Analogue and digital electronics for engineers

AN INTRODUCTION

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1

Principles of amplifiers

1.1 Introduction

The amplifier is a basic building block of electronic systems. The contents of the block may change over the years but we will always need to know how one amplifier will load another when they are connected in series (cascade). Also we will need to know how an amplifier will be affected by the capacitance of the wires bringing its input to it and taking its output from it.

Consider the amplifier shown in fig. 1.1. We will assume that the input X_i is related to the output X_o by a constant. The stage is said to have a gain, A, given by:

$$A = \frac{\text{amplifier output}}{\text{amplifier input}} = \frac{X_o}{X_i}.$$

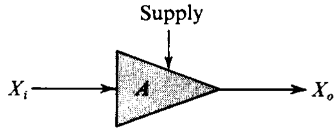

Fig. 1.1. Amplifier symbol.

Note that, in fig. 1.2, as the input X_i is increased, there will come a time when X_o cannot rise any more due to limitations of the supply. Thus every amplifier will become non-linear for very large output demands, as shown in fig. 1.2. Also all amplifiers will be non-linear to some extent, even for small signals: i.e. the ratios X_{o1}/X_{i1} and X_{o2}/X_{i2} may be different. However there will be a restricted working range of the amplifier where the ratios are nearly constant, say within a few per cent of each other. In this chapter, we consider amplifiers working in their linear range and we take A to be a constant.

Having said that the gain of an amplifier is $A = X_o/X_i$, we can write $X_o = AX_i$. If this is so, what is the output in the arrangement in fig. 1.3? Or what is the output of the mechanical system with levers shown in

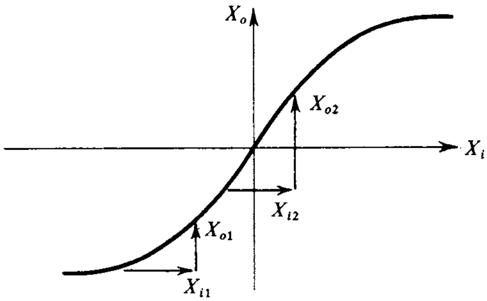

Fig. 1.2. Linearity of gain.

fig. 1.4? Is the output always $16x$ for an input x? It is clear that the output will be $16x$ sometimes; but if the output point is not entirely free to move, i.e. the output could be compressing a spring, then the beams may bend and some deflection less than 16 times the input will result. The lever example shows a realisation of $A = -4$ on no load.

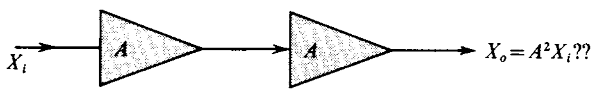

Fig. 1.3. Two amplifiers in cascade.

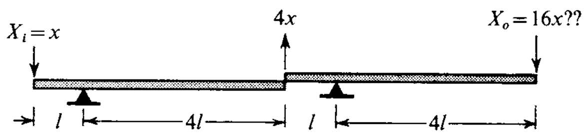

Fig. 1.4. Lever system.

Returning to the amplifiers, let us assume that they are voltage amplifiers each of gain $= -4$. That is, for a unit voltage *rise* at the input, there will be a 4 unit voltage *fall* at the output. It is again clear that the output could be $16v$ for an input v, but it may be less. This will be caused by the second amplifier drawing current by having a path of impedance Z_i at its input terminal. If the first amplifier has an impedance Z_o in its output path then a voltage drop will occur across Z_o and something less than $-4v$ will be the input of the second amplifier of fig. 1.5.

Thus we are interested in the 'coupling' between stages. Every amplifier has these input and output impedances. Can we say what are desirable values for best voltage, or current or power coupling? These impedances may be reactive; for instance, Z_i may be $10^5\ \Omega$ in parallel with a capacitor

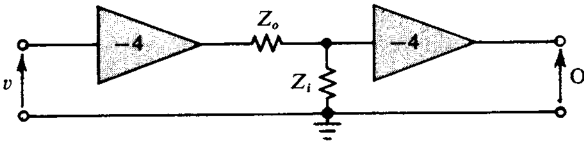

Fig. 1.5. Amplifiers showing coupling circuit.

of 10^{-10}F. We wish to know what such a value for Z_i will do to the coupling between amplifiers or between an amplifier and some signal source.

1.2 Coupling between voltage amplifiers

The circuit of fig. 1.6 contains the following parts:

(*a*) is a source of voltage v_1 and internal impedance Z_1. We know that the source may be a circuit including many devices, but Thévenin's theorem says that this can be reduced to a single voltage generator and a series impedance. We could have reduced the source to the Norton form of a single current generator and a parallel impedance; identical answers would be obtained, but less easily.

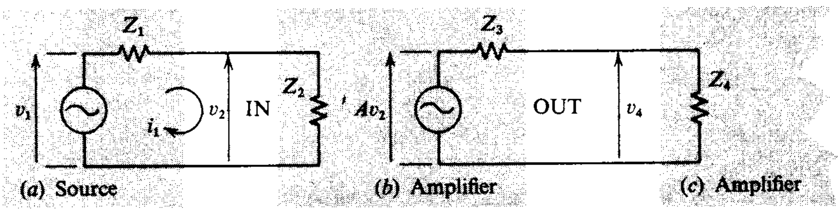

Fig. 1.6. Voltage coupling.

(*b*) is an amplifier whose input draws some current. One side of the input may be ground, or may be a power supply voltage rail which would be in common with one side of the amplifier output but, in this general case, this is not assumed. The two terminals are shown at which the input voltage v_2 may be developed and the impedance Z_2 across these terminals is the input impedance of the amplifier.

Again the amplifier may contain many components but the Thévenin representation is used whereby the output circuit is reduced to one voltage generator Av_2 and one series impedance Z_3. If no load current flows and so no voltage drop occurs in Z_3, Av_2 will be the output for an input v_2. So A is the no-load voltage gain of the amplifier and Z_3 is the amplifier's output impedance.

(*c*) is either a load of impedance Z_4, or is a further amplifier which may draw current from the amplifier (*b*) and which we represent as having an input impedance Z_4. The voltage developed across this load is v_4.

Considering the input circuit of the first amplifier, we wish to know what voltage v_2 is developed there compared to v_1 which is the voltage that would be available from the source if no current was drawn from it.

Kirchhoff's laws allow us to solve circuit problems in two ways. The first way is to label the voltages appearing between different points of a circuit and some reference point. We can then write the currents in each path of the circuit to be equal to the voltage difference between the ends of the path divided by the impedance of the path. Lastly we write down that the currents into any point of a circuit must equal the currents out of that point. Thus we end with as many equations as points and we can solve these equations. The second way is to mark in the unknown circulating currents in each loop and write down that the voltage drops round a loop must equal the source voltages.

In our circuit the voltages are already labelled, and at the upper input terminal of the amplifier, the current out of the source must equal that flowing into the amplifier (we have shown no other paths) thus:

$$\frac{v_1 - v_2}{Z_1} = \frac{v_2}{Z_2}$$

or

$$v_1 Z_2 = v_2 Z_1 + v_2 Z_2,$$

so

$$v_2 = v_1 \left(\frac{Z_2}{Z_1 + Z_2}\right). \tag{1.1}$$

Note that this could have been obtained by putting in i_1 as an unknown circulating current and from

$$v_1 = i_1 Z_1 + i_1 Z_2 \quad \text{and} \quad v_2 = i_1 Z_2$$

the same relation for v_2 and v_1 results.

Equation (1.1) is sometimes called the potential divider expression for the circuit of fig. 1.7.

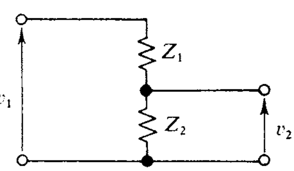

Fig. 1.7. Potentiometer.

The output voltage is a fraction $Z_2/(Z_1 + Z_2)$ of the input voltage; but note that this is only true when no other current is drawn from the point joining Z_1 and Z_2. This equation is easy to remember and can be used

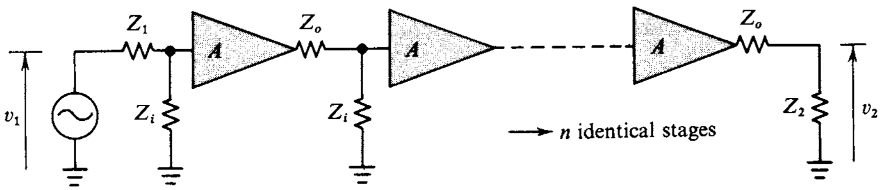

Fig. 1.8. Multistage voltage amplifier.

if all the circuits across which v_2 is developed are combined to give one effective impedance Z_2 which is used in the equation.

Under some circumstances, we may want perfect voltage coupling or $v_2 \to v_1$. From (1.1) this is achieved if

$$\frac{Z_2}{Z_1+Z_2} \to 1 \quad \text{or} \quad Z_1 \ll Z_2.$$

Note that we do not want Z_1 to be 0 but merely very much smaller than Z_2, the input resistance of the amplifier. A factor making Z_1 from 20 to 100 times smaller than Z_2 is suitable for most engineering purposes although for high quality instrumentation and computing, a factor making Z_1 from 10^3 to 10^6 smaller may be appropriate.

Referring again to the circuit, fig. 1.6, we can write an equation similar to (1.1) for the second part of the circuit:

$$v_4 = Av_2\left(\frac{Z_4}{Z_3+Z_4}\right) = A\left(\frac{Z_2}{Z_1+Z_2}\right)\left(\frac{Z_4}{Z_3+Z_4}\right)v_1;$$

or for the multistage amplifier with n identical stages shown in fig. 1.8, we may write generally:

$$v_2 = A^n\left(\frac{Z_i}{Z_i+Z_1}\right)\left(\frac{Z_i}{Z_i+Z_o}\right)^{n-1}\left(\frac{Z_2}{Z_o+Z_2}\right)v_1, \qquad (1.2)$$

where Z_i is the input impedance and Z_o is the output impedance of each amplifier.

Thus as a general method of working out the overall relation, we have separated the 'attenuation' (which is a gain of less than unity) of each coupling section between amplifier blocks from the gain of the amplifier blocks.

1.3 Worked example

A sine wave generator whose internal source is represented by a phasor of voltage $10\angle -10°$ at a certain instant has an output impedance $600-\text{j}100\ \Omega$. It is connected to a circuit of impedance $4000-\text{j}700\ \Omega$; what input will be developed there?

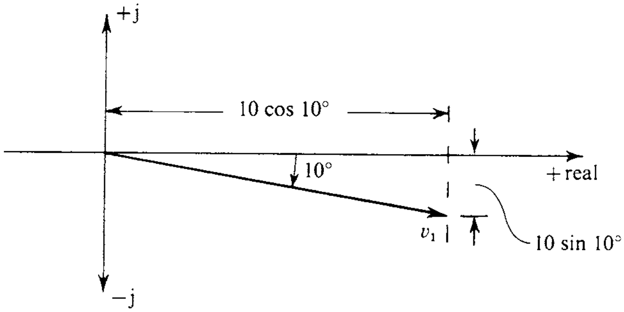

Fig. 1.9. Phasor diagram of v_1.

These data refer to a circuit similar to the left-hand part of fig. 1.6 where:

$$v_1 = 10 \angle -10° \text{ or } 10\cos 10° - \text{j}10\sin 10° \text{ volts (see fig. 1.9)},$$

$$Z_1 = 600 - \text{j}100\,\Omega,$$

$$Z_2 = 4000 - \text{j}700\,\Omega.$$

Equation (1.1) gives:

$$v_2 = \frac{v_1 Z_2}{Z_1 + Z_2} = 10 \angle -10° \frac{4000-\text{j}700}{4600-\text{j}800}$$

$$= 10 \angle -10° \frac{4061 \angle -9°56'}{4669 \angle -9°51'}$$

$$= \frac{10 \times 4061}{4669} \angle \{-10° + (-9°\,56') - (-9°\,51')\}$$

$$= 8.70 \angle -10°\,5' \text{ volts.}$$

This is 87 per cent of the unloaded output of the generator and lags it by 5′.

This arithmetic, which consists of taking the *product* and *quotient* of complex quantities, is more easily handled in polar form. Each step can be checked roughly and numerical mistakes are less likely to creep in, i.e. one would expect an impedance to 4000 Ω resistance and 700 Ω capacitative reactance to have a magnitude of a little over 4000 Ω, and so on.

1.4 Logarithmic expression for gain: the decibel (dB)

The logarithmic expressions for gain have several uses. To work out the gain of a multistage amplifier from an expression such as (1.2), one just *adds* the logarithmic gain for each coupling and for each amplifier to get the overall gain of many stages in cascade.

In fig. 1.8, we considered how the output voltage, v_2, was related to the input voltage v_1 and an expression of the form, $v_2 =$ (voltage gain) v_1 was obtained for an amplifier. If we know what resistances the input and output voltages appear across, say R_1 and R_2 respectively, then we can write that the input power, $p_1 = v_1^2/R_1$. Similarly the output power, $p_2 = v_2^2/R_2$, where v_1 and v_2 are the RMS voltages. Then a figure for power gain could be obtained from

$$p_2 = (\text{power gain})p_1.$$

An alternative to the dimensionless figure for power gain is that expressed in decibels (dB) and it is defined by:

$$\text{Power gain (dB)} = 10\log_{10}\frac{p_2}{p_1}. \qquad (1.3)$$

Thus an amplifier giving 4 watts output for 4mW input will have a power gain of 30 dB. If we write the powers in terms of voltages we get

$$\text{Power gain (dB)} = 10\log_{10}\frac{v_2^2/R_2}{v_1^2/R_1}$$

$$= 20\log_{10}\frac{v_2}{v_1}+10\log_{10}\frac{R_1}{R_2}. \qquad (1.4)$$

In any circuits where $R_2 = R_1$, then and *only* then:

$$\text{Power gain (dB)} = 20\log_{10}\frac{v_2}{v_1}$$

$$= 20\log_{10}(\text{voltage gain}). \qquad (1.5)$$

Many communication circuits are designed with standard source and load resistors of 600 Ω and transmission lines of 50 and 75 Ω. In these systems, (1.5) expressing the *voltage gain* in dB is often used.

The decibel unit of power gain is very useful for two reasons:

(*a*) The response curves of many circuits have simple forms when the gain in decibels is plotted against the frequency on a logarithmic scale.

(*b*) When power (or voltage or current) amplifiers are followed by further circuits with gain (+dB) or attenuation (−dB), then the overall gain is obtained algebraically by *summing* the gains and attenuations of all parts in a path between the input and output.

Thus if our amplifier of input 4 mW and output 4 W is followed by a passive circuit of power gain 0.5 (or attenuation of 2) and this is followed by a further amplifier of power gain 400, then in terms of decibels:

$$\text{Power gain of } 0.5 = 10\log_{10}0.5 = -10\log_{10}2 = -3 \text{ dB}.$$

$$\text{Power gain of } 400 = 10\log_{10}400 = 10\times 2.6 = +26 \text{ dB},$$

and therefore the overall gain $= 30-3+26 = +53$ dB (or 200000 times).

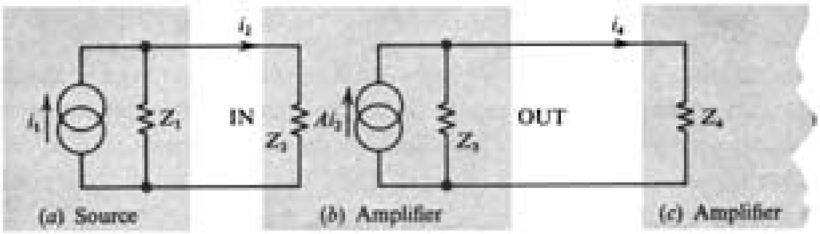

Fig. 1.10. Current coupling.

Readers will see some amplifier data sheets with the gain quoted in decibels *without* the output load being specified. These data should be treated with caution.

1.5 Coupling between current amplifiers

The circuit of fig. 1.10 contains the following parts.

(*a*) is a source of current i_1 and internal impedance Z_1. Norton's theorem allows the source, whatever its internal complexity, to be reduced to a single current generator and parallel impedance Z_1. The conduction path Z_1 means that the source can waste some of its current internally which is the dual of a voltage source dropping some of its voltage internally.

(*b*) is an amplifier whose input impedance is Z_2: thus to drive a current i_2 into the input terminals, some voltage must exist there. The amplifier is reduced to an equivalent circuit of a single generator Ai_2 and a parallel impedance Z_3 at the output terminals. Here A is the current gain of the amplifier when it is loaded by a short circuit and Z_3 is its output impedance.

In the ideal current amplifier, $Z_2 = 0$. It may appear unreal to terminate any source by a short circuit, but certain devices are good current amplifiers and we shall show that, for best current coupling, they should be followed by low impedance stages.

(*c*) is a further amplifier which has an input impedance Z_4 or it is a load.

Considering the input circuit of the first amplifier, we wish to know what current, i_2, flows into the first amplifier compared with i_1, which is apparently available from the source. Kirchhoff's current law allows us to write that the current going down Z_1 is $i_1 - i_2$, whence we can write expressions for the voltage at the amplifier input as

$$Z_1(i_1 - i_2) \quad \text{or} \quad i_2 Z_2.$$

Since these are identical,

$$Z_1(i_1 - i_2) = i_2 Z_2,$$

so

$$i_2 = i_1 \left(\frac{Z_1}{Z_1+Z_2}\right). \tag{1.6}$$

Note that this is the dual of the voltage coupling expression. Now it is the load impedance Z_2 which should be much smaller than the source impedance to give good current coupling, i.e. when,

$$Z_2 \ll Z_1, \qquad (Z_1+Z_2) \approx Z_1,$$

so

$$i_2 \approx i_1.$$

Consider next the current i_4 coupled from the amplifier output to the next stage input in fig. 1.10; here

$$i_4 = Ai_2\left(\frac{Z_3}{Z_3+Z_4}\right) = A\left(\frac{Z_1}{Z_1+Z_2}\right)\left(\frac{Z_3}{Z_3+Z_4}\right)i_1.$$

So the overall gain, i_4/i_1, is the product of the gain of an amplifier and the efficiency of its input and output coupling; namely $Z_1/(Z_1+Z_2)$ and $Z_3/(Z_3+Z_4)$ respectively. If these figures are known or calculated in decibels, the terms are added rather than multiplied.

It is interesting that bipolar transistors whose input resistances are usually much smaller than their output resistances give good current coupling between each stage when they are connected in cascade. (Typical figures for a small transistor are an input resistance of 1 kΩ and an output resistance of 30 kΩ or, for a power transistor, 10 Ω and 200 Ω respectively.)

1.6 Loading of a source for maximum power output

The circuit of fig. 1.11 shows:

(*a*) a source whose voltage on no load is v_1 and whose internal impedance Z_1, has resistive and reactive components, R_1 and X_1.

(*b*) a load whose impedance, Z_2, we wish to determine to get maximum power output from the source. The load is considered generally to be made up of a resistive part R_2 in series with a reactive part of impedance X_2.

We can write the value of the current into the load, i_2, as:

$$i_2 = \frac{\text{e.m.f.}}{\text{impedance}} = \frac{v_1}{Z_1+Z_2} = \frac{v_1}{R_1+R_2+\mathrm{j}(X_1+X_2)}. \tag{1.7}$$

The expressions for power in any load are $|v_2|\,|i_2|\cos\phi$ or $|i_2|^2R_2$. Both

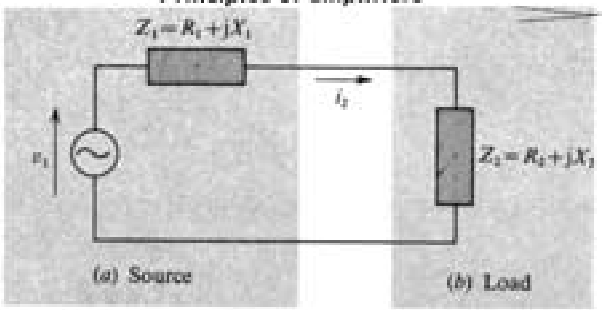

Fig. 1.11. Power coupling.

will give the same result. Here we do not know the load voltage, v_2, but we do know the load current i_2 and the resistive part of the load R_2 through which the current flows. Hence the power into the load, p_2 is given by:

$$p_2 = |i_2|^2 R_2 = \left| \frac{v_1}{R_1 + R_2 + \mathrm{j}(X_1 + X_2)} \right|^2 R_2$$

which cannot exceed
$$\frac{v_1^2 R_2}{(R_1 + R_2)^2}. \tag{1.8}$$

This is because the complex denominator can be made a minimum if $X_1 = -X_2$ and this will make the power a maximum. Thus the first condition for getting maximum power output is that the load reactance, X_2, should be the conjugate of the source internal reactance, X_1; i.e. if one is inductive the other should be capacitative and vice versa.

Differentiating the expression for power, p_2, with respect to R_2 gives:

$$\frac{\mathrm{d}p_2}{\mathrm{d}R_2} = \frac{(R_1 + R_2)^2 v_1^2 - 2v_1^2 R_2 (R_1 + R_2)}{(R_1 + R_2)^4}$$

$= 0$ for a maximum or minimum. This is given by setting the numerator to zero,

$$0 = (R_1 + R_2) - 2R_2,$$

hence
$$R_1 = R_2. \tag{1.9}$$

This is a well known result that the resistive part of the load impedance must be equal to the resistive part of the source impedance for maximum power output. Substituting the value $R_2 = R_1$ into the expression for power p_2, (1.8), gives:

$$p_2\,(\text{maximum}) = \frac{1}{4}\frac{v_1^2}{R_1}.$$

Note that v_1^2/R_1 is the power that could be dissipated internally in the source if the output terminals were loaded only by a reactance X_2 equal to $-X_1$. Thus the maximum power output is a quarter of that that could be dissipated internally in the source.

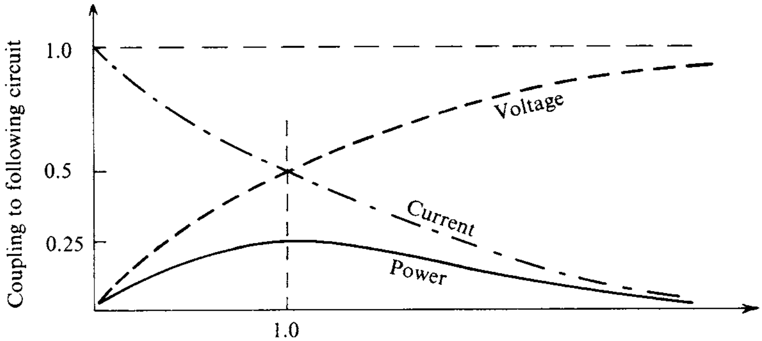

Fig. 1.12. Variation of coupling with the ratio of load to source resistance.

Power matching is used mainly in three situations:

(*a*) Where the signal levels are very small so any power lost gives a much worse signal to noise ratio: for example, it is used in all aerial to receiver connections in television, radio and radar equipment.

(*b*) Where the signal at high frequency is connected through lines of appreciable self capacity and self inductance to a load. Then it is possible to get large standing waves due to reflections from the load which can make the source to load power transfer low.

(*c*) Where signals are very large, say at the output stage of a transmitter, and where the maximum efficiency is desirable on economic grounds.

Fig. 1.12, summarises how the ratio of load to source resistance influences the efficiency of voltage, current and power coupling between circuits.

1.7 Frequency characteristics of coupling circuits and amplifiers

So far, the gain and other properties of the amplifier blocks and the coupling between the blocks have not been related in any way to frequency. There will always be some high frequency at which the gain of any amplifier is less than its gain at low frequency. The effects are akin to mechanical inertia; on requiring an output, it takes a finite time for current flowing at an input terminal of a device to pass through it and reach its output – this is called transit time and it is naturally very short if the device is small. Also, it takes time for the voltage to build up in an output circuit after current has started to flow into it, due to its capacitance. Thus we can show some features of an amplifier's response by showing the output of an amplifier for a step input, fig. 1.13.

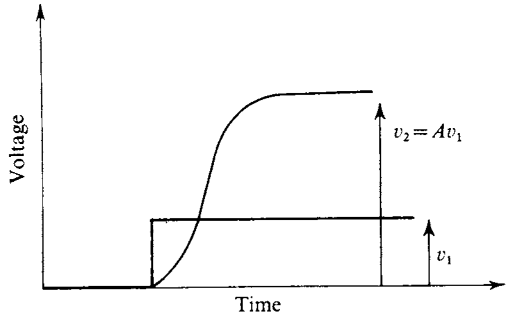

Fig. 1.13. Amplifier output for step input.

To a rough approximation, the output may rise exponentially; the time constant of the amplifier can be obtained from the time taken for the output to get to 63 per cent or $1 - 1/e$ of the state to which it eventually settles. If this time is τ_2, then we have approximated the output as a function of time to $v_2(t) = Av_1(1 - e^{-t/\tau_2})$.

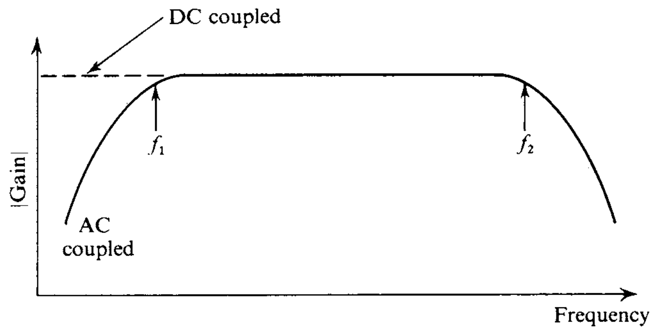

Fig. 1.14. Frequency response of a typical circuit.

An alternative representation to show the gain of an amplifier or of a coupling circuit is to plot the magnitude of its gain against frequency, as in fig. 1.14. The approximate relationship between f_2 when the gain starts dropping and τ_2 is simple and it will be developed later.

The important features of an amplifier can be seen from a plot of its |gain| against frequency. It will amplify signals at frequencies between f_1 and f_2 by very nearly the same amount. Thus a musical note with certain harmonics at the source should be almost faithfully amplified if the fundamental and the main harmonics present are in the range of frequencies between f_1 and f_2. The waveform of the electrical signal to make a television picture is very complex. It contains components between 25 Hz and 5 MHz and thus 'video' amplifiers need to have a flat response over this band.

The methods of Fourier series and Fourier transforms allow pulses, ramp waveforms and square waves to be broken up into a spectrum of frequencies. The circuit should have approximately the same gain and approximately the same phase shift over this spectrum for the waveform at the output to closely resemble that at the input.

We wish therefore to know which components in the coupling and amplifier circuits will give rise to the frequencies f_1 and f_2 at which the gain drops from a largely constant figure called the *midband gain*. It is possible to have coupling circuits whose gain stays constant down to zero frequency – such circuits are called direct current coupled or 'DC coupled'. With many circuits this is not necessary and they can be simplified by being coupled for alternating signals only and so are called 'AC coupled'. The difference between these two is shown at the left-hand end of the response plot, fig. 1.14.

We want to construct such a plot with the minimum of computational effort. We wish to know which components in our amplifiers and coupling circuits give rise to the drop in gain at the extremes of frequency. Then we can design circuits to handle just the frequency band required for the signals in which we may be interested.

1.8 Coupling circuits at low frequency

The following analysis explores the effect of a simple coupling circuit between a source and load. Exactly the same effect is produced by coupling circuits between stages of amplifiers and by the decoupling circuits needed by amplifiers (decoupling circuits are mentioned in the chapters on amplifier realisation with field-effect and bipolar transistors). In those cases one must first identify the resistors in series with the reactive element and the analysis then reduces to the same as that now given.

The circuit, fig. 1.15, shows:

(*a*) is a sinusoidal source of voltage of amplitude v_1 and frequency ω rad/s $= 2\pi f$ where f is in hertz (Hz). The source internal impedance Z_1 is shown to be partly resistive and partly reactive. The reactive part is due to a capacitor C_1 which may be blocking the internal supply voltages of the source from appearing at its output terminal. Alternatively it may be stopping the source circuit from upsetting the DC supply voltages of an amplifier which is forming the first stage of the load. Then C_1 may actually be within the load but it can be considered as having an effect similar to that of R_1; i.e. a voltage drop will occur across it so that part of v_1 is dropped and not usefully developed across the resistive part of the load.

At frequency $\omega = 2\pi f$

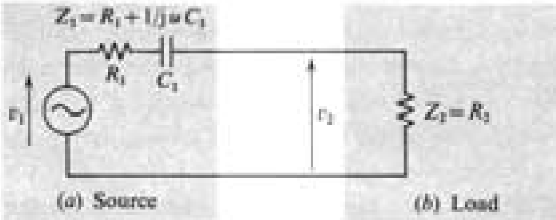

Fig. 1.15. Coupling circuit with series capacitor.

(*b*) is the load which is considered to be resistive only, so $Z_2 = R_2$. This is an approximation because there will be stray capacities in all circuits, but the fact is separately investigated in § 1.9.

We need to obtain an expression for the voltage v_2 appearing at the load in terms of the source voltage v_1. The expression for voltage coupling gives

$$v_2 = v_1\left(\frac{Z_2}{Z_1+Z_2}\right) = v_1\left(\frac{R_2}{R_1+1/\mathrm{j}\omega C_1+R_2}\right) = \frac{v_1 R_2/(R_1+R_2)}{1+1/\mathrm{j}\omega C_1(R_1+R_2)}. \quad (1.10)$$

By studying the dimensions of each term of the final expression, it can be seen that the denominator must be dimensionless. Since ω has the units of s^{-1}, the product $C_1(R_1+R_2)$ must have the units of seconds and is called the time constant, τ_1. (This seems odd until the units of farads × ohms are looked at more closely and are found to be seconds!)

Now we wish to find out how the expression for v_2 changes with the frequency, ω. We consider three cases.

(*a*) Frequency is high so ω in the denominator of the expression makes the complex part $\ll 1$.

$$\text{So gain } = \frac{v_2}{v_1} \approx \frac{R_2}{R_1+R_2} = B, \quad \text{say.} \quad (1.11)$$

Note that this is real and not imaginary. Therefore there is no phase shift associated with the circuit gain at high frequency. This is plotted in fig. 1.17 as the region (*a*).

(*b*) At a frequency $\omega = \omega_1$, such that

$$\omega_1 C_1(R_1+R_2) = 1 \qquad \left(\text{i.e. } \omega_1 = \frac{1}{C_1(R_1+R_2)} = \frac{1}{\tau_1}\right). \quad (1.12)$$

$$\text{Gain } = \frac{v_2}{v_1} = \frac{R_2/(R_1+R_2)}{1+1/\mathrm{j}.1} = \frac{B}{1-\mathrm{j}}$$

$$= \frac{B(1+\mathrm{j})}{2} = 0.707B \angle +45°.$$

Fig. 1.16 shows the conversion from cartesian to polar co-ordinates.

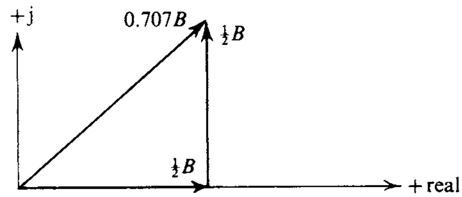

Fig. 1.16. Phasor diagram for gain at low frequency turnover.

At this particular frequency, ω_1, the gain is 70.7 per cent of its value at the higher frequency and the phase shift is such that the output *leads* the input by 45°. This is plotted on fig. 1.17 in the region (*b*). ω_1 is called the *turnover frequency*.

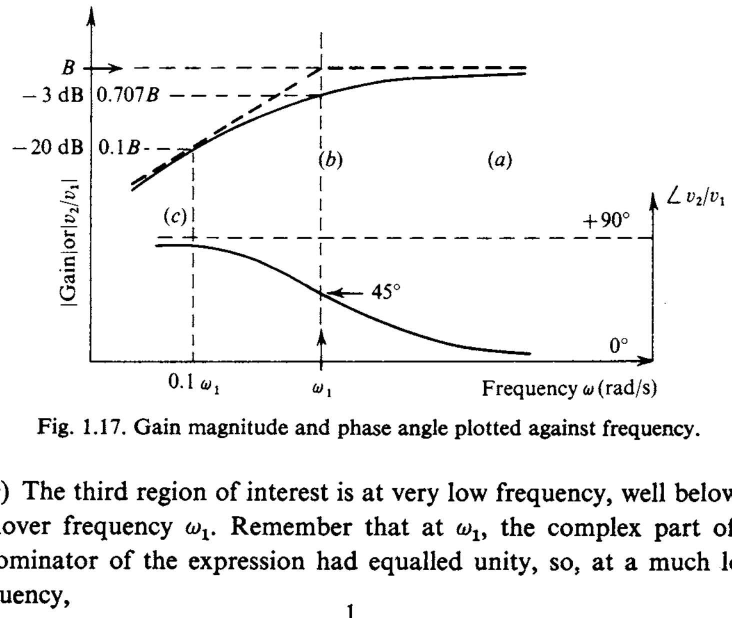

Fig. 1.17. Gain magnitude and phase angle plotted against frequency.

(*c*) The third region of interest is at very low frequency, well below the turnover frequency ω_1. Remember that at ω_1, the complex part of the denominator of the expression had equalled unity, so, at a much lower frequency,

$$\frac{1}{\omega C_1(R_1+R_2)} \gg 1.$$

Using

$$C_1(R_1+R_2) = \frac{1}{\omega_1},$$

then from (1.10)

$$\text{Gain} = \frac{v_2}{v_1} \approx \frac{R_2/(R_1+R_2)}{\omega_1/\mathrm{j}\omega} = 0+\mathrm{j}B\frac{\omega}{\omega_1} = B\frac{\omega}{\omega_1}\angle+90°. \qquad (1.13)$$

The final expression has been converted from complex terms to polar co-ordinates to give a magnitude and phase angle for the gain and these are shown in region (*c*) of figure 1.17.

Thus at a frequency one tenth of the turnover frequency, $\omega = 0.1\omega_1$, the magnitude of the gain is $0.1B$, and at one hundredth of the turnover frequency, the gain is $0.01B$. Thus on a logarithmic plot of gain against frequency, the relation will be linear (note $0.1B = 20 \log_{10} B - 20$ decibels and $0.01B = 20 \log_{10} B - 40$ decibels because these are voltage ratios).

The broken lines on fig. 1.17 where the gain tends to a constant value B at frequencies above the frequency ω_1 and where the gain tends to fall linearly at frequencies below ω_1 show an approximate response curve which is called the asymptotic approximation to the frequency response. More accurately, the gain has dropped to $0.707B$ at ω_1 and the response curve really passes through this point and is an asymptote to the two broken lines which are accurate for low or high frequencies. The reason for calling ω_1 the turnover frequency or break frequency can now be seen; clearly the gain has stopped being a constant figure B and drops steeply once the frequency drops below ω_1.

The expression for gain gave an angle as well as a magnitude for the ratio v_2/v_1. Rewriting the expression as $v_2 = \text{gain} \times v_1$, we see that if the gain had a positive angle between 0 and 90° associated with it then v_2 will lead v_1 in phase by between 0 and 90°, the actual value depending on the frequency. The cartesian plot of log |gain| and ∠gain against log frequency is called the Bode plot for the circuit and fully specifies the gain of the circuit.

The alternative scaling on the |gain| axis shows how simple the relation becomes in decibels. If the output of Bv_1 at higher frequencies is taken as the normal level, then when the frequency has dropped to ω_1, the output is 0.707 of Bv_1.

$$\begin{aligned}\text{Voltage ratio } 0.707 &= 20\log_{10}0.707\\ &= -20\log_{10}1.414\\ &\approx -3 \text{ decibels.}\end{aligned}$$

When the frequency is well below ω_1, if the frequency is halved, or dropped by an octave, the gain will also be halved.

$$\begin{aligned}\text{Voltage ratio } 0.5 &= 20\log_{10}0.5\\ &= -20\log_{10}2.0 \approx -6 \text{ decibels.}\end{aligned}$$

So the slope of the asymptote is -6 decibels/octave. It can alternatively be expressed as -20 decibels/decade.

The analysis from fig. 1.15 showed the source as a voltage generator. A similar analysis could be done if the source was a current generator, i_1, and had a parallel conductance G_1. However it is easier to use the conversion from current to voltage source shown in fig. 1.18.