THE
INTERNAL CONSTITUTION OF THE STARS

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BY

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WITH A NEW FOREWORD BY

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FOREWORD

Eddington's *The Internal Constitution of the Stars*, when it was published in 1926, was immediately heralded as one of the great masterpieces of astronomical literature. It was easily one of the books that most influenced the growth of astronomy during the two decades following its publication. It was always referred to in the literature as "*ICS*" without further qualification. I can still remember the gusto with which Henry Norris Russell used to quote *ICS* to clinch an argument!

The Internal Constitution of the Stars continues to have interest for students of astrophysics, not so much for particular developments that are relevant for the active researcher of today, but rather for the manner in which some of the basic elements of our present understanding first emerged in Eddington's writings. These elements are:

(1) Radiation pressure must play an increasingly important role in maintaining the equilibrium of stars of increasing mass (§14, p. 15 and Chapter VII).

(2) In parts of the star in which radiative equilibrium, as distinct from convective equilibrium, obtains, the temperature gradient is determined jointly by the distribution of energy sources and of the opacity of the matter to the prevailing radiation field. Precisely,

$$\frac{dp_R}{dr} = -k\frac{L_r}{4\pi c r^2}\rho, \quad p_R = \tfrac{1}{3} a T^4 \qquad (1),$$

and

$$L_r = 4\pi \int_0^r \epsilon \rho r^2 dr \qquad (2),$$

where p_R, k, ϵ and ρ denote, respectively, the radiation pressure, the coefficient of stellar opacity, the rate of energy generation per gram of the stellar material and the density. Also, a is Stefan's radiation constant and c is the velocity of light (Chapter V).

(3) The principal physical process contributing to the opacity, k, is determined by the photo-electric absorption coefficient in the soft X-ray region, that is, by the ionisation of the innermost K- and L-shells of the highly ionised atoms (Chapter IX).

(4) With electron scattering as the ultimate source of stellar opacity, there is an upper limit to the luminosity, L, that can support a given mass M. The maximum luminosity, set by the inequality (§82, p. 115).

$$L < \frac{4\pi c G M}{\sigma_e} \qquad (3),$$

where σ_e denotes the Thomson scattering-coefficient, is now generally referred to as the *Eddington limit*. This limit plays an important role in current investigations relating to X-ray sources and the luminosity of accretion discs around black holes.

(5) In a first approximation, in normal stars (that is, in stars along the main sequence) the (mass, luminosity, effective temperature)-relation is not very sensitive to the distribution of the energy sources through the star. Therefore, a relation is available for comparison with observations even in the absence of a detailed knowledge of the energy sources of the star (Chapter VII).

(6) The burning of hydrogen into helium is the most likely source of stellar energy (Chapter XI).

(7) The phenomenon of Cepheid variability is due to the adiabatic radial pulsations of stars (Chapter VIII).

These deductions, which Eddington made some sixty years ago, continue to be valid – then, as now.

I should like to expand on two of these major deductions to illustrate Eddington's approach to problems of this kind.

Eddington establishes the increasing importance of radiative pressure as a factor in the equilibrium of stars of increasing mass in that marvellous paragraph (p. 16) opening with his imagining a physicist calculating on a cloud-bound planet and ending with the dramatic conclusion, "What 'happens' is the stars." The calculations of the "cloud-bound physicist" are included in Table 2 (p. 16); and they are made on the assumption that the ratio of the gas pressure (p_G) to the radiation pressure (p_R) is constant through the star; and that the mean molecular weight, μ, has the value 2·5. A value of $\mu = 1{\cdot}0$ is more realistic (in view of the present estimates of the abundance of hydrogen), in which case the mass of each of the globes would have to be increased by a factor $(2{\cdot}5)^2 = 6{\cdot}25$. This factor is not of much consequence though it would have "spoiled the agreement". But there are two important aspects of the argument on which Eddington is silent. One is that while a combination of the natural constants of the dimensions of a mass and of stellar magnitude is clearly implied by the calculations, Eddington does not isolate it – a surprising omission in view of his later preoccupations with natural constants. Actually, the combination of natural constants which determines the masses of the globes, in the interesting range, is (see equations (84·4) and (84·5), p. 117),

$$\left[\frac{3}{a}\left(\frac{\mathfrak{R}}{\mu}\right)^4\right]^{\frac{1}{2}} \frac{1}{G^{\frac{3}{2}}} \qquad \text{.................................} (4),$$

where $\mathfrak{R}$ is the "gas constant", G is the constant of gravitation, and a is Stefan's radiation constant. On inserting for Stefan's constant its value (equation (40·8), p. 55),

$$a = \frac{8\pi^5}{15} \frac{(\Re H)^4}{h^3 c^3} \quad \text{.............................(5)},$$

where H denotes the mass of the hydrogen atom and h is Planck's constant, we find that the combination of the natural constants of the dimensions of a mass that is involved is

$$\left(\frac{hc}{G}\right)^{\frac{3}{2}} \frac{1}{(H)^2} \simeq 29{\cdot}2 \text{ solar masses} \quad \text{.....................(6)}.$$

Eddington implicitly makes the case that the successes of the theories of stellar structure and stellar evolution must derive in large measure from this combination of the natural constants providing a mass of the correct stellar magnitude.

A second aspect of the calculations, presented in Table 2 (p. 16), on which Eddington is also silent, is: why is the extent to which radiation pressure provides support against gravity relevant to the "happening of the stars"? On this question, Eddington, instead of basing his arguments on his "standard model" (in which the ratio of the radiation pressure to the gas pressure is a constant through the star), could have used a theorem of his own (Problem I, p. 91) – namely, that for *stability*, the pressure at the centre of a star must be less than that at the centre of a configuration of uniform density of the same mass and the same central density – to show that the ratio of the radiation pressure to the total pressure at the centre of a star must be less than a certain fraction dependent on the mass of the star only. Eddington's Table 2 would then be replaced by a similar table (in which, for example, "radiation pressure" is respectively 0·01, 0·10 and 0·40 times the total pressure for $M = 0{\cdot}56$, 2·14 and 9·62 solar masses) and the conclusion of the physicist on the cloud-bound planet would not have been different.

With regard to the other key points, the one relating to the source of stellar opacity reveals best the manner of Eddington's acceptance of physical theories when confronted with his astronomical deductions. In his considerations relating to the problem of stellar opacity, he had the benefit of consultations with C. D. Ellis, who was an expert in X-ray and γ-ray physics. Also, a famous paper by H. A. Kramers (1923, *Phil. Mag.* **46**, 836) provided the first theoretical evaluation of the atomic cross-sections for photo-electric ionisation which Eddington needed. (Prior to the publication of Kramers' paper, Eddington had developed a theory of his own based on the physically untenable hypothesis of the direct capture of electrons by atomic nuclei (§170, p. 245).) But the comparison of astronomical observations with the theoretical mass–luminosity relation, adopting Kramers' opacity law, left a discrepancy of a factor exceeding ten (§167, p. 242).

Eddington (and, independently, B. Strömgren) eliminated this discrepancy in 1932 by adopting the large abundances of hydrogen and helium which had, by then, been established by Russell from the analysis of the abundances of the elements in the solar atmosphere. Eddington was, however, slow to accept the non-universality of the composition of the stars that followed. Eddington had in fact realised much earlier that the assumption that hydrogen was abundant and that a mean molecular weight of the stars close to unity would resolve the opacity discrepancy. But this assumption (by lowering the values from his preferred value 2·2 to 1) would have spoiled his argument for the "happening of the stars" (Table 2, p. 16); and it is characteristic of Eddington that he should have concluded "I would much prefer to find some other explanation of the discordance" (p. 245).

Among Eddington's predictions, that of the source of stellar energy is perhaps the most spectacular. His address, on 24 August 1920 to the British Association meeting in Cardiff, contains some of the most prescient statements in all of astronomical literature (1920, *Observatory*, **43**, 353–5); and it is worth quoting in full.

> Only the inertia of tradition keeps the contraction hypothesis alive – or rather, not alive, but an unburied corpse. But if we decide to inter the corpse, let us frankly recognise the position in which we are left. A star is drawing on some vast reservoir of energy by means unknown to us. This reservoir can scarcely be other than the subatomic energy which, it is known, exists abundantly in all matter; we sometimes dream that man will one day learn how to release it and use it for his service. The store is well-nigh inexhaustible, if only it could be tapped. There is sufficient in the Sun to maintain its output of heat for 15 billion years....
>
> Aston has further shown conclusively that the mass of the helium atom is even less than the sum of the masses of the four hydrogen atoms which enter into it – and in this, at any rate, the chemists agree with him. There is a loss of mass in the synthesis amounting to 1 part in 120, the atomic weight of hydrogen being 1·008 and that of helium just 4. I will not dwell on his beautiful proof of this, as you will no doubt be able to hear it from himself. Now mass cannot be annihilated, and the deficit can only represent the mass of the electrical energy set free in the transmutation. We can therefore at once calculate the quantity of energy liberated when helium is made out of hydrogen. If 5 per cent of a star's mass consists initially of hydrogen atoms, which are gradually being combined to form more complex elements, the total heat liberated will more than suffice for our demands, and we need look no further for the source of a star's energy.
>
> If, indeed, the subatomic energy in the stars is being freely used to maintain their great furnaces, it seems to bring a little nearer to fulfillment our dream of controlling this latent power for the well-being of the human race – or for its suicide.

Eddington's interest in the internal constitution of the stars arose from his efforts to find an explanation for stellar variability and the period–luminosity relation exhibited by the Cepheids (Chapter VIII). Eddington first generalised Ritter's earlier analysis of the adiabatic pulsations of gaseous stars in convective equilibrium to the case of a star in radiative equilibrium built on his standard model (§§127–30, pp. 186–92). Then combining the resulting formula for the period with his mass–luminosity relation, Eddington was able to account, in a general way, for the observed period–luminosity relation of the Cepheids. The pulsation theory of stellar variability thus came to be established.

Eddington's preliminary analysis of Cepheid variability did not provide the correct phase relationships among the various variables such as the brightness, the effective temperature and the radial velocity of the star. However, he clearly realised that these phase relationships can be understood only by a careful examination of the mechanism of energy transfer in the outer layers of the star where the abundant elements, hydrogen and helium, get ionised and zones of convection are formed. He returned to this problem several times in later years. Indeed, one of his last published papers is devoted to this problem. But the final solution was found only later by the combined investigations of M. Schwarzschild, P. Ledoux and R. Christy.

While Eddington's major contributions to astrophysics lay in the domain of stellar structure, his contributions to other areas of astrophysics are by no means insignificant. He devised a method of approximation (§226, p. 322) – the "Eddington approximation" – for solving problems in radiative transfer. His solution for the problem of line formation in stellar atmospheres (§234, p. 339), for example, was much in use during the pioneering years in the theory of stellar atmospheres. He also considered the effect of reflection in close binaries – an effect which one must allow for in analysing the light curves of eclipsing binaries for the purposes of determining the masses of the components (§143, p. 210). The problem Eddington considered in this latter context is the prototype of the larger problem of the diffuse reflexion and transmission of light by plane-parallel atmospheres – a subject which grew to maturity in later years.

In these other areas of astrophysics, perhaps the most important is Eddington's introduction of a "dilution factor" (p. 382) – a term he coined and which is still in current usage – to allow for the reduced intensity of the prevailing radiation field in determining the state of ionisation in interstellar space. All these and much else are described in this volume which, to quote H. N. Russell, "has every claim to be regarded as a masterpiece of the first rank".

S. Chandrasekhar

(For a more complete assessment of Eddington's contributions to stellar dynamics, astrophysics and relativity see S. Chandrasekhar, *Eddington: The Most Distinguished Astrophysicist of His Time*, Cambridge University Press, 1983.)

PREFACE

THE study of the mechanical and physical conditions in the deep interior of the stars is undertaken primarily in the hope that an understanding of the internal mechanism will throw light on the external phenomena accessible to observation. More than fifty years have gone by since the general mode of attack was first developed; and the scope of the inquiry has grown so that it now involves much of the recently won knowledge of atoms and radiation, and makes evident the ties which unite pure physics with astrophysics. It would be hard to say whether the star or the electron is the hero of our epic.

The reader will judge for himself whether solid progress has been made. He may, like Shakespeare, take a view less optimistic than my own—

> The heaven's glorious sun
> That will not be deep-searched with saucy looks;

but I hope he will not be so unkind as to continue the quotation—

> Small have continual plodders ever won
> Save base authority from others' books.

Re-reading this work I find passages where I have been betrayed into too confident assertion. It is only too true that the most patent clues may mislead, and observational tests of the rough kind here possible sometimes flatter to deceive. But the subject is a fair field for the struggle to gain knowledge by scientific reasoning; and, win or lose, we find the joy of contest.

The last two chapters trespass beyond the ground indicated by the title of the book. This extension can perhaps be justified; but I am aware that, whatever excuse I might make, the real reason was that I could not forgo a desire to collect and review some of the remarkable researches of those who have investigated the outer layers of a star.

The book was written between May 1924 and November 1925. Time was occupied by a number of minor investigations made to fill the gaps that disclosed themselves as the material was brought together. Anyone writing on a theme which many workers are actively investigating is liable to find his pen unable to overtake the rate of growth of the subject. During the above-mentioned period the theoretical papers on stellar constitution in the *Monthly Notices* alone amounted to more than 400 pages. It has been still more difficult to cope with modifications and progress in the theory of the atom, on which astronomical developments must rest. As we go to press a "new quantum theory" is arising which may have important reactions on the stellar problem when it is more fully developed.

An effort has been made to include everything judged important up to November 1925; but naturally the whole book could not be rewritten in the last month and late developments had to be grafted on to an earlier foundation. Further additions have been made in proof up to March 1926; footnotes in *square brackets* show information received too late to be used in the text.

The question of notation has caused me much perplexity since branches of physics ordinarily remote from one another are here brought together. Apart from the inadequacy of the alphabet, the abolition of overlapping symbols and adoption of a consistent notation throughout the book has been deemed impracticable. There are limits to our toleration of change of familiar symbols. A rose by any other name would smell as sweet, but the equation $p\omega = U_2 - U_1$ would lack the familiar savour of the quantum relation. The use of m for absolute magnitude and the mass of an electron, of R for Boltzmann's constant and the radius of a star, of a for a radiation constant and the semiaxis of an orbit, of e for the charge of an electron and the Napierian base, may cause momentary confusion but, it is hoped, no serious difficulty. Similarly we refer to the astronomical equation of areas in its most recognisable form in sections where $h^2 = \mu l$ is not likely to be misconstrued as a relation between Planck's constant and molecular weight. I would suggest to the reader in difficulty that there is a chance that the symbol which puzzles him is included in the list of natural constants in Appendix I.

I have derived help from many colleagues. Mr R. H. Fowler has generally been my referee in difficulties over points of theoretical physics, and I have similarly had recourse to Dr C. D. Ellis for experimental questions. I thank especially Prof. E. A. Milne who has read the proof sheets and eliminated a number of errors and obscurities. My acknowledgments are also due to the staff of the University Press for their care and attention in the printing.

A. S. E.

July, 1926