Algorithmic graph theory

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To my children
Gabrielle, Chantal and Rosalind,
with love.

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Preface

In the last decade or so work in graph theory has centred on algorithmic interests rather than upon existence or characterisation theorems. This book reflects that change of emphasis and is intended to be an introductory text for undergraduates or for new postgraduate students.

The book is aimed primarily at computer scientists. For them graph theory provides a useful analytical tool and algorithmic interests are bound to be uppermost. The text does, however, contain an element of traditional material and it is quite likely that the needs of a wider audience, including perhaps mathematicians and engineers, will be met. Hopefully, enough of this material has been included to suggest the mathematical richness of the field.

Prerequisites for an understanding of the text have been kept to a minimum. It is essential however to have had some exposure to a high-level, procedural and preferably recursive programming language, to be familiar with elementary set notation and to be at ease with (for example, inductive) theorem proving. Where more advanced concepts are required the text is largely self-contained. This is true, for example, in the use of linear programming and in the proofs of *NP*-completeness.

There is rather more material than would be required for a one-semester course. It is possible to use the text for courses of more or of less difficulty, or to select material as it appeals. For example an elementary course might not include, amongst other material, that on branchings (in chapter 2), minimum-cost flows (in chapter 4), maximum-weight matchings (in chapter 5), postman problems (in chapter 6) and proofs of *NP*-completeness (all of chapter 8). Whatever the choice of material, any course will inevitably reflect the main preoccupation of the text. This is to identify those important problems in graph theory which have an efficient algorithmic solution (that is, those whose time-complexity is polynomial in the problem

size) and those which, it is thought, do not. In this endeavour the *most* efficient of the known polynomial time algorithms have not necessarily been described. These algorithms can require explanations that are *too* lengthy and may have difficult proofs of correctness. One such example is graph planarity testing in *linear*-time. It has been thought preferable to go for breadth of material and, where required, to provide references to more difficult and stronger results. Nevertheless, a body of material and quite a few results, which are not easily available elsewhere, have been presented in elementary fashion.

The exercises which appear at the ends of chapters often extend or motivate the material of the text. For this reason outlines of solutions are invariably included. Some benefit can certainly be obtained by reading these sections even if detailed solutions are not sought.

Thanks are due to Valerie Gladman for her cheerful typing of the manuscript. Primary and secondary sources of material are referenced at the ends of chapters. I gratefully acknowledge my debt to the authors of these works. However, I claim sole responsibility for any obscurities and errors that the text may contain.

A. M. Gibbons *Warwick*, January 1984