UNROLLING TIME

Christiaan Huygens and the mathematization of nature

Joella G. Yoder

The right of the University of Cambridge to print and sell all manner of books was granted by Henry VIII in 1534. The University has printed and published continuously since 1584.

Cambridge University Press

Cambridge

New York   Port Chester   Melbourne   Sydney

PUBLISHED BY THE PRESS SYNDICATE OF THE UNIVERSITY OF CAMBRIDGE
The Pitt Building, Trumpington Street, Cambridge, United Kingdom

CAMBRIDGE UNIVERSITY PRESS
The Edinburgh Building, Cambridge CB2 2RU, UK
40 West 20th Street, New York NY 10011–4211, USA
477 Williamstown Road, Port Melbourne, VIC 3207, Australia
Ruiz de Alarcón 13, 28014 Madrid, Spain
Dock House, The Waterfront, Cape Town 8001, South Africa

http://www.cambridge.org

First published 1988
Reprinted 1990
First paperback edition 2002

A catalogue record for this book is available from the British Library

Library of Congress Cataloguing in Publication data
Yoder, Joella Gerstmeyer, 1944–
Unrolling time: Christiaan Huygens and the mathematization of
nature / Joella G. Yoder.
p. cm.
Bibliography: p.
ISBN 0 521 34140 X
1. Mathematics – Early works to 1800 – Collected works. 2. Science –
Mathematics – Early works to 1800 – Collected works. I. Title.
QA3.Y63 1988
510′.92′4–dc19 88-6975 CIP

ISBN 0 521 34140 X hardback
ISBN 0 521 52481 4 paperback

Contents

1

Introduction

In the era bounded by Galileo's *Dialogo* of 1632 and Newton's *Principia* of 1687, science changed. Observation, even when performed with enough care to be called experimentation, gave way to rigorous mathematical analysis as the primary approach to physical phenomena. Whereas Galileo aimed to instruct laymen about his view of the world order by means of plausible arguments and analogies, only an experienced mathematician could hope to understand the world picture envisioned by the *Principia.* This mathematization of physics was a defining element of that intellectual upheaval we call the Scientific Revolution, and the requirement, still imposed today, that a theoretical physicist be an able mathematician stems from a tradition that flowered in the seventeenth century.

Such sweeping change cannot be attributed to one particular moment or person. Yet the development of this interrelationship between mathematics and physics has remained too long in the realm of vague generalizations, whose validity has yet to be substantiated by a careful comparison with actual events. A new difficulty arises, however, because the particulars against which any generalization must be tested are not well documented. It is the latter deficiency that this book addresses by focusing on a specific person and event in the development of mathematical physics during the seventeenth century. This is a modest endeavor, designed not to explain the greater phenomenon but to provide a case study that any general account must encompass. The person is Christiaan Huygens; the event is his creation of the theory of evolutes.[1]

Preeminent mathematician, physicist, and astronomer, Christiaan Huygens (1629–95) was one of the major figures of the Scientific Revolution. Second son of the great Dutch poet and diplomat Constantijn Huygens, he was introduced at a very young age into a learned, cosmopolitan society and rapidly distinguished himself in mathematics and observational astronomy. His early achievements included studies on classical mathematics in the Archimedean and Apollonian traditions, an approximation of pi, the first printed treatise on probability, the discovery of one of Saturn's moons (Titan), the correct explanation of Saturn's varying profile (his ring hypothesis), and an unpublished treatise on the mechanics of impact. By his midthirties he had acquired such international acclaim that he was called to the court of Louis XIV to participate in the formation of the Académie Royale des Sciences, and there he remained for almost twenty years, except for trips home during periods of debilitating illness. Cited in modern histories of science primarily for his wave theory of light, his work on centrifugal force, his analysis of percussion, and the Huygens ocular for telescopes, he is regarded as the last great proponent of the mechanical philosophy (usually equated with Cartesianism).[2]

The design and development of clocks was one problem that interested Huygens throughout his life. In 1657, he created a clock whose advance was regulated by a pendulum, and consequently he has usually been designated the inventor of the pendulum clock.[3] Galileo Galilei had also considered using the pendulum as a timekeeper and had even given his son instructions on how to build a clock regulated by a swinging rod, a task never completed. Both Huygens and Galileo hoped that the new design would greatly improve the accuracy of astronomical measurements and make possible a precise determination of longitude at sea. Both scientists have partisans claiming priority of invention for their candidate's design, although still other enthusiasts argue that Leonardo da Vinci invented the pendulum clock.[4] As with most mechanical devices, priority of invention depends on whether the emphasis and value are placed on the basic design, on the construction of a physical model, or on the accuracy of the mechanism once constructed. Huygens's claim rests on the last criterion, for in addition to mounting the pendulum on the clock in a better manner, he instituted features that guaranteed far greater accuracy – for example, the endless chain, which made it possible to wind the clock without disturbing its progress. In 1658, Huygens published his *Horologium,* a description of his most recent design incorporating these advances, and thereby popularized the pendulum clock.[5]

A little more than a year later, Huygens began planning a second edition of this *Horologium* in order to incorporate a wealth of discoveries and consequent designs that he had developed in the meantime. Many years passed before this new work, one of the masterpieces of seventeenth-century scientific literature, was finally published in 1673 under the title *Horologium Oscillatorium* (The pendulum clock). Much more than a mere description of a clock, as the earlier work had been, it was in fact a treatise on the accelerated motion of a falling body, as exemplified by the bob of a pendulum clock.[6]

The book is divided into five parts, the first describing the mechanical features of a clock designed by Huygens. This clock (Fig. 1.1) included the endless chain, a lens-shaped bob that minimized air resistance, the *curseur* weight that allowed fine adjustment of the period of swing, and a pair of plates that were curved in the shape of an inverted cycloid and mounted on either side of the pendulum.

The second part of the *Horologium Oscillatorium* is a series of propositions on gravitational fall, beginning with free fall, including linear fall along inclined planes, and ending with fall along a curved path. The culminating proposition is Huygens's proof that a body falling along an inverted cycloid (Fig. 1.2) reaches the bottom in a fixed amount of time, irrespective of the point on the path at which it begins its fall. In other words, the cycloid is isochronous.[7]

The third section of Huygens's great work introduces his theory of evolutes, a mathematical correspondence between curves that, among other applications, allows one to find the length of a curve. Using evolutes, Huygens justifies his introduction of the curved plates to the clock that he describes in Part 1, for he proves mathematically that the cycloidal-shaped plates will force the bob of the pendulum to move along the isochronous cycloidal path. Thus, ideally, the pendulum will keep uniform time regardless of how wide it swings, as he has shown in Part 2.

The fourth, and longest, section of the *Horologium Oscillatorium* deals with the physical, rather than the ideal, pendulum. Here Huygens presents his theory of the compound pendulum, in which the motion of a pendulum with mass distributed along its length is compared with that of an ideal simple pendulum of weightless cord and point-mass bob.

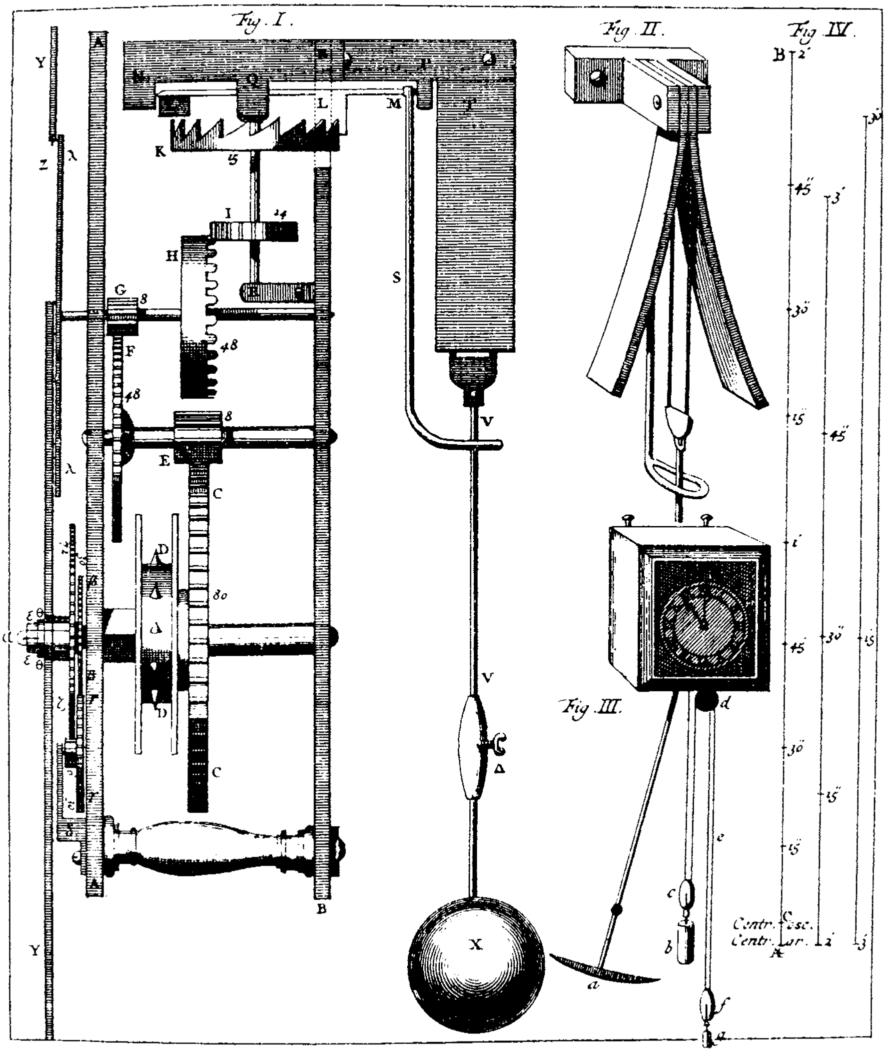

Figure 1.1. The clock of the *Horologium Oscillatorium.*

The last part of the book introduces a second timepiece, one that is a variant of a conical clock in which the pendulum, instead of swinging, rotates about a vertical axis. As with the cycloidal pendulum of Part 1, the bob is kept on an isochronous path by a curved plate whose shape is also determined by the theory of evolutes.

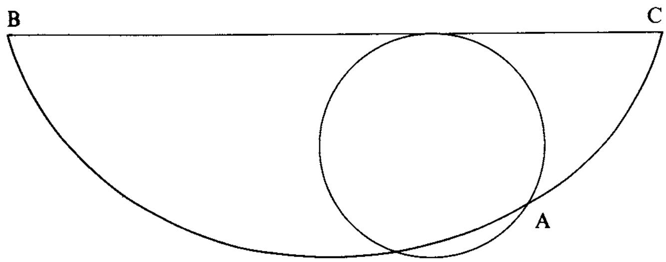

Figure 1.2. An inverted cycloid, drawn by tracing the path of a fixed point *A* on the generating circle's rim as it rolls counterclockwise along a straight line.

Following the description of this clock Huygens lists, without proofs, thirteen theorems on centrifugal force that form the theoretical justification of the pendulum's motion, in the same way that the theorems of Part 2 validate the clock of Part 1. The proofs of these theorems eventually appeared in the posthumously published *De Vi Centrifuga.*

The contents of the *Horologium Oscillatorium* are tied together much more closely than has ever been hinted in previous literature. Except for Part 4, which was not written until 1664, the entire treatise was essentially developed in a three-month period beginning in October 1659. During that time, Huygens proceeded rapidly, almost inexorably, from one creative event to another until the major theorems of the *Horologium Oscillatorium,* and *De Vi Centrifuga,* were revealed. This progression of ideas can be reconstructed in considerable detail by a careful examination of the massive evidence in the *Oeuvres complètes de Christiaan Huygens* and by a return to the original manuscripts, some of which are not included in the *Oeuvres complètes.*[8] A reconstruction of this period of great creativity is first and foremost a study of Huygens's method of research, and as such affords a detailed examination of the interaction between his mathematics and his physics.

Huygens's theory of evolutes, the key mathematical concept to emerge from this work of 1659, provides a natural focal point for such a study, since it was as much a physical as a mathematical idea, both in its roots and in its applications. This duality is evident in the definitions of the evolute and its companion, the involute. Although viewed today as a feature of pure mathematics, the evolute was originally conceived by Huygens in mechanical terms, and

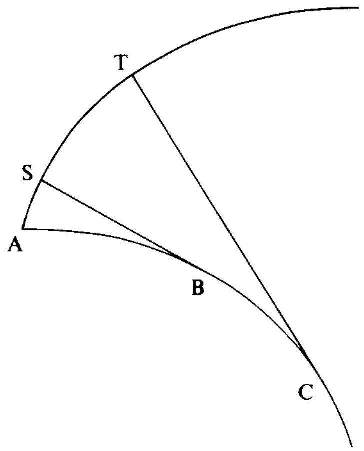

Figure 1.3. The mechanical evolution of an involute from the unrolling of its evolute.

its origin is reflected in its very name, which comes from the participle *evolutus* (unrolled) of the Latin verb *evolvere* (to unroll).[9] Given a curve *ABC* (Fig. 1.3), called the evolute, it is "unrolled" by the simple expedient of fitting a thread (*filum*) to its shape and carefully unwinding the thread from one end, with the freed end of the thread always pulled taut. The end of the thread, which begins at *A* and moves out toward *S* and thence to *T* as the thread unrolls, traces out a companion curve, which Huygens always referred to as "that drawn by the unrolling" (*descripta ex evolutione*) and which modern mathematics labels the involute.[10]

Although Huygens defined the relationship between evolute and involute mechanically, in practice he derived the evolute mathematically from a given involute. Given a curve *MPQ* (Fig. 1.4), where *Q* is infinitesimally close to *P*, the intersection *N* of the normal to *P* (the line perpendicular to the tangent at *P*) and the normal to *Q* is presumed to lie on the evolute. In modern terminology, never used by Huygens, the evolute is the locus of the instantaneous centers of rotation or curvature of the involute. The two approaches - mechanically deriving the involute and mathematically deriving the evolute - are equivalent, and one of Huygens's first tasks in Part 3 of the *Horologium Oscillatorium* is to show the uniqueness of the relationship between evolute and involute, by proving that the tangents to the evolute (the mechanical approach) are normals to the involute (the mathematical avenue).[11] Note that any curve can function as either an evolute or an involute; its role depends on the circumstances of the problem at hand.

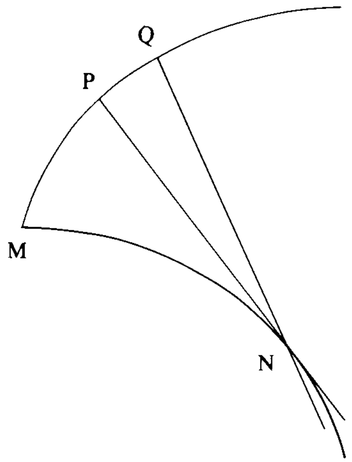

Figure 1.4. The mathematical derivation of an evolute from the normals to its involute.

Evolutes grew out of Huygens's studies of the pendulum; the complete theory was a product of his mathematical response to a physical question. However, evolutes formed only one part of a rich matrix of mathematical techniques that Huygens used to attack the problems presented him in late 1659. Those techniques and how they helped him deal successfully with physical questions are a major concern of this book.

A caveat is therefore in order: Mathematics ahead! I have tried to avoid introducing extraneous mathematical material, and in particular I have minimized the introduction of modern equivalents to the techniques used by Huygens, with most of the exceptions relegated to the notes as shorthand aids. This procedure should (1) aggravate those mathematical adepts who seek an easy understanding of his solutions, (2) please my fellow purists who feel that historical accuracy and insight are lost when results are couched in modern terms (see the Preface for a confession regarding compromises), and (3) bore those who cannot understand the mathematics in whatever format it is presented. For the sake of the last group, I have tried to make sure that the derivations can be skipped or at least skimmed without loss of the underlying story. However, the purpose of this book is to demonstrate Huygens's mathematics at work in the formation of his physics and vice versa, and thus the mathematics cannot be relegated to the notes. On the contrary, focusing on his original derivations yields a rare glimpse of creativity in action.

The question that initiated Huygens's intensive period of research at the end of 1659, which culminated in the theory of evolutes and the cycloidal-pendulum clock, is easy to state, appearing almost inconsequential. How it leads to such fertile results reveals a master of seventeenth-century science in the process of discovery. The question: What is the constant of gravitational acceleration?