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K. E. Bullen and Bruce A. Bolt

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**An introduction to the  
theory of seismology**

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# An introduction to the theory of seismology

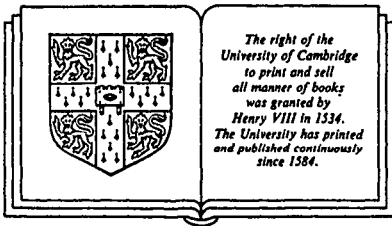
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*To*  
*Sir Harold Jeffreys*

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## Preface

About sixty per cent of this edition is new material. Changes have been introduced throughout the text to shorten and up-date discussions and explanations and to give more emphasis to modern research methods, computer related algorithms, and strong ground-motion problems.

Bullen's death in 1976 prevented him from revising his famous book on the theory of seismology, first published in 1947. The only substantial revision, made in 1963, was quite limited because of strong constraints on the format and length allowed. In the 1970s he was occupied with writing his treatise on the Earth's density, but he had already formed the intention of returning to *Theory of Seismology* at the end of the decade. To this end, he produced extensive subsidiary notes while giving seismology courses in 1971 and 1972 at the International Institute of Seismology and Earthquake Engineering, Tokyo, Japan, and these have been used in making the present revisions.

Any rewriting at this time must take account of the remarkable strengthening and broadening of seismological work in the last two decades. Seismological theory is now so extensive that a reasonably comprehensive introduction in one volume demands a somewhat invidious selection of methods and topics, many of which were hardly even formulated in 1947. Yet the attempt appears worthwhile because Bullen's book has continued as a valuable text for over thirty years.

I have seen my task in undertaking a radical revision as keeping what remains most useful of the 1963 text, but incorporating at the appropriate introductory level the necessary fundamental ideas to fill the most notable gaps. In this edition there are new chapters on the theory of seismic sources, seismic waves through anomalous zones, eigen-vibrations of the Earth, and strong-motion seismology. Other new material has been added, particularly on the theory of the seismograph, damping, density estimation in the Earth, and analysis of travel-times and eigen-vibration data using statistical inverse theory. To make room for this expanded theory, the

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chapters in the 1963 edition on nuclear explosions and extraterrestrial seismology have been omitted and these topics incorporated more briefly elsewhere. Key references have been reselected and updated and there are new sections giving problems, drawn from many sources, as exercises for the reader.

The principal aim of the book remains as before: to provide an introduction to seismological *theory*. For this reason, the treatment of the extensive observational side of earthquake studies is quite restricted and is introduced only to provide the necessary background. Practical applications of seismology are indicated only briefly, and the more difficult advanced theoretical problems (involving, for example, Cauchy–Riemann theory) are referred to other texts and publications. Several admirable, advanced textbooks on theoretical seismology and elastic wave theory have been published in recent years, among which must be mentioned the works of W. Pilant (1979), K. Aki and P. Richards (1980), J. Hudson (1980), A. Ben-Menahem and S.J. Singh (1981), and B. Kennett (1983). I have endeavoured to select material so that this book meets the requirements of teachers and professionals for a development between elementary physics and advanced mathematics. Based on lecture courses at the University of California, Berkeley, the selection given here should be helpful to upper division undergraduates and beginning graduate students of geophysics and earthquake engineering.

The reader should note several points on format. Equations within each chapter are referred to in that chapter by a single number; equations are referred to from other chapters by two numbers denoting chapter and equation, respectively (e.g. equation (2.4)).

The references have, for overall convenience, been separated at the back of the book into two parts: a short ‘Selected bibliography’, and ‘References’. Entries in both lists are cited in the text by author and year of publication. The reader should therefore consult both lists where necessary.

Because seismology, like a number of large areas of physics, still uses cgs units they have been retained in this edition. A short conversion table to SI(mks) units is printed at the back of the book.

To the best of my ability, the overall approach is meant to remain close to that of my former professor and colleague, K.E. Bullen, and, ultimately, to our common mentor, Sir Harold Jeffreys. I am indebted to many colleagues and research students for help. Indeed, parts of the text are derived directly from joint research articles. W.J. Hannon and R.A. Uhrhammer helped greatly by reading critically through much of the text. Thanks are also due in particular to R.D. Adams, D.R. Brillinger,

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*Preface*

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*Berkeley, California 1984*

BRUCE A. BOLT