CHAPTER I

SURVEY OF THE PROBLEM

1. At first sight it would seem that the deep interior of the sun and stars is less accessible to scientific investigation than any other region of the universe. Our telescopes may probe farther and farther into the depths of space; but how can we ever obtain certain knowledge of that which is hidden behind substantial barriers? What appliance can pierce through the outer layers of a star and test the conditions within?

The problem does not appear so hopeless when misleading metaphor is discarded. It is not our task actively to "probe"; we learn what we do learn by awaiting and interpreting the messages dispatched to us by the objects of nature. And the interior of a star is not wholly cut off from such communication. A gravitational field emanates from it, which substantial barriers cannot appreciably modify; further, radiant energy from the hot interior after many deflections and transformations manages to struggle to the surface and begin its journey across space. From these two clues alone a chain of deduction can start, which is perhaps the more trustworthy because it is only possible to employ in it the most universal rules of nature—the conservation of energy and momentum, the laws of chance and averages, the second law of thermodynamics, the fundamental properties of the atom, and so on. There is no more essential uncertainty in the knowledge so reached than there is in most scientific inferences.

We should be unwise to trust scientific inference very far when it becomes divorced from opportunity for observational test. We do not, however, study the interior of a star merely out of curiosity as to the extraordinary conditions prevailing there. It appears that an understanding of the mechanism of the interior throws light on the external manifestations of the star, and the whole theory is ultimately brought into contact with observation. At least that is the goal which we keep in view.

2. The gravitational field emanating from the interior and the radiant energy streaming out from the interior together control the conditions in the shallow layer or atmosphere examined with the telescope and spectroscope. We believe that they are by far the most important controlling factors. Spectrum analysis detects in the stellar atmospheres chemical substances which differ from one star to another; in some helium is prominent, in others oxygen, hydrogen, calcium, iron, titanium oxide, and so on. But it is not to be supposed that this is an indication of the relative abundance of the chemical elements—that a star showing strongly the iron spectrum is richer in that element than other stars; it is rather an indication of physical conditions of temperature and density favourable for exciting the respective spectra. Without entirely denying the possibility of differences of chemical composition, which may be necessary to account for some of the more unusual types of spectrum, we assume that in the main the observed differences of surface phenomena are not connected with chemical constitution.

We have thus to consider an atmosphere of material of fixed composition, with free upper surface and density increasing downwards. Its physical state—distribution of density, temperature and pressure; hence also its radiative and optical properties—will then depend entirely on the extraneous controlling influences to which it is subjected; and these extraneous influences are, as already stated, the force of gravity holding it down to the star and the stream of radiant heat poured into it from below. In order to remain in a steady state the atmosphere must adjust itself to let the radiant heat pass through. Thus the surface conditions depend on two parameters, viz. the value of g at the surface and the "effective temperature" T_e. The effective temperature is a conventional measure specifying the rate of outflow of radiant heat per unit area; it is not to be regarded as the temperature at any particularly significant level in the star.

By varying the controlling factors g and T_e the state of the stellar atmosphere can be varied in two directions. Accordingly we must expect that the possible varieties of stellar spectrum will form a twofold sequence, that is to say, will be capable of arrangement in two-dimensional order. This is in fact the case. For a long time only a one-dimensional order was recognised, viz. the well-known Draper sequence of types. But the spectroscopic method of determining absolute magnitudes, due to Adams and Kohlschütter in 1914, introduces a classification of spectra transverse to the Draper classification. Roughly speaking the Draper criterion follows the parameter T_e and the absolute magnitude criterion the parameter g; but the correspondence is probably not so close as was at one time supposed. The observational criteria divide the two-dimensional distribution of states into one system of meshes, and the parameters T_e and g into another system. There is no reason to anticipate any close coincidence of the two methods of partition.

The same twofold sequence of possible states appears when we consider the star as a whole. Evidently one sequence is obtained by considering stars of different mass. A transverse sequence is formed by stars of the same mass but different radius (or mean density). Thus a third way of dividing into meshes the two-dimensional distribution of states is obtained by taking the mass and radius of the star, M and R, as parameters.

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3. Consider now the connection between our three pairs of parameters —g and T_e; Draper Type and Absolute Magnitude criterion; M and R—any pair defining a unique state of the star. The connection of the spectral criteria with g and T_e is a problem of great importance in which much recent progress has been made; but it is not a problem of the stellar interior and lies outside the main lines of our investigation. As regards the connection of g and T_e with M and R, the connection of g needs no comment; the main question is, How is T_e, or equivalently the rate of outflow of radiation, determined by the mass and radius of the star? That is the central problem of this book. Various branches of inquiry will diverge from it; but it supplies the continuous thread in the discussion, so long as we are studying the stellar interior.

This is essentially a problem of the stellar interior and not of superficial conditions. The sun does not radiate $6 \cdot 10^{10}$ ergs per square centimetre per second *because* its photosphere is at 6000° C.; its photosphere is maintained at 6000° because $6 \cdot 10^{10}$ ergs are streaming through it. It is under the temperature gradient in the interior that the radiant stream gathers way; the surface layers cannot dam the flow since their capacity for storing energy is insignificant; they can only adjust themselves to let it through. Qualitatively the radiant stream is greatly transformed in passing through the last few thousand kilometres of the star, and the actual waves that spread through space are born in the photospheric layers; but quantitatively it is one continuous stream passing from the interior into outer space.

The intensity of this outward flow of energy through the interior depends on two factors, the one helping and the other hindering. Heat flows from a higher to a lower temperature, and the cause of the flow within the star must be a gradually increasing temperature from the surface to the centre. The hindering factor is the obstruction opposed by matter to the transmission of this stream of heat. We shall find that in a star the heat is transmitted almost entirely by radiation, and the obstruction to the flow of radiation is the *opacity* or *absorption coefficient* of the stellar material. Our problem is, therefore, firstly to find the distribution of temperature inside a star so as to determine the temperature gradient urging the flow; secondly, to determine the opacity of matter under the physical conditions prevailing in the interior.

4. Here at the outset we must deal with a criticism urged by Nernst, Jeans and others. It has been argued that this procedure for calculating the outward flow of radiation is necessarily doomed to failure, because the star's output of heat energy is determined by entirely different considerations. The supply of heat replenishing that which the star radiates into space must come from the conversion of other forms of energy; and since the star remains apparently steady for exceedingly long periods of time, the radiation of the star must be just equal to the amount of energy converted in the interior. It is now believed that this conversion process is the liberation of subatomic energy. The critic contends that, since the outflowing heat represents the energy liberated by subatomic processes, the amount can only be calculated if we know the laws of liberation of subatomic energy, and any procedure which evades this difficult problem begs the question.

Now it is quite true that a theory of the rate of liberation of subatomic energy is a conceivable approach to the problem of stellar radiation. In the present state of our knowledge such theories are little more than guess-work and results are rudimentary. But it is unsound to argue that no other procedure is permissible. The amount of water supplied to a town is the amount pumped at the waterworks; but it does not follow that a calculation based on the head of water and diameter of the mains is fallacious because it evades the problems of the pumping station.

It may seem puzzling to understand how two radically different ways of calculating the theoretical radiation from a star can be made to agree. Appealing again to the analogy, the two modes of calculating the water supplied to a town may not agree; but in that case there will be a flood at the pumping station. Similarly in a star a disagreement would involve the blowing up or collapse of the star. Accepting it as a fact that the stars generally are in a nearly steady state, we must infer that for actual stars (but not necessarily for a model star of arbitrarily assigned constitution) the two modes of calculating the radiation would give the same result; and in Chapter XI we shall try to follow up the question how the adjustment has occurred by which the supply of subatomic energy just meets the demand. Meanwhile we note that, flood or no flood, the flow of water must conform to the pressure gradient and diameter of the pipe; and so also the radiation from a star must in any case conform to the temperature gradient and opacity in the interior.

We may thus proceed with our method of determining the expenditure of radiation by the star without reference to the supply of subatomic energy. How the star manages to accommodate its supply to balance its expenditure, and so avoid collapse or expansion, is an independent problem.

Lane's Theory.

5. The pioneer investigation of the distribution of temperature within a star is contained in a paper published in 1870 by J. Homer Lane entitled, "On the Theoretical Temperature of the Sun, under the Hypothesis of a Gaseous Mass maintaining its Volume by its Internal Heat, and depending on the Laws of Gases as known to Terrestrial Experiment*."

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* *American Journ. of Sci. and Arts*, Series 2, 4, p. 57.

This was followed and amplified by investigations on similar lines by A. Ritter*, Lord Kelvin†, and others, culminating in the systematic and exhaustive research of R. Emden. Although we find it necessary to break away from these earlier investigations on a fundamental point, viz. the mode of transfer of heat within the star, they contain much that is sufficiently general to be adapted to present theories. The calculations and tables in Emden's remarkable book *Gaskugeln* (Teubner, 1907) have been extensively used by the author.

Lane reached the striking result that if a star contracts the internal temperature rises so long as the material is sufficiently diffuse to behave as a perfect gas. Until recently it was believed that the gravitational energy converted into heat by contraction was the only important source of maintenance of a star's heat. In that case the star through radiating heat must contract, and the heat generated by the falling in of material must be sufficient not only to replace the radiation lost but to raise the internal temperature to a higher level. Lane's result thus took the paradoxical form that a star by losing heat automatically grows hotter.

Lane's investigation is not, however, bound up with any particular views as to the source of a star's heat. It sets forth the change of temperature necessary to preserve equilibrium. The star has the option to obey Lane's law or to collapse; it is obvious that actual stars have not chosen the latter alternative, but the reason lies outside Lane's theory. Accepting the modern belief that the heat is supplied by liberation of subatomic energy, we still suppose that stars are formed by gradual condensation of primordial matter; so that the course of evolution is from low to high density and therefore by Lane's law from low to high temperature. At least in the earlier stages the internal temperature of a star is gradually rising. If in the later stages of high density the material no longer behaves as a perfect gas the temperature may ultimately fall again.

6. In Lane's time there was no evidence that any star existed for which the theory of a perfect gas would be applicable. The mean density of the sun is 1·41 gm. per cu. cm., and long before reaching such a density terrestrial gases cease to conform to the perfect gas law. There was at that time no reason to doubt that the sun's density was typical of stars in general. But we now know that there exist stars ("giant stars") with mean densities comparable to that of air or even to the density in an ordinary vacuum tube. These at least can be treated as composed of perfect gas; so that there will be no lack of opportunity for application to actual stars of results obtained for perfect gas.

The existence of stars of low density is now a commonplace of astronomy, and it is unnecessary to survey the abundant proofs derived indirectly

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* *Wiedemann's Annalen*, 1878–1889.

† *Phil. Mag.*, Series 5, **23**, p. 287 (1887).

from studies of absolute magnitude and spectral type and more directly from the calculated densities of eclipsing variables. The confirmation that is most easily grasped is afforded by the recent interferometer measurements of the angular diameters of stars at Mount Wilson. These show that certain stars such as Betelgeuse, Antares and *o* Ceti, are of enormous bulk, capable of containing the whole orbit of the earth inside them. We are therefore compelled to extend our ideas of the nature of stars beyond anything that would be suspected from knowledge of the sun.

The great bulk of these giant stars is due to low density rather than great mass. Betelgeuse for example has a radius of the order 250 million km. and a volume 50 million times greater than the sun. But the mass, or amount of matter contained in it, is probably between 10 and 100 times greater, so that the density is about a million times less. It is rather interesting to notice that Einstein's theory of gravitation has something to say on this point. According to it a star of 250 million km. radius could not possibly have so high a density as the sun. Firstly, the force of gravitation would be so great that light would be unable to escape from it, the rays falling back to the star like a stone to the earth. Secondly, the red-shift of the spectral lines would be so great that the spectrum would be shifted out of existence. Thirdly, the mass would produce so much curvature of the space-time metric that space would close up round the star, leaving us outside (i.e. nowhere). The second point gives a more delicate indication and shows that the density is less than 0·001; for even at that density there would be a red-shift of the spectrum too great to be concealed by any probable Doppler effect.

Lest this argument should be regarded by our more conservative readers as ultra-modern, we hasten to add that it is to be found in the writings of Laplace—

> A luminous star, of the same density as the earth, and whose diameter should be two hundred and fifty times larger than that of the sun, would not, in consequence of its attraction, allow any of its rays to arrive at us; it is therefore possible that the largest luminous bodies in the universe may, through this cause, be invisible*.

7. For many years Lane's discovery had little effect on the accepted theories of stellar evolution. Sir Norman Lockyer accepted it and accordingly classified the stars in an ascending and descending temperature sequence; but he was almost alone in his views. Astrophysicists in general regarded the hottest stars as the earliest and the coolest stars as the latest in order of development†. Probably they did not realise that any of the

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* Laplace, *Système du Monde*, Book 5, Cp. VI. I am indebted to Dr H. Jeffreys for this reference.

† The expressions "early" and "late" type of spectrum are still commonly employed for high-temperature types (*B* and *A*) and low-temperature types (*K* and *M*) respectively.

ordinary types of spectrum could be produced in bodies diffuse enough to behave as a perfect gas, and supposed that Lane's theory, if it had any astronomical significance, must refer to some pre-stellar stage of development.

About 1913 a revolution of ideas occurred and the "Giant and Dwarf Theory" of E. Hertzsprung and H. N. Russell soon gained general acceptance. Setting aside certain misgivings which have arisen since 1924, we shall summarise the main points of the theory. In principle it was a revival of the ideas of Lane and Lockyer; the novel point was the adaptation of these ideas to the observational data, so that each star could be assigned its particular place in the scheme. The stars start to be visible as cool red stars of type *M* with low density and enormous bulk. They contract and in obedience to Lane's condition rise in temperature*, passing up the spectral series *K*, *G*, *F* to *A* and *B*—i.e. the reverse of the previously accepted order. At some stage of the contraction the density becomes too great for the perfect gas laws to apply, the rise of temperature is checked, and ultimately the star cools down again as a solid or liquid would do; in this last stage it returns down the spectral series to type *M* and ends in extinction. On this theory the stars which had been classed together indiscriminately as type *G*, for example, must be divided into two groups, the one making the ascent, the other on the descent, the one a nearly perfect gas, the other a very imperfect gas behaving similarly to a liquid. The surface conditions being similar, as evidenced by the spectral type, the outstanding distinction is that the ascending series or *giants* have much greater volume than the descending series or *dwarfs*. The greater volume and surface of the giant stars gives them greater luminosity, and when the absolute magnitudes are studied the division into two groups is easily seen. The separation is shown in the types *M*, *K*, *G* and *F*; it is not to be expected in type *A*, which marks the turning-point for most stars. Naturally it is most striking in type *M*, where the stars in the most diffuse and most concentrated state are brought into contrast; the one group clusters about absolute magnitude $+ 1^{m}\cdot 5$, the other about $+ 10^{m}\cdot 5$, and there is a clear gap of about 6^{m} in which no *M* star has yet been detected.

According to the statistics there is little or no change of absolute brightness with type along the giant series; this would be expected since the rising temperature and decreasing surface area will keep the total light about the same. In descending the dwarf series the decreasing temperature and decreasing surface combine to give a rapid falling off of brightness.

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* The theory applies to internal temperature, and it was generally taken for granted that the observed photospheric temperature would keep step; but this is by no means inevitable.

Much additional confirmation is obtained. The required bifurcation of density has been verified by the researches of Russell and Shapley on eclipsing variable stars. The sun and a number of other dwarf stars of type G have densities near that of water; but at least three eclipsing variables of type G are found to have densities less than that of air. There is evidence that this is not due to continuous range of density but is a definite bifurcation; intermediate densities belong to the higher types F, A, B which are traversed between the two stages of G. As already mentioned, the startling bulk ascribed by this theory to the giant stars has been verified by interferometer measurements.

The giants and dwarfs can now be distinguished by special differences in their spectra of a kind not considered in the Draper classification into types. This is a particular application of the spectroscopic method of determining absolute magnitude.

We shall find later that it is difficult to accept the giant and dwarf theory in its entirety. The ascending series presents no difficulty; but the descending series does not seem to be explicable in the manner that Lockyer, Russell and Hertzsprung supposed, because we now have evidence that the sun and other stars assigned to this branch behave as though constituted of perfect gas, notwithstanding that their densities are greater than water. In fact, the conditions in the stellar interior are such that the gas laws should continue to hold at much higher densities than under terrestrial conditions. The theory of stellar evolution is now in a very confused state, and the difficulties will be considered in due course.

8. The broad principles used by Lane in calculating the internal distribution of temperature have been followed in all later researches. We consider the case of a star composed of perfect gas. Then any one of the three variables, pressure (P), density (ρ), temperature (T), can be calculated from the other two by the law

$$P = \mathfrak{R}\rho T/\mu \quad \text{.........................(8·1)},$$

where $\mathfrak{R}$ is the universal gas constant $8{\cdot}26 \,.\, 10^7$ and μ the molecular weight in terms of the hydrogen atom. Thus effectively there are only two independent variables determining the state of the material. The differential equations satisfied by them are obtained by expressing two conditions: (1) the *mechanical equilibrium* of the star, which requires that the pressure at any internal point is just sufficient to support the weight of the layers above, and (2) the *thermal equilibrium* of the star, which requires that the temperature distribution is capable of maintaining itself automatically notwithstanding the continual transfer of heat from one part of the star to another. It is necessary to formulate and integrate the two equations expressing these conditions; and they suffice to determine the two independent variables specifying the condition of the material at any point.

Hence the distribution of pressure, density and temperature is found. The general scheme of distribution is (in the first approximation) homologous from star to star; that is to say, all gaseous stars copy the same model each on its own appropriate scale of mass, length, temperature, etc. The heavy work of the solution can be done once for all, and it is then only a question of adapting it to the scale of the particular star considered. We do not here enter into details; the problem is fully treated in Chapter IV.

In order to obtain definite numerical values of the temperature inside a star according to Lane's theory it was necessary to have the following data—

M the mass,
R the radius,
μ the mean molecular weight of the material,
γ the ratio of specific heats of the material.

The first two define the star under consideration; but we might suppose that the values of the last two in any star could only be guessed by considering the probable chemical composition of the interior—as to which we know practically nothing. We shall explain how this difficulty has been surmounted.

Actually the value of γ gave no serious trouble. It cannot exceed $\frac{5}{3}$, the value for a monatomic gas; and it cannot be less than $\frac{4}{3}$ without rendering the star unstable—which we know it is not. The difference in temperature distribution corresponding to the limits $\frac{5}{3}$ and $\frac{4}{3}$ is of some account; but there is no important change in its general character, and either limit gives an approximation good enough for many purposes. The constant γ, however, no longer concerns us. We shall abandon that part of Lane's theory responsible for its introduction, replacing Lane's hypothesis of *convective* equilibrium by *radiative* equilibrium. In all the earlier researches it was supposed that heat was carried from the interior to the surface of the star by convection currents, so that the interior was kept thoroughly stirred and followed the same law of thermal equilibrium as the lower part of the earth's atmosphere. But it appears now that the heat is transferred by radiation and the temperature distribution is controlled by the flow of radiation; convection currents, if they exist, will strive to establish a different distribution, but the temperature continually slips back to radiative equilibrium since the transfer by radiation is much more rapid. Radiative equilibrium was first adopted by R. A. Sampson[*] in 1894; but it could not be developed fully without the more recent progress of thermodynamics. K. Schwarzschild[†] brought it into prominence in a famous paper on the condition of the sun's atmosphere. Our task is to apply the same principle to the interior of the sun and stars.

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[*] *Memoirs R.A.S.*, 51, p. 123.

[†] *Göttingen Nachrichten*, 1906, p. 41.

With the substitution of radiative for convective equilibrium the constant γ disappears; its place is taken by the numerical constant $\frac{4}{3}$ which from one point of view can be regarded as the ratio of specific heats of the aether, aether having now replaced matter as the agent of transport of heat.

9. It remains to fix the appropriate value of μ, and it is necessary to do this with fair accuracy because μ is raised to a high power in many important formulae of the theory. We may assume that all chemical bonds are dissolved at the high temperature in the stellar interior, so that the atoms are isolated. Our first impulse is to adopt for μ the average atomic weight of the elements which we think likely to be most abundant. Since iron is often supposed to be the commonest element and is moreover an element of medium weight, a value about 50 is suggested. The author's first investigations* were made on this assumption. It was, however, suggested to him independently by Newall, Jeans and Lindemann that in stellar conditions the atoms themselves would break up to a considerable degree, many of the satellite electrons being detached.

The atom is often compared to a miniature solar system. Nearly all the mass is concentrated in a nucleus carrying positive charge; negative electrons of small mass, in number sufficient to balance the positive charge, describe orbits round the nucleus. At high temperatures a process known as ionisation occurs by which these satellite electrons are successively set free and travel about in the material as independent particles. The molecular weight μ appearing in our formulae (e.g. in (8·1)) is the average mass per independent particle. We use the term *molecule* to denote the particles moving independently of one another whether they are combinations of atoms or portions of atoms. If the ionisation is carried to the extreme limit a remarkable simplification occurs; *the molecular weight becomes approximately equal to* 2 *whatever the chemical composition of the material*, provided only that there is not an excessive proportion of hydrogen.

The number of satellite electrons is equal to the atomic number Z of the element, so that if all of them are set free there will be $Z + 1$ independent particles or molecules. Hence if A is the atomic weight

$$\mu = A/(Z + 1).$$

It is a well-known rule that the atomic number is about half the atomic weight, so that μ is near to 2. Some illustrations are given in Table 1.

Evidently the uncertainty of chemical composition is a much less serious matter when we realise that it is column 4 of the table which concerns us instead of column 3.

In the actual conditions of a star the ionisation is not quite complete, and for the heavier elements some of the satellite electrons remain undetached. This raises the molecular weight a little. It is now possible to

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* *Monthly Notices*, **77**, p. 16.