Bibliography and author index

The numbers in square brackets following each entry give the pages of this book on which the entry is cited.


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Motion picture index

Various films about solitons have been made. Some 16mm ones, which may be loaned or bought, are listed below. It is very instructive to see, in particular, computer-made animations of solitons and their interactions.

Eilbeck, J. C. (F1982). Kink Interactions in the φ⁴ Model. Silent, colour, 6 min.

The above films are also available on video. Enquiries about hiring or buying any of these should be made to Professor J. C. Eilbeck, Department of Mathematics, Heriot–Watt University, Riccarton, Edinburgh EH14 4AS, U.K.


The above film may be loaned from Bell Telephone Laboratories, Inc. Film Library, Murray Hill, New Jersey 07971, U.S.A.
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