Dual arguments have, in recent years, become standard tools for analysis of
problems involving optimization by consumers and producers. The principal aim
of this book is to provide a fairly systematic yet simple exposition of the basic
structure of such arguments. The emphasis is not on providing mathematically
general proofs; instead, a geometric approach is used to provide, in an informal
way, an intuitive understanding of duality theory. This book introduces the most
common alternative ways of representing preferences and technologies, such as
indirect utility and distance functions, expenditure and cost functions, and profit
and revenue functions, and it discusses the assumptions under which alternative
formulations contain precisely the same information. Results such as Roy’s identi-
tity, the Hotelling-Wold identity, and Shephard’s lemma are fully explained, as
are their roles in facilitating analysis of behavior.

In addition to his exposition of duality, Professor Cornes discusses the several
applications in which dual arguments have proved particularly helpful. These
include the modeling of quantity-constrained choice, productive efficiency and
welfare measurement, index numbers, aggregation theory, externalities, and pub-
lic goods theory. At various points, the usefulness of dual representations in
providing a richer menu of econometric specifications is stressed.
Duality and modern economics

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Preface

Since the early 1970s, the use of dual techniques has become widespread among economists. Familiarity with basic duality theory is now beginning to be taken for granted among graduate students, whether their interest is in economic theory or in empirical applications of that theory. In view of this, the paucity of simple introductory expositions of these techniques is both surprising and disappointing. The existing microeconomic textbooks understandably mix duality in with a lot of other material, which is fine for the current generation of graduate students, but inconvenient for those who have a strong orthodox microeconomic background and who wish to catch up with recent developments in the techniques of modeling. In addition, the textbook treatment typically is limited to presenting a few basic definitions and standard results, together with a demonstration of how the new techniques can facilitate the derivation of well-known standard results. There is little indication of the potential attraction of duality as part of a graduate student’s research kit, and therefore little motivation to acquire more than a rudimentary familiarity with dual formulations.

My aim in this book is to provide a brief and informal introduction to the fundamental ideas lying behind duality theory and to show not only the principal results but also how they can be applied to a variety of economic problems. The emphasis is on geometric and economic intuition rather than on rigorous development of general results. Part III, in particular, shows that dual formulations have further uses in addition to the establishment of downward-sloping compensated demand curves, upward-sloping supply curves, and symmetric compensated substitution responses. I hope that readers of this book will subsequently be able to read theoretical and empirical research without being fazed by the presence of dual methods of analysis and will also have the interest, competence, and confidence to exploit dual formulations in their own work.

During the long gestation period of this book, I have incurred a number of debts that I am happy to acknowledge.
Preface

In 1977, my colleague Robert Albon introduced me to some of the problems of modeling quantity-constrained choice in the context of work that he was doing on rent control. It was in the course of thinking about these problems that I became interested in the power of dual techniques for handling quantity constraints and in their application to externalities and public goods. Since then I have learned much from collaborative work with Avinash Dixit, Masaaki Homma, Ted Bergstrom, Frank Milne, and, over a prolonged period, Todd Sandler. Both Frank Milne and Todd Sandler have read parts of this manuscript and have made many helpful and encouraging comments, as have Edward Greenberg and Norman Ireland, who were long ago asked by the publishers to read an early version of the manuscript. My colleague Ngo Van Long has also helped to eliminate some sloppy and inaccurate statements from earlier drafts. Patricia Apps made some valuable comments on my treatment of econometric applications and drew my attention to a number of significant references. I want to express special thanks to Ray Rees, who, during a brief visit to the Australian National University, made many helpful and constructive suggestions. I appreciate having a reader who is both sufficiently interested to spend as much time as he did reading another author’s manuscript and also sufficiently bold to advise me, gently but firmly, to tear up the initial draft of a whole chapter. Rolf Färe also gave both time and comments generously during his visit to the Australian National University. His contributions significantly improved my treatment of the logical foundations of duality.

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