1 Fundamental principles

Introduction

In this opening chapter we introduce a variety of topics and basic principles, related to the mechanics of development of geological structures, which will be used throughout the book. As all the various aspects introduced cannot be treated in detail in an introductory chapter, some will be considered in greater detail later in appropriate chapters.

The elements of force and stress, which play such important roles in the mechanics of the development of geological structures, are considered at some length in the first section of this chapter. The derivation of the various stress equations is presented in some detail, so that the reader should quickly become familiar with the concepts used.

The second section is given to a relatively brief exposition of strain which, in the third section, leads to a treatment of stress-strain relationships. In this third section, Linear Elastic theory is dealt with at some length, but Viscosity and Plasticity theory receive relatively brief mention. The topic of fluids in the Earth's crust is then introduced and the importance of the concept of effective stress is then applied to criteria of brittle failure in compressive and also in tensile stress conditions. This is followed by an introduction to certain pertinent aspects of rock mechanics and the way in which various environmental factors and parameters influence the differential stress which can be supported by rock. The final section of the chapter is given to a very brief description of the terms and concepts relating to the various mechanisms of crystal deformation involved in the deformation of rock masses.

Forces and stress

Thermally and gravitationally activated movements within the mantle and crust are the prime causes of the force and stress fields which result in the development of folds, faults and minor structures of various kinds. In order to understand the mechanical processes which give rise to these events, the student of structural geology must have some grasp of the concepts of force and stress. The treatment given in this chapter is extremely elementary and most of the necessary steps in the mathematical arguments are presented in some detail. By such treatment it is hoped that those readers whose natural inclination is to 'skip the maths' may be induced to overcome their aversion and discover how simple these concepts are.

**Force**

Force is usually defined as any action which alters, or tends to alter, a body's state of rest or uniform motion in a straight line. When a force acts on a body it can be specified completely if one knows (i) its direction of action in space and (ii) its magnitude (Fig. 1.1(a)). Consequently, it is a *vector quantity*.

The magnitude of a force is measured by its effect. Thus, one can measure a force by the weight it will support. In dynamics, which is that aspect of mechanics which deals with motion, the magnitude of a force is measured by the motion it will induce in a given time. This is best seen from Newton's Second Law of Motion which states that 'rate of change of momentum is proportional to the force applied and takes place in the direction in which the force acts'. Thus, if a force \( F \) acts on a mass \( M \) there is a proportional change in momentum (where momentum is \( Mv \) and \( v \) is the velocity). As the mass does not
change when a force is applied, the applied force produces a change in velocity \( (F) \), i.e. it produces an acceleration \( (a) \) so that the Second Law of motion can be written as:

\[
F = \text{const} \, Ma.
\]

It is convenient to choose units of force such that the constant in this equation equals unity; i.e. \( F = 1.0 \) when \( M = 1.0 \) and \( a = 1.0 \). This fundamental equation then becomes:

\[
F = Ma. \tag{1.1}
\]

When the unit of mass is the gram, and the units of space and time are the centimetre and the second respectively, the force unit is known as the dyne, which is termed an absolute unit because its value is not dependent upon gravitational attraction. Alternatively, if the acceleration owing to Earth’s gravity is defined as \( g = 9.81 \, \text{m s}^{-2} \), the force exerted by a mass of 1 kg at rest on Earth is 9.81 Newtons.

If two forces act at a single point then, because they are vector quantities, they may be combined graphically (Fig. 1.1(b)) by the parallelogram of forces. Similarly, a single force may be resolved into two or more components. There are, of course, an infinite number of ways in which this may be done, but in most analyses it is necessary, or convenient, to resolve the force into two directions at right angles to one another. Such an example is indicated in Fig. 1.2, in which a rectangular particle, of mass \( M \), is resting on a planar surface inclined at an angle \( \phi \) to the horizontal. The force \( F \) generated by this mass \( (M) \) in the gravitational field acts vertically as indicated. The component of tractive force \( (F_t) \) which acts parallel to the inclined surface is clearly given by:

\[
F_t = F \sin \phi. \tag{1.2}
\]

Similarly, the component \( (F_c) \) acting normal, or perpendicular, to the slope is:

\[
F_c = F \cos \phi. \tag{1.3}
\]

If the angle \( \phi \) is small, the component of traction is also small and the particle will not slide down the slope, because of the resistance to movement provided by a frictional force. If, however, the angle \( \phi \) is gradually increased, \( F_t \) also increases, while \( F_c \) decreases. When the angle \( \phi \) reaches the critical value \( (\phi_c) \), the frictional resistance to movement is overcome and the particle slides down the slope. This critical angle is a characteristic of the material, or materials, of which the particle and slope are made. It has been found by experiment, that when one body is in contact with another along a planar surface and these are moved laterally relative to one another, the frictional force tending to prevent motion is proportional to the normal reaction, or force, which acts on the surface along which sliding takes place. This constant ratio is termed the coefficient of dynamic or sliding friction \( (\mu) \) and for the given example, shown in Fig. 1.2, is obtained from Eqs. (1.2) and (1.3), so that:

\[
\mu = F_s/F_n = F \sin \phi / F \cos \phi = \tan \phi. \tag{1.4}
\]

As we shall see later, this concept of sliding friction plays an important role in the mechanics of fracture and fault movement.

**Stress**

If a cube of granite with sides of 25 cm is submitted to an evenly distributed, compressive force of 10 tonnes (i.e. 10,000 kg) only infinitesimal deformation (strain) would be observed in the cube. If, however, the same load were applied to a cube of the same material with sides of one twentieth the length of the larger cube, the smaller granite cube would be pulverised by the action of the force. The magnitude of the applied force is the same in both instances and the difference in the behaviour of the two cubes is the result of the different stresses induced by the force, where the term stress \((S)\) is defined as the force \((F)\) per unit area \((A)\), or:

\[
S = F/A. \tag{1.5}
\]

With reference to the example given above, the stress in the larger cube was 16 kg cm\(^{-2}\), i.e. about 16 atmospheres of pressure (16 bar or 1.6 megapascals (MPa)), while in the smaller cube it was 400 times larger at 6400 kg cm\(^{-2}\), or 64 kbar, which greatly exceeds the uniaxial crushing strength of granite.

In the example cited, it was tacitly assumed that the direction of application of force was perpendicular to the surface of the cube, so that there was no component of force acting parallel to the loading surfaces. The corresponding stress which acts perpendicular to a surface is defined as a principal stress when the shear stress acting on that surface is zero.

If only one principal stress acts on a body, as indicated in Fig. 1.3(a), this is termed uniaxial compression. When two or three principal stresses act, as shown in Fig. 1.3(b) and (c), the conditions are called biaxial and triaxial compression respectively.

Fig. 1.2: Vertical, layer normal and layer parallel components of force of a mass, \( M \), on a rigid, inclined plane at an angle \( \phi \).
shown later that the axes of the principal stresses, i.e. the directions in which the principal stresses act, are always at right angles to one another.

If one considers a plane within a body which is inclined to the direction of the applied force, or principal stress (Fig. 1.4), it is apparent that the force \( F \) has a component acting normal to, and also one parallel to, the internal plane. There are corresponding normal and shear stresses which are designated \( S_n \) and \( \tau \) respectively.

It is often convenient to refer stresses to a coordinate system in which one of those coordinates is vertical and the other two are in a horizontal plane. Convention has it that the suffix \( z \) is used to indicate the vertically-acting stress \( S_z \). The other stresses, oriented parallel to the \( x \) and \( y \) directions are then \( S_x \) and \( S_y \). It is emphasised that, when using this terminology, \( S_z, S_x \) and \( S_y \) need not be, and in general are not, principal stresses. Shear stresses may also be referred to the coordinate system. The nomenclature used in a biaxial field is indicated in Fig. 1.5. Taken together, the two subscripts indicate the plane in which the shear stresses act; while the last of the subscripts indicates their direction of action.

Some of the earliest tests of materials were conducted on specimens (usually metal) which were subjected to a tensile force, thereby causing an elongation of the test specimen. The extension, and hence the strain, was considered to be a positive quantity. It therefore became the convention among engineers and physicists to regard the tensile stresses associated with this ‘positive’ strain to be positive also. Conversely, compressive stresses were considered to be negative quantities. Geologists, however, tend to use the opposite convention for, they argue, stresses in the

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**Fig. 1.3.** Uniaxial, biaxial and triaxial compression; (a), (b) and (c) respectively.

**Fig. 1.4.** Normal and shear forces on an internal plane within a cube subjected to uniaxial compression by force \( F \). \( a = \) area of cube face and \( a = \) area of internal plane.

**Fig. 1.5.** Nomenclature for normal and shear stress in biaxial compression in the \( x \) and \( y \) directions.
4 Analysis of geological structures

Fig. 1.6. (a) Sign convention for shear stresses when compressive stresses are positive. (b) Moments induced in an element by shear stresses.

Moment = Force × Distance

Clockwise moment about centre O

\[ M_O = \tau \times \text{area} = \pi a^2 \]

Anticlockwise moment about centre O

\[ M_O = \tau \times \text{area} = \pi a^2 \]

Earth’s crust is usually compressive and it is more convenient to deal with positive quantities. These sign conventions refer specifically to normal stresses. There are also corresponding sign conventions regarding shear stresses. The signs for shear stresses, when compressive normal stresses are taken as positive (the convention used throughout this book) are indicated in Fig. 1.6(a).

In analysing mechanical processes pertaining to the development of geological structures, it is usual to assume, when dealing with stresses, that strain is rotational or that an element is rotating so slowly that one may consider it to be rotational. If the sides of the element (Fig. 1.6(b)) have a length \( a \) and there is a shear stress \( \tau \) acting on all sides, the shear force on sides \( AB \) and \( CD \) has a magnitude \( \tau \times a \), and because these forces act at a distance \( a \) from each other they form a couple of \( \tau a^2 \), which tends to rotate the element in a clockwise direction. However, the same argument may be applied to the pair of shear stresses \( \tau \), which provide a couple \( \tau a^2 \) which tends to rotate the element in an anticlockwise direction. But, as it is assumed that the element is in equilibrium, \( \tau a = -\tau a \). Thus, in two-dimensional problems, only three stresses \( S, S_n, \) and \( \tau \) are required to define completely the stress system acting on any two-dimensional element.

It can be readily shown that if \( S, S_n, \) and \( \tau \) acting on two surfaces are known, then it is possible to calculate the orientation and magnitude of the two principal stresses. Consider a small, triangular prism of unit thickness and lengths of sides \( a’, b’, \) and \( c’ \), as shown in Fig. 1.7, with \( S_n, S_1, \) and \( \tau \) acting perpendicular to sides \( a’ \), \( b’ \), and \( c’ \) respectively. Suppose that side \( c’ \) makes an angle \( \theta \) with \( b’ \) and that the stress \( S \), acting on \( a’ \) is wholly normal, then \( S_1 \) is a principal stress. Because the thickness of the prism is unity, the areas of the sides are respectively \( a, b, \) and \( c \). Moreover, it will be evident from Eq. (1.5) that if one multiplies a stress by an area, the resulting quantity is a force (which one can consider to act at the centre of the area). Hence, the force acting on face \( b’ \) is \( S_1 b \). Now, both the horizontal and vertical components of the force action on the prism must be in equilibrium. Equating the horizontal components one obtains:

\[ S_n \sin \theta = S_1 a + \tau \]

or

\[ S_n \sin \theta = S_1 a/b + \pi/b. \]

But \( a/b = \sin \theta \) and \( c/b = \cos \theta \), so:

\[ S_n \sin \theta = S_1 \sin \theta + \cos \theta \]

Therefore,

\[ S_n - S_1 = \tau \cot \theta. \quad (1.6) \]

Equating the vertical components one obtains:

\[ S_n \cos \theta = S_1 c + \pi \]

or

\[ S_n \cos \theta = S_1 c/b + \pi/b. \]
so that
\[ S_y \cos \theta = S_x \cos \theta + \tau \sin \theta. \]

Therefore:
\[ S_y = S_x \tan \theta. \]  \hspace{1cm} (1.7)

If one multiplies Eq. (1.6) by Eq. (1.7) (remembering that \( \tan \theta \cot \theta = 1 \)), one obtains:
\[ \tau = (S_x - S_y)(S_y - S_z) \]
or, by multiplying out and rearranging the terms:
\[ S_y^2 - (S_x + S_z)S_y + (S_y - \tau) = 0. \]  \hspace{1cm} (1.8)

Eq. (1.8) is a quadratic in \( S_y \) and the two roots may be obtained in the usual manner, so that:
\[ S_y = \frac{[(S_x + S_z) \pm \sqrt{(S_x + S_z)^2 + 4\tau^2}]}{2}. \]  \hspace{1cm} (1.9)

Thus, provided the roots are real, the values of the two principal stresses can be calculated.

In order to ascertain the orientation of these principal stresses, it is necessary to determine the two corresponding values of \( \theta \). To obtain the required relationship between \( \theta \) and the stresses, subtract Eq. (1.6) from Eq. (1.7), so that:
\[ S_y - S_z = \tau \cot \theta - \tan \theta. \]

It can be shown by simple trigonometrical manipulation that:
\[ \cot \theta - \tan \theta = 2 \cot 2\theta \]
therefore:
\[ S_y - S_z = 2 \cot 2\theta \]
or
\[ \cot 2\theta = \frac{(S_y - S_z)}{2\tau}. \]  \hspace{1cm} (1.10)

Also, from trigonometrical relationships it is known that
\[ \cot 2\theta = \cot(2\theta - 180^\circ) \]
and from Eq. (1.10) it follows that:
\[ \cot(2\theta - 180^\circ) = \frac{(S_z - S_x)}{2\tau}. \]  \hspace{1cm} (1.10a)

Therefore, the stress data give rise to two angles \( \theta \) and \( \theta - 90^\circ \) which define the orientation of the principal stresses.

Thus, knowing \( S_x, S_y \), and \( \tau \), the values and orientations of the corresponding principal stresses can be calculated. Alternatively, if the orientation and magnitude of the principal stresses are known, one can easily calculate the values of the normal and shear stresses acting on any plane which makes an angle \( \theta \) with the axis of principal stress.

Consider first the uniaxial compression of a rectangular prism, represented in Fig. 1.8. A force \( F \) acts perpendicular to the end surfaces of the prism, which has a cross-sectional area \( A \), so that the principal stress \( (S_z) \) is given by:
\[ S_z = F/A. \]

Now consider an internal surface, GHIJ, which is oriented so that it makes an angle \( \theta \) with the axis of principal stress. It will be seen that the force \( F \) has a component \( F_z \) which is normal to the internal plane, so that:
\[ F_z = F \sin \theta. \]

Similarly, the force \( F \) has a component of traction \( F_t \), parallel to the surface, where:
\[ F_t = F \cos \theta. \]

It is clear from Fig. 1.8 that the area \( A' \) of the internal plane is greater than the area \( A \) of the end sections of the prism and that:
\[ A' = A/\sin \theta. \]

Using these equations, it is clear that the normal stress \( (S_z) \) acting on the internal plane is given by:
\[ S_z = F_z/A' = F \sin^2 \theta/A = S_z \sin^2 \theta \]
therefore
\[ S_z = S_z \sin^2 \theta. \]  \hspace{1cm} (1.11)

Similarly, the shear stress \( (\tau) \) on this same plane is given by:
\[ \tau = F_t/A' = F \sin \theta \cos \theta/A = S_z \sin \theta \cos \theta \]
therefore,
\[ \tau = S_z \sin \theta \cos \theta. \]  \hspace{1cm} (1.12)

These uniaxial stress equations (Eqs. (1.11) and (1.12)) give the normal and shear stress on any plane inclined at an angle \( \theta \) to the principal stress.
Consider now the condition of biaxial compression (Fig. 1.9). The normal stress \( S_n \) acting on the internal surface is compounded from two components of normal stress as the result of \( S_1 \) and \( S_2 \). From Eq. (1.11), the component of normal stress that results from \( S_2 \) (designated as \( S_{2n} \)) is:

\[
S_{2n} = S_2 \sin^2 \theta.
\]

Similarly, the component of normal stress \( S_{1n} \) that results from \( S_1 \) is:

\[
S_{1n} = S_1 \sin^2(90^\circ - \theta) = S_1 \cos^2 \theta.
\]

Hence,

\[
S_n = S_1 + S_2 = S_1 \cos^2 \theta + S_2 \sin^2 \theta. \tag{1.13}
\]

If the principal stresses are both compressive and \( S_n \) is greater than \( S_1 \), then it will be seen that the component of shear stress on the internal plane that result from these two principal stresses will have a different sense. The total shear stress, therefore, will be:

\[
\tau = -\tau_n,
\]

Therefore, from Eq. (1.12),

\[
\tau = (S_1 - S_2) \sin \theta \cos \theta. \tag{1.14}
\]

These two equations (Eqs. (1.13) and (1.14)) are the biaxial stress equations.

As we shall see, it is often convenient to express Eqs. (1.13) and (1.14) in terms of the double angle \( 2\theta \).

It can be shown from elementary trigonometry that:

\[
\sin \theta \cos \theta = \frac{\sin 2\theta}{2}.
\]

Consequently, Eq. (1.14) may be written as:

\[
\tau = \frac{(S_1 - S_2)}{2} \sin 2\theta. \tag{1.15}
\]

It can also be shown that:

\[
\cos^2 \theta - \sin^2 \theta = \cos 2\theta
\]

and

\[
\cos^2 \theta + \sin^2 \theta = 1.
\]

Therefore,

\[
\cos^2 \theta = \frac{1 + \cos 2\theta}{2}
\]

and

\[
\sin^2 \theta = \frac{1 - \cos 2\theta}{2}.
\]

Substituting these last two expressions in Eq. (1.13), gives:

\[
S_n = \frac{1}{2}[(S_1 - S_2) \cos 2\theta + (S_1 + S_2) \cos 2\theta]
\]

or

\[
S_n = \frac{(S_1 + S_2)}{2} - \frac{(S_1 - S_2)}{2} \cos 2\theta. \tag{1.16}
\]

Eqs. (1.15) and (1.16) are particularly important because they lend themselves to graphical solutions of stress problems by the use of a technique developed by Otto Mohr.

Mohr expressed the stress equations (Eqs. (1.15) and (1.16)) graphically by plotting shear stress against normal stress. Knowing the magnitude of the principal stresses, the normal and shear stresses on any plane, with values of \( \theta \) between 0° and 180°, can be determined using these equations. If the normal and shear stresses for all values of \( \theta \) are plotted, they form a circle, known as the Mohr’s stress circle, Fig. 1.10(a). Because there are no shear stresses acting on a surface perpendicular to an axis of principal stress, the stresses \( S_1 \) and \( S_2 \) plot on the \( S_n \) axis, at points \( A \) and \( B \) respectively. Clearly, the distance from the origin \( O \) to the mid-point \( C \) between \( A \) and \( B \) is \( S_1 + S_2)/2 \) and the radius of the Mohr’s circle \( BC = (S_2 - S_1)/2 \). The magnitude of \( \tau \) is represented by \( XP \). It will be seen that \( XP/XC = \sin 2\theta \). It will also be noted that \( XC = (S_1 - S_2)/2 \), therefore the value of \( \tau \) is as given by Eq. (1.15). Similarly, the magnitude of \( S_2 \) equals \( CO \) minus \( CP \), where:

\[
CO = \frac{(S_1 + S_2)}{2}
\]

and

\[
CP = \frac{(S_1 - S_2)}{2} \cos 2\theta.
\]

Thus the value of \( S_1 \) is that given by Eq. (1.16). Hence, knowing the two principal stresses, points \( A \) and \( B \) on the diagram enables the Mohr’s circle to be con-
Fig. 1.10. (a) The Mohr’s circle: a graphical representation of Eqs. (1.15) and (1.16). See text. (b) Construction of the Mohr’s circle from values of normal and shear stress acting on two surfaces of given orientation, see text. (c) Diagrammatic representation of Eq. (1.13), known as the stress ellipse.

structured, and the normal and shear stresses acting on any plane making an angle $\theta$ with the maximum principal stress can be determined simply and quickly. Conversely, if the values of the normal and shear stresses acting on two surfaces of given orientations are known, it is possible to determine the values and orientations of the related principal stresses. Thus, the data for surface EF of the inset diagram shown in Fig. 1.10(b) are plotted (let this define point Y) and the data for surface FG plot at point Z. (It should be noted that the shear stresses on EF and FG are of opposite sign.) Connect points Y and Z, and where this line cuts the $S_3$ axis, at C, one has the point equivalent to $\frac{1}{2}(S_1 + S_3)$. Also CY = $\frac{1}{2}(S_1 - S_3)$. Hence, if a circle of radius CY is drawn, the values of $S_1$ and $S_3$ can be read off. The orientation of the axis of maximum principal stress relative to surface EF can be determined by measuring angle OCY ($2\theta$) and taking half the measured angle.

The properties of stress, and the implication of the stress equations, can be further understood by determining the values of normal stress of various values of $\theta$, using Eq. (1.13) and representing this quantity by an arrow, pointing in the appropriate direction, whose length is proportional to its magnitude. The two principal stresses are at right angles and correspond to the maximum and minimum values of normal stress. As can be seen from Fig. 1.10(c), the normal stress vectors define an ellipse, the stress ellipse (ellipsoid in 3-D) whose axes coincide with the principal stresses.

If the principal stresses are equal the resulting situation is sometimes termed a hydrostatic stress, because it is the stress experienced by a fluid at rest (however, the term is also widely used to describe the corresponding stresses in a solid). It is often convenient to consider a non-hydrostatic stress as being made up of two parts. These are the mean stress ($S$) and the deviatoric stress ($S''$). The mean stress is defined as:

$$S = \frac{(S_1 + S_2 + S_3)}{3}$$

and, because it is only responsible for volumetric changes, can be considered to be the hydrostatic part of the stress system. The deviatoric stress is defined as:

$$S'' = S_3 - S'$$

and is a measure of how much the normal stress in any direction deviates from the mean, or hydrostatic, stress. The deviatoric stress is responsible for distortional deformation. A useful indication of a stress field’s ability to cause deformation is the difference in magnitudes between the maximum and minimum principal stresses ($S_1 - S_3$), which is known as the differential stress.

Strain

There are a number of reasons why it is useful to determine the state of strain in rocks. For example, strain determination can be used to assess the original stratigraphic thickness of a deformed sedimentary sequence, or to calculate the amount of displacement across a shear zone. Moreover, by studying deformed
objects of known original shape, strain determination may enable one to establish the amount of deformation necessary for the formation of various rock fabrics such as slaty cleavage. In addition, if the strain distribution within and around a geological structure, such as a fold, is known, it can be compared with the strain pattern predicted from, or assumed to exist in, a particular theoretical treatment of folding. The relevance of the theory to the formation of that fold can thereby be assessed.

When a rock is subjected to stress, the particles of the rock are displaced. The types of displacement fall into four categories; two rigid body displacements and two non-rigid body displacements. These are:

(i) rigid body translation,
(ii) rigid body rotation,
(iii) distortion (shape change), and
(iv) volume changes (Fig. 1.11).

Rigid body translation is the movement of a body through space without any change of shape of that body and in such a way that any line drawn on the body maintains the same orientation throughout the displacement (Fig. 1.11(a)). The displacements parallel to the x and y axes are termed u and v respectively and the resultant displacement vector \( R \) is the same for all parts of the body.

Rigid body rotation also involves the movement of the body through space without any change in shape of the body, but the displacement vectors of all points on the body are not the same and there is a single "stationary" point about which the body rotates (Fig. 1.11(b)).

Distortion involves the movement of the particles with respect to each other and causes a change in shape of the body (Fig. 1.11(c)). The displacements of the particles are not the same and displacement gradients are set up, the magnitude of which gives a measure of the distortion or strain. For example, the strain \( \epsilon_x \) (extension) in the x direction is defined as a displacement gradient:

\[
\epsilon_x = \frac{\partial u}{\partial x} \tag{1.17}
\]

Volume changes cause no change in shape, and although often termed dilation, can be either positive (Fig. 1.11(d)) or negative.

Displacement in rocks generally involves all four types but frequently one type of behaviour is dominant. For example, the movement on an overthrust may be predominantly rigid body translation but locally, at or near the decollement horizon, considerable distortion of the rock mass can occur. Taking another example, volume changes dominate during compaction and the dewatering of sediments associated with burial and diagenesis.

All the types of displacements shown in Fig. 1.11, can be represented mathematically, singly or in combination, by displacement or transformation equations which relate the undisplaced body to the displaced body by giving new coordinates for a point \((x', y')\) in terms of the old coordinates \((x, y)\). For example, the transformation equations for rigid body translation are:

\[
x' = x + u \tag{1.18}
\]
\[
y' = y + v \tag{1.19}
\]

when \( u \) and \( v \) are constants.

Generally, it is not possible to determine the exact amount of rigid body displacement that a rock has experienced since its formation. However, if it contains objects of known, original shape or size, then the strain in the rock can be measured precisely. A considerable amount of effort has been directed towards determining the state of strain in rocks, and for a more detailed discussion of strain and techniques of measuring it, the reader is referred to books by...
Ramsay (1967), Means (1976) and Ramsay & Huber (1983). The following brief discussion is presented to cover aspects of the subject commented on and used in this book.

Strain is homogeneous where it is constant throughout a body and heterogeneous where the displacement-gradient varies across a body (Fig. 1.12). Although strain in rocks is generally non-uniform, it is possible to consider a small domain over which the strain is approximately homogeneous. By determining the states of strain in these domains, the more complex, heterogeneous strain state of a region can be inferred. The size of the domains over which the strain can be considered to be homogeneous varies enormously. For example, in a slate belt the domain may extend over many kilometres, whereas in a folded region, the domains may be only a few metres, or even less, in extent.

The sub-division of a heterogeneous strained region into smaller domains across which the strain is approximately homogeneous considerably simplifies the problem of strain determination. This problem can be simplified even further by considering strain in only two dimensions, i.e., on a rock face. The state of three-dimensional strain can be subsequently determined by combining two-dimensional strain data from three non-parallel faces.

During two-dimensional, homogeneous strain, straight lines remain straight, parallel lines remain parallel and a circle is deformed into an ellipse, the strain ellipse (Fig. 1.13). However, the length of a line and the angle between two lines generally change. Changes in angle indicate the amount of shear deformation in a particular direction and changes in length give a measure of the elongation in that direction. A variety of parameters have been used to measure these changes.

Changes in the length of a line
There are several parameters used to measure changes in the length of a line and each has its own advantage. The most commonly used is the extension (ε), which is defined as:

$$\epsilon = \frac{l_i - l_0}{l_0} = \frac{dl}{l_0}$$

(1.20)

where $l_0$ and $l_i$ are the original and final length of a line. A more complex parameter used to measure length changes is the quadratic elongation:

$$\lambda = (1 + \epsilon)^2$$

(1.21)

As we shall see, this parameter is used because it enables the strain equations (Eqs. (1.31) and (1.32)) to be expressed more simply.

Natural or logarithmic strain ε defined as:

$$\epsilon = \log(1 + \epsilon)$$

(1.22)

gives a measure of the change in length of a line. Ramsay (1967) points out that Eq. (1.22) expresses the changes in length more realistically than ε. For example, in a comparison of the two parameters along two lines, one of which is contracted to half its original length and the other expanded to twice its original length, the extensions, ε, of the two lines are: $\epsilon = -0.5$, and 1.0 respectively, whereas the natural strains are $\epsilon = -\log_22$ and $\epsilon = +\log_22$. With very great contractions, ε approaches $-1$, whereas $\epsilon$ approaches $\infty$.

Changes in the angle between lines
The change in angle between two lines that originally intersected at right angles is known as the angular shear ($\phi$) (Fig. 1.15) and the shear strain γ is defined as:

$$\gamma = \tan \phi$$

(1.23)

The angular shear and shear strain along a line may be either positive or negative depending upon whether
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the original normal to the line is deflected to the right (Fig. 1.15) or left respectively.

Two types of homogeneous strain that have been studied extensively by structural geologists are pure shear and simple shear. Extension parallel to the x axis involving no area change (Fig. 1.14) is an example of pure shear and the transformation equations representing this deformation are:

\[
x' = (1 + \varepsilon_x)x, \quad (1.24)
\]

\[
y' = (1 + \varepsilon_y)y, \quad (1.25)
\]

where \(\varepsilon_x\) and \(\varepsilon_y\) are related by \(1/(1 + \varepsilon_x) = 1/(1 + \varepsilon_y)\).

The deformed state (Fig. 1.14(b)) is a state of finite strain which can be considered as being made up of a large number of incremental or infinitesimal strains (Fig. 1.14). The circle representing the undeformed state is deformed into an ellipse, the strain ellipse, whose major and minor axes indicate the magnitude and orientation of the maximum and minimum principal strains. During pure shear deformation, the axes of the strain ellipse do not rotate and the incremental and finite strain ellipses are coaxial. Such a deformation is known as an irrotational deformation. Simple shear results when a body is subjected to a uniform shear, parallel to some direction, involving no area change (Fig. 1.15) and the transformation equations that describe this deformation are:

\[
x' = x + y\tan \phi, \quad (1.26)
\]

\[
y' = y, \quad (1.27)
\]

It can be seen from Fig. 1.15 that the axes of the strain ellipse rotate during simple shear. Such a deformation is known as a rotational deformation and except during the first increment of strain, the incremental and finite strain ellipses are non-coaxial.

The two sets of transformation equations (Eqs. 1.24 to 1.27) represent two specific examples of two-dimensional, homogeneous strain. They are both examples of the more general transformation equations:

\[
x' = ax + by \quad (1.28)
\]

\[
y' = cx + dy \quad (1.29)
\]

which represent any two-dimensional homogeneous strain.

Although it can be argued that geological deformations generally involve area changes (volume in 3-D) and that therefore the models of pure and simple shear deformations are geologically unrealistic, field observations show that deformation closely related to these two types of deformation (i.e. homogeneous flattening involving contraction in one direction and extension at right angles to this direction and localised shear deformation) are extremely common. For example, the deformation within large areas of some slate belts approximates very closely to homogeneous flattening. Alternatively, the deformation in other areas is found to be localised along bands of high shear strain. The development of ‘shear’ instabilities in rocks is discussed extensively in this book in the chapter on faulting and folding and it is instructive to consider the properties of simple shear in a little more detail.

Although we are interested in the effects of simple shear on homogeneous, isotropic material, the properties of this deformation can be studied conveniently with the aid of card deck models (Fig. 1.16). After the first increment of shear parallel to the cards, the original, undeformed circle is transformed into an ellipse whose axes are inclined at 45° to the shearing direction (Fig. 1.16(b)). The amount of shear strain