (Batchelor, 1967). In most engineering devices, except those that work at pressures several orders of magnitude below standard atmosphere or are of very small scale, the characteristic length scale of the motion in a gas will be many times the size of the mean free path (the mean distance between collisions for a molecule). This is not a very restrictive condition since the mean free path in a gas at standard temperature and pressure is approximately 10^{-7} m. In such situations we can ignore the detailed molecular structure and discuss the properties “at a point” as if the fluid were a continuous substance or *continuum*. In this context, we will use the term *fluid particle*, which can be defined as the smallest element of material having sufficient molecules to allow the continuum interpretation. For a liquid the corresponding condition is that the particle be much larger than the molecular size, which is of order 10^{-9} m for water (Lighthill, 1986a), again this is most typically the case.¹ In summary, at pressures, temperatures, and device dimensions commonly encountered, variations due to fluctuations on the molecular scale can be ignored and the fluid treated as a continuum.

1.3 Dynamic and thermodynamic principles

The principles that define the motion of a fluid may be expressed in a number of ways, but can be stated as follows: conservation of mass, conservation of momentum (Newton’s second law of motion), and the first and second laws of thermodynamics. These must also be supplemented by the equation of state of the fluid, a relation between the thermodynamic properties, generally derived from observation. These conservation and thermodynamic laws are statements about *systems*, or *control masses*, which are defined here as collections of material of fixed identity. For example, conservation of mass is a statement that the mass of a fluid particle remains constant no matter how it is deformed. Newton’s second law, force equals rate of change of momentum, also applies to a particle or to a given collection of particles.

In general, however, interest is not in fixed mass systems but rather in what happens in a fixed volume or at a particular position in space. For this reason, we wish to cast the equations for a system into a form which applies to a *control volume*, V , of arbitrary shape, bounded by a control surface, A ,

¹ As an example, in a cube of air which is 10^{-3} mm (1 μ m) on a side there are roughly 3×10^7 molecules at standard conditions. For water in a cube of these dimensions there are roughly 10^{10} molecules.

3 1.3 Dynamic and thermodynamic principles

i.e. to transform the system (control mass) laws into control volume laws.² We will carry out these transformations in several steps. The concept of differentiation following a fluid particle, or sum of particles, is first introduced. This is then employed to express the conservation laws explicitly in a form tied to volumes and surfaces moving with the fluid. We then derive the relation between changes that occur in a volume moving with the fluid and changes in a volume fixed in an arbitrary coordinate system. This leads to expressions for the equations of motion in integral (control volume) as well as differential form.

1.3.1 The rate of change of quantities following a fluid particle

To describe what happens at a fixed volume or point in space we must inquire how the time rate of change for a particle can be described in a fixed coordinate system. For definiteness we take Cartesian coordinates x , y , z , and fluid velocity components u_x , u_y , and u_z . Suppose that c is some property of the fluid and we visualize a field of values of c continuously distributed throughout space. For small arbitrary and independent increments dx , dy , dz , and time, dt , the change in property c is

$$dc = \frac{\partial c}{\partial x} dx + \frac{\partial c}{\partial y} dy + \frac{\partial c}{\partial z} dz + \frac{\partial c}{\partial t} dt. \quad (1.3.1)$$

For a given particle, the increments dx , dy , and dz are related to the local instantaneous velocity components and the time increment, dt , by:

$$dx = u_x dt, \quad dy = u_y dt, \quad dz = u_z dt, \quad (1.3.2)$$

where u_x , u_y , and u_z are velocity components in the three spatial directions. Dividing each term by dt , the rate of change of c following a fluid particle can be written as

$$\text{rate of change of } c \text{ following a fluid particle} = \frac{Dc}{Dt} = u_x \frac{\partial c}{\partial x} + u_y \frac{\partial c}{\partial y} + u_z \frac{\partial c}{\partial z} + \frac{\partial c}{\partial t}. \quad (1.3.3)$$

In (1.3.3), the notation $D()/Dt$ has been used to indicate a derivative defined following the fluid particle. This notation is conventional, and the quantity $D()/Dt$, which occurs throughout the description of fluid motion, is known variously as the substantial derivative, the material derivative, or the convective derivative. Noting that in Cartesian coordinates the first three terms of the derivative are formally equivalent to $\mathbf{u} \cdot \nabla c$, the substantial derivative can be written more compactly as

$$\frac{Dc}{Dt} = \frac{\partial c}{\partial t} + (\mathbf{u} \cdot \nabla) c = \frac{\partial c}{\partial t} + u_i \frac{\partial c}{\partial x_i}. \quad (1.3.4)$$

In (1.3.4), and throughout the book, we use the convention that a repeated subscript implies summation over the appropriate indices. In (1.3.4),

$$u_i \frac{\partial c}{\partial x_i} = u_1 \frac{\partial c}{\partial x_1} + u_2 \frac{\partial c}{\partial x_2} + u_3 \frac{\partial c}{\partial x_3}.$$

In this notation the derivative of the velocity following a fluid particle, which is the acceleration, is (for the i^{th} component): $Du_i/Dt = \partial u_i/\partial t + u_j(\partial u_i/\partial x_j)$. In vector notation the acceleration is $D\mathbf{u}/Dt = \partial \mathbf{u}/\partial t + (\mathbf{u} \cdot \nabla)\mathbf{u}$.

² The terms *system* (or *control mass*) and *control volume* are used here in describing the two different viewpoints; these concepts are also referred to as *closed system* and *open system* respectively.

4 Equations of motion

1.3.2 Mass and momentum conservation for a fluid system

We can use the derivative following a fluid particle to obtain expressions for the conservation laws, starting with the simplest, conservation of mass. If dm is the mass of a fluid particle, conservation of mass is obtained by taking c to be dm ; i.e.

$$\frac{D}{Dt}(dm) = 0. \quad (1.3.5)$$

To obtain an expression valid for an assemblage of particles, i.e. a fluid system, we sum over the different particles in the system. In the continuum limit this can be represented by an integral over the masses:

$$\frac{D}{Dt} \int dm = 0. \quad (1.3.6)$$

In interpreting (1.3.6), it is important to keep in mind that the integral is taken over a fixed mass, which implies a volume fixed to fluid particles and moving with them.

Newton's second law can also be written for an assemblage of fluid particles as

$$\sum \mathcal{F}_{ext} = \frac{D}{Dt} \int \mathbf{u} dm. \quad (1.3.7)$$

In (1.3.7) \mathcal{F}_{ext} represents the *external* forces acting on the particles and the summation includes all the forces that act on this mass. The forces can be body forces, which act throughout the mass, or can be surface forces exerted at the boundary of the system. Coriolis, gravity, and centrifugal forces are examples of the first of these; pressure and shear forces, which are exerted by the fluid or by bodies that bound the system, are examples of the second.

1.3.3 Thermodynamic states and state change processes for a fluid system

To describe the thermodynamics of fluid systems, we need to introduce the idea of a system state and define two classes of state change processes. The thermodynamic state of a system is defined by specifying the values of a small set of measured properties, such as pressure and temperature, which are sufficient to determine all other properties. In flow situations it is useful to express properties such as volume, V , or internal energy, E , which depend on the mass of the system, as a quantity per unit mass. The properties on this unit mass basis are referred to as specific properties and denoted here by lower case letters (v , e , for specific volume and specific internal energy respectively).

The state of a system in which properties have definite (unchanged) values as long as external conditions are unchanged is called an equilibrium state. Properties describe states only when the system is in equilibrium. For thermodynamic equilibrium of a system there needs to be: (i) mechanical equilibrium (no unbalanced forces), (ii) chemical equilibrium (no tendency to undergo a chemical reaction or a transfer of matter from one part of the system to another), and (iii) thermal equilibrium (all parts of the system at the same temperature, which is the same as that of the surroundings).

Fluid devices typically have quantities such as pressure which vary throughout, so that there is no single value that characterizes all the material within the device. If so the conditions for the three types of equilibrium to hold on a *global* basis (e.g. the absence of finite pressure differences or unbalanced forces) are not satisfied when we view the complete region of interest as a whole. To deal with this situation we can (conceptually) divide the flow field into a large number of small

5 1.3 Dynamic and thermodynamic principles

(differential) mass elements, over which the pressure, temperature, etc. have negligible variation, and consider each of these elements a different system with its own *local* properties.³ In defining the behavior of the different systems the working assumption is that the local instantaneous relation between the thermodynamic properties of each element is the same as for a uniform system in equilibrium.⁴

Processes that change the state of a system can be classed as reversible or irreversible. Fluid process that are *irreversible* (also referred to as natural processes) include motions with friction, unrestrained expansion, heat transfer across a finite temperature difference, spontaneous chemical reaction, and mixing of matter of different composition or state. These processes have the common characteristic that they all take place spontaneously in nature. A further aspect is that “a cycle of changes $A \rightarrow B \rightarrow A$ on a particular process, where $A \rightarrow B$ is a natural process, cannot be completed without leaving a change in some other part of the universe” (Denbigh, 1981).

A central role in thermodynamic analysis is played by *reversible* processes, defined as a process “whose direction can be reversed without leaving more than a vanishingly small change in any other system” (Denbigh, 1981). This means that the departures from thermodynamic equilibrium at any state in the process are also vanishingly small. In the case of forces, for instance, the internal forces exerted by the system must differ only infinitesimally from the external forces acting on the system. Similarly, for reversible heat transfer between surroundings and system, there can only be infinitesimal temperature differences between the two. A reversible process must also be *quasi-static*, i.e. slow enough that the time for the fluid to come to equilibrium when subjected to a change in conditions is much shorter than any time scale for the process, again so that the system essentially passes through a series of equilibrium states during the process. As with the continuum approximation this is not restrictive for the situations of interest: for example, equilibration times for air at room conditions are on the order of 10^{-9} seconds (Thompson, 1984).⁵ All real fluid processes are in some measure irreversible although, as we will see, many processes can be analyzed to a high degree of accuracy assuming they are reversible.

Recognition of the irreversibility in a real process is vital in fluids engineering. A perspective on its effect is that “Irreversibility, or departure from the ideal condition of reversibility, reflects an increase in the amount of disorganized energy at the expense of organized energy” (Reynolds and Perkins, 1977). Organized energy is illustrated by a raised weight. Disorganized energy is represented by the random motions of the molecules in a gas (the internal energy of the gas). The importance of the distinction is that *all* the organized energy can, in principle, produce work, whereas a consequence of the second law of thermodynamics (Section 1.3.4) is that only a fraction of the disorganized energy is available to produce work. The transition from organized to disorganized energy brought about by irreversibility thus corresponds to a loss in opportunity to produce work (and hence power or propulsion) from a fluid device. In this connection Section 1.3.4 introduces the thermodynamic property

³ A consequence is that the state definition requires specification of several functions rather than several variables. In addition, although we refer to the temperature and pressure *at a point*, the division into differential elements is made with the caveat expressed in Section 1.2.

⁴ From a macroscopic point of view this assumption must be assessed by experience, which shows that its appropriateness is extremely well borne out for the flows of interest. The approximation made, referred to as the principle of local state, is discussed further by Kestin (1979) and Thompson (1984).

⁵ For more complex molecules or temperatures much higher than room temperature, the equilibration time can be several orders of magnitude larger (times of 10^{-5} seconds are given by Thompson (1984) for gases at 3000 K). If so, the relaxation of the gas to the equilibrium state may need to be included. We do not examine these regimes.

6 Equations of motion

entropy, which provides a quantitative measure of irreversibility; Section 1.10 discusses entropy generation in a flowing fluid; and Sections 5.1 and 5.2 examine the relation between irreversibility and the loss in capability for work production.

1.3.4 First and second laws of thermodynamics for a fluid system

The first law of thermodynamics can be expressed for a system as

$$\Delta E_t = Q - W \quad (1.3.8)$$

where ΔE_t is the change in the total energy of the system, Q is the heat received, and W is the work done by the system on the environment. In differential form (1.3.8) is

$$dE_t = dQ - dW. \quad (1.3.9)$$

The notations $d()$ and $d\langle \rangle$ denote conceptual and physical differences between the terms in (1.3.9). The total energy, E_t , is a property. Changes in E_t (dE_t or its integral ΔE_t) represent state changes which do not depend on the path taken to achieve the change. Work and heat are not state variables and are only defined in terms of interactions with the system. For a specified change of state (specified initial and final states) ΔE_t is given, but the individual amounts of heat and work transfer to the system can vary, depending on the path by which the change is accomplished.⁶ To emphasize the difference between the two types of quantities, we use $d\langle \rangle$ for small changes in properties and $d()$ for the small amounts of heat and work transfer that bring these changes about.

For the systems we are concerned with, the total energy can be written as an integral, over the system mass, of the sum of the internal energy, e , per unit mass, and the kinetic energy, $u^2/2$, per unit mass. For flow situations the items of interest are generally the rates at which quantities change so it is useful to cast the first law as a rate equation:

$$\frac{DE_t}{Dt} = \frac{D}{Dt} \int \left[e + \frac{u^2}{2} \right] dm = \frac{dQ}{dt} - \frac{dW}{dt}. \quad (1.3.10)$$

In (1.3.10) dQ/dt is the rate of heat transfer to the system and dW/dt is the rate of work done by the system.

The second law of thermodynamics can be expressed in two parts.⁷ The first part is a definition of the thermodynamic property entropy of the system, denoted as S . If dQ_{rev} is the heat transferred to the system during a reversible incremental state transformation, and T is the temperature of the system,

$$dS = \frac{dQ_{rev}}{T}. \quad (1.3.11)$$

For a finite change from state 1 to state 2,

$$S_2 - S_1 = \int_1^2 \frac{dQ_{rev}}{T}. \quad (1.3.12)$$

⁶ Discussion of this point is given in many texts. See, for example, Denbigh (1981), Kestin (1979), Reynolds and Perkins (1977) and Sonntag, Borgnakke, and Van Wylen (1998).

⁷ See, for example, Abbott and Van Ness (1989), Denbigh (1981), and Kestin (1979) for additional discussion.

7 1.3 Dynamic and thermodynamic principles

The second part of the second law states that for *any* process the change in entropy for the system is

$$dS \geq \frac{dQ}{T}. \quad (1.3.13)$$

The equality occurs only for a reversible process. A consequence of (1.3.13) for a system to which there is no heat transfer is

$$dS \geq 0 \quad (\text{for a system with } dQ = 0). \quad (1.3.14)$$

Equation (1.3.13) can also be written as a rate equation in terms of the heat transfer rate and temperature of the fluid particles which comprise the system. With s the *specific entropy* or entropy per unit mass,

$$\frac{DS}{Dt} = \frac{D}{Dt} \int s dm \geq \sum \frac{1}{T} \frac{dQ}{dt}. \quad (1.3.15)$$

In (1.3.15), the summation is taken over all locations at which heat enters or leaves the system. Equation (1.3.15) will be developed in terms of fluid motions and temperature fields later in this chapter.

The fluids considered in this book are those described as simple compressible substances. The thermodynamic state of such fluids is specified when two independent intensive thermodynamic properties (pressure and temperature, for example) are given and the only reversible work mode is that associated with volume change (Reynolds and Perkins, 1977).

For incremental reversible processes in a simple compressible substance, the heat addition to the fluid is

$$dQ = TdS. \quad (1.3.16a)$$

If kinetic energy changes can be neglected (the change is in thermal energy only) the work done is

$$dW = pdV. \quad (1.3.16b)$$

Although the association of work with pdV and heat addition with TdS is only true for a reversible process, the sum of these, as expressed by the first law, is a relation between thermodynamic properties. For negligible kinetic energy changes, this relation is

$$de = Tds - pdv, \quad (1.3.17)$$

where s and v are the entropy and volume per unit mass. Equation (1.3.17), known as the Gibbs equation, can be regarded as a combined form of the first and second laws. It is a relation between thermodynamic properties and is not restricted to reversible processes.

A thermodynamic property which will be seen to occur naturally in flow processes is the *enthalpy*, denoted by h and defined as

$$h = e + p/\rho. \quad (1.3.18)$$

8 Equations of motion

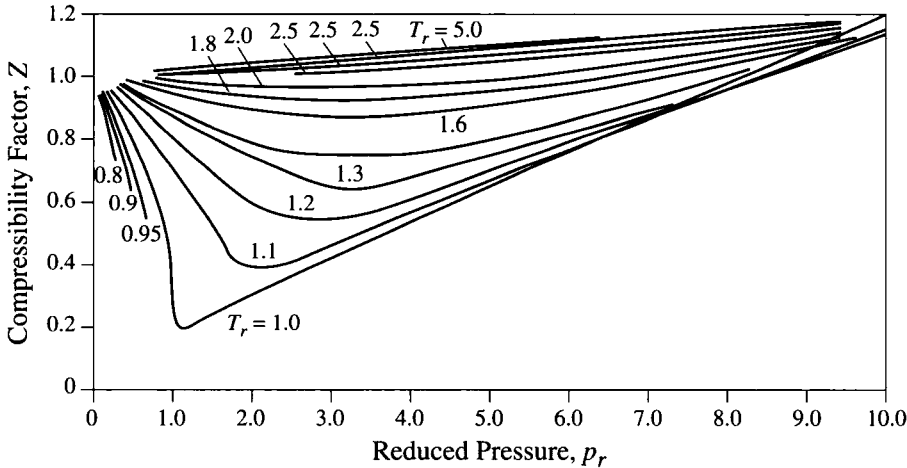


Figure 1.1: Compressibility factor $Z = p/\rho RT$, at low pressures; reduced temperature $T_r = T/T_c$, values of critical temperature, T_c , given in Table 1.1 (Lee and Sears, 1963).

A form of the Gibbs equation useful for flow processes can be written in terms of enthalpy changes, using the definition $v = 1/\rho$, as

$$dh = T ds + \frac{1}{\rho} dp. \quad (1.3.19)$$

As with (1.3.17), (1.3.19) is not restricted to reversible processes.

1.4 Behavior of the working fluid

1.4.1 Equations of state

The equations relating the intensive thermodynamic variables of a substance are called the equations of state. The flows examined in this book are very well represented using one of two equations of state. The first is for a *perfect* gas,

$$p = \rho RT, \quad (1.4.1)$$

where $R = \mathcal{R}/M$, with \mathcal{R} the universal gas constant ($\mathcal{R} = 8.3145 \text{ kJ}/(\text{kmol K})$)⁸ and M the molecular weight of the gas. Equation (1.4.1) holds for air and other gases over a wide range of temperatures and pressures.

The ratio $p/\rho RT$ is called the compressibility factor, and its variation from unity gives a good measure of the applicability of (1.4.1). This quantity is plotted in Figure 1.1. The curves are averaged from experimental data on a number of monotonic and diatomic gases, plus hydrocarbons (Lee and Sears, 1963). The compressibility factor is given as a function of the reduced pressure,

⁸ A kmol is a mass equal to the molecular weight of the gas in kilograms.

9 **1.4 Behavior of the working fluid****Table 1.1** *Critical pressures and temperatures for different gases (Lee and Sears, 1963)*

Substance	p_c (MPa)	T_c (K)
He	0.23	5.3
H ₂	1.30	33.6
Air	3.77	132.7
O ₂	5.04	154.5
CO ₂	7.39	304.3
H ₂ O	22.1	647.4

defined as pressure/critical pressure⁹ (p/p_c) for different reduced temperatures, T_r , defined as temperature/critical temperature (T/T_c). For reference, several values of p_c and T_c are listed in Table 1.1. For reduced temperatures between 1.6 and 5.0 and reduced pressures of less than approximately 3, the perfect gas approximation is valid to within 5%. For example, air at a pressure of 30 atmospheres and a temperature of 1650 K (conditions representative of the exit of the combustor in a gas turbine) corresponds to $p/p_c = 0.8$ and $T/T_c = 12.5$. Even at these conditions, the compressibility factor would be approximately 1.03.

The second equation of state that will be used is for an incompressible fluid, i.e. a fluid in which the volume of a given fluid mass (density) is constant. This is suitable for liquids. It is also a very good approximation for gases at low speeds. In Chapter 2 this statement is made more precise but, to give a numerical appreciation for the approximation, in air at standard temperatures the assumption of constant density holds within 3% for speeds of 100 m/s or less. Incompressible denotes that the volume of a fluid particle remains constant; it does not necessarily mean uniform density throughout the fluid.

1.4.2 Specific heats

Two important thermodynamic properties are the *specific heat at constant volume* and the *specific heat at constant pressure*. These quantities, denoted by c_v and c_p respectively for the values per unit mass, have a basic definition as derivatives of the internal energy and enthalpy. For a simple compressible substance, the energy difference between two states separated by small temperature and specific volume differences, dT and dv , can be expressed as

$$de = \left(\frac{\partial e}{\partial T} \right)_v dT + \left(\frac{\partial e}{\partial v} \right)_T dv. \quad (1.4.2)$$

The derivative $(\partial e/\partial T)_v$ is c_v . It is a function of state, and hence a thermodynamic property.

The name specific heat is somewhat of a misnomer because only in special circumstances is the derivative $(\partial e/\partial T)_v$ related to energy transfer as heat. For a constant volume reversible process, no work is done. Any energy increase is thus due only to energy transfer as heat, and c_v represents the

⁹ The critical pressure and temperature correspond to p and T at the critical point, the highest pressure and temperature at which distinct liquid and gas phases of the fluid can coexist.

10 Equations of motion

energy increase per unit of temperature and per unit of mass. In general, however, it is more useful to think of c_v in terms of the definition as a partial derivative, which is a thermodynamic property, rather than a quantity related to energy transfer as heat.

Just as c_v is related to a derivative of internal energy, c_p is related to a derivative of enthalpy. Writing the enthalpy as a function of T and p ,

$$dh = \left(\frac{\partial h}{\partial T} \right)_p dT + \left(\frac{\partial h}{\partial p} \right)_T dp. \quad (1.4.3)$$

The derivative $(\partial h / \partial T)_p$ is called the specific heat at constant pressure and denoted by c_p . For reversible constant pressure heat addition, the amount of heat input per unit mass is given by $dq = c_p dT$.

Values of c_v and c_p are needed often enough that they have been determined for a large number of simple compressible substances. Numerical values of c_p for several gases are shown in Figure 1.2 (Sonntag, Borgnakke and Van Wylen, 1998).

For a perfect gas, the internal energy and enthalpy are defined to depend only upon temperature. Thus

$$de = c_v(T) dT, \quad (1.4.4a)$$

$$dh = c_p(T) dT, \quad (1.4.4b)$$

where c_v and c_p can depend on T . Further, $dh = de + d(pv) = c_v dT + R dT$. Hence, for a perfect gas (sometimes also referred to as an ideal gas (Reynolds and Perkins, 1977)),

$$c_v = c_p - R. \quad (1.4.5)$$

For other substances, e and h depend on pressure as well as temperature and, in this respect, the perfect gas is a special model.

Depending on the application, the variation in specific heat with temperature may be able to be neglected so that c_p and c_v can be treated as constant at an appropriate mean value. If so

$$e_2 - e_1 = c_v(T_2 - T_1), \quad (1.4.6a)$$

$$h_2 - h_1 = c_p(T_2 - T_1). \quad (1.4.6b)$$

Equations (1.4.6) hold only for a perfect gas with constant specific heats as do the relations that have been derived between changes in energy (or enthalpy) and temperature in (1.4.4).

For an *incompressible fluid*, the volume of a given fluid particle is constant and the internal energy is a function of a single thermodynamic variable, the temperature. The specific heat at constant volume is thus also a function of temperature but the change in internal energy of an incompressible fluid undergoing a temperature variation is

$$e_2 - e_1 = \int_{T_1}^{T_2} c_v(T) dT. \quad (1.4.7)$$

From the definition of enthalpy, $h = e + p/\rho$, the enthalpy change of an incompressible fluid for a specified pressure and temperature change is

$$h_2 - h_1 = e_2 - e_1 + \frac{1}{\rho} (p_2 - p_1). \quad (1.4.8)$$