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978-0-521-20016-5 - The Large Scale Structure of Space-Time

S. W. Hawking, F.R.S. and G. F. R. Ellis

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THE LARGE SCALE STRUCTURE OF SPACE-TIME

S. W. HAWKING, F.R.S.

Lucasian Professor of Mathematics in the University of Cambridge and
Fellow of Gonville and Caius College

AND

G. F. R. ELLIS

Professor of Applied Mathematics, University of Cape Town



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To
D.W. SCIAMA

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Preface

The subject of this book is the structure of space-time on length-scales from 10^{-13} cm, the radius of an elementary particle, up to 10^{28} cm, the radius of the universe. For reasons explained in chapters 1 and 3, we base our treatment on Einstein's General Theory of Relativity. This theory leads to two remarkable predictions about the universe: first, that the final fate of massive stars is to collapse behind an event horizon to form a 'black hole' which will contain a singularity; and secondly, that there is a singularity in our past which constitutes, in some sense, a beginning to the universe. Our discussion is principally aimed at developing these two results. They depend primarily on two areas of study: first, the theory of the behaviour of families of timelike and null curves in space-time, and secondly, the study of the nature of the various causal relations in any space-time. We consider these subjects in detail. In addition we develop the theory of the time-development of solutions of Einstein's equations from given initial data. The discussion is supplemented by an examination of global properties of a variety of exact solutions of Einstein's field equations, many of which show some rather unexpected behaviour.

This book is based in part on an Adams Prize Essay by one of us (S. W. H.). Many of the ideas presented here are due to R. Penrose and R. P. Geroch, and we thank them for their help. We would refer our readers to their review articles in the *Battelle Rencontres* (Penrose (1968)), Midwest Relativity Conference Report (Geroch (1970*c*)), Varenna Summer School Proceedings (Geroch (1971)), and Pittsburgh Conference Report (Penrose (1972*b*)). We have benefited from discussions and suggestions from many of our colleagues, particularly B. Carter and D. W. Sciama. Our thanks are due to them also.

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