

Introduction to the Physics of Waves

Balancing concise mathematical analysis with the real-world examples and practical applications that inspire students, this textbook provides a clear and approachable introduction to the physics of waves.

The author shows through a broad approach how wave phenomena can be observed in a variety of physical situations and explains how their characteristics are linked to specific physical rules, from Maxwell's equations to Newton's laws of motion. Building on the logic and simple physics behind each phenomenon, the book draws on everyday, practical applications of wave phenomena, ranging from electromagnetism to oceanography, helping to engage students and connect core theory with practice. Mathematical derivations are kept brief and textual commentary provides a non-mathematical perspective. Optional sections provide more examples together with higher-level analyses and discussion.

This textbook introduces the physics of wave phenomena in a refreshingly approachable way, making it ideal for first- and second-year undergraduate students in the physical sciences.

Tim Freegarde is a Senior Lecturer in Physics at the University of Southampton, where his research explores the use of light to trap, cool and manipulate atoms and particles. He has taught wave-related subjects to physics undergraduates of all levels for over 15 years.

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Preface

When we revised our Southampton undergraduate programme to draw into a single course the wave phenomena hitherto distributed among optics, electromagnetism, thermal physics, quantum mechanics and solid-state physics, there seemed to be no single text to recommend. This book is an expanded version of the lecture notes that resulted, and its aim, beyond covering wave physics in its own right, is to introduce the common phenomena and analytical methods that are encountered in these individual fields as well as in the disciplines such as oceanography from which we have always drawn a further audience.

There were nonetheless some excellent textbooks for individual aspects. Coulson's classic [15] provides a concise and elegant mathematical introduction; French's once ubiquitous volume [29] is admirable for its clarity and brevity; and Crawford's brilliant and popularly acclaimed approach through everyday examples [16] suffers only from being long out-of-print. Many other texts are highly satisfactory in the areas they cover, and references to their recent editions are included throughout the following chapters.

One privilege for the author of any new volume is to have a new range of scientific and technological examples upon which to draw. Oscillations in the circulations of the oceans have been quite recently recognized; the extraction of power from ocean waves and tides is only now emerging as practical and necessary; the electronic control of holographic arrays has been possible for just a few years; and the quantum mechanics of coherent systems now underpins major research fields and devices that not long ago appeared impossible.

Yet a particular delight in the physics of waves is that it is readily encountered in so many everyday phenomena – from mountain clouds to surface ripples and from sound and music to the colours of feathers and oil films – to which Pretor-Pinney's brilliantly approachable guide [72] is a joy that takes few scientific liberties. Many seminal studies were therefore performed centuries ago by scientists who are now venerated, and it is a miracle of our age that their original writings are often available, via the Internet, without having to don white gloves in the vaults of a national library. For those like me to whom it brings a particular thrill to read these classic works, I have given a number of references. While scholars of old were often handicapped by the mathematical limits of their eras, their writings generally prove much more profound and insightful than we are commonly led to believe.

Acknowledgements

It is a great pleasure to express my heartfelt thanks to those who have helped me to compile this book. My Southampton colleagues benevolently allowed me to divert them when I repeatedly demonstrated that only by attempting to explain something does one discover the limits of one's own understanding; the students and post-docs in my research group provided a good-humoured sounding-board; and it was Rob Eason who suggested that my lecture notes might make a textbook in the first place. My editors John Fowler and Claire Poole at Cambridge University Press offered invaluable technical guidance and bore my repeated deadline-busting with great forbearance; Steven Holt's meticulous copy-editing immeasurably improved the 'final' manuscript; and Sehar Tahir of Aptara/CUP's T_EXline provided help and advice with exceptional speed and efficiency.

My gliding friends Dave Aknai and Alison Randle enthusiastically offered photographs to illustrate mountain waves and boat wakes. For permission to reproduce other photographs I am grateful to Walter Spaeth of ARTside, Arthur Barlow and James Kennedy of New College, Oxford, John Teufel of NIST, and the companies Holoeye, OpenHydro, Pelamis and Raytheon. Other figures are reproduced with permission from the journal *Nature*, and from the US Navy and Air Force, JPL, Minnesota Sea Grant, NASA and NOAA, all of which generously place their materials in the public domain.

Final thanks go to my parents and Deirdre for their constant encouragement and deliveries of tea to the distant garret, and in particular to Dad for painstakingly proof-reading a late draft of the entire manuscript.

Symbols

The following symbols are used throughout this book for the variables and constants given below. Physical constants and scalar variables are set in *italics*; vectors are in **bold**. Operators and unit vectors are indicated by a circumflex (e.g. $\hat{\mathbf{k}}$). The accepted values of physical and mathematical constants are given, and SI units are indicated where appropriate.

a	amplitude, coefficient, distance (slit width)		m
b	amplitude, coefficient		
c	speed of light	$2.997\,924\,58 \times 10^8$	m s^{-1}
	specific heat capacity		$\text{J K}^{-1} \text{kg}^{-1}$
d	distance (between slits)		m
d	differential, infinitesimal increment		
∂	partial differential		
e	elementary charge	$1.602\,176 \times 10^{-19}$	C
	Jones vector or component		
e	Euler's number	2.718 282	
f	function,		
	frequency,		Hz
	focal length		m
g	acceleration due to gravity		m s^{-2}
h	height, elevation		m
	Planck's constant	$6.626\,069 \times 10^{-34}$	J s
$\hbar$	Planck's constant/(2π)	$1.054\,572 \times 10^{-34}$	J s
i	index		
i	imaginary unit ($\sqrt{-1}$)		
$\hat{\mathbf{i}}$	unit vector along the x axis		
j	index		
$\hat{\mathbf{j}}$	unit vector along the y axis		
k	index,		
	wavenumber, wavevector		rad m^{-1}
k_{B}	Boltzmann's constant	$1.380\,650 \times 10^{-23}$	J K^{-1}
$\hat{\mathbf{k}}$	unit vector along the z axis		
l	index,		
	length		m

xii	Symbols	
<i>m</i>	index, mass	kg
<i>n</i>	index, number of moles	
<i>p</i>	pressure, momentum, dipole moment,	Pa kg m s ⁻¹ C m
<i>p</i>	image–lens distance	m
<i>q</i>	index, charge	C
<i>r</i>	radius, radial coordinate, position vector amplitude reflectivity	m
<i>s</i>	coordinate variable along arbitrary path, object–lens distance	m m
<i>t</i>	time, amplitude transmission	s
<i>u</i>	transformed position, e.g. $x - vt$	
<i>v</i>	velocity	m s ⁻¹
<i>v_p</i>	phase velocity	m s ⁻¹
<i>v_g</i>	group velocity	m s ⁻¹
<i>w</i>	grating slit width	m
<i>x</i>	coordinate variable	m
<i>y</i>	coordinate variable	m
<i>z</i>	coordinate variable	m
<i>A</i>	cross-sectional area	m ²
A	vector potential	V s m ⁻¹
<i>B</i>	magnetic flux density	T
<i>C</i>	capacitance, capacitance per unit length, specific heat capacity	F F m ⁻¹ J K ⁻¹ kg ⁻¹
<i>D</i>	electric displacement field	C m ⁻²
<i>E</i>	electric field strength, modulus of elasticity	N C ⁻¹ ≡ V M ⁻¹ Pa
<i>ℰ</i>	energy	J
<i>F</i>	force	N
<i>G</i>	gravitational constant	6.674 28 × 10 ⁻¹¹ m ³ kg ⁻¹ s ⁻²
<i>H</i>	magnetic field strength	A m ⁻¹
<i>ℋ</i>	Hamiltonian operator	J
<i>I</i>	current	A
<i>ℐ</i>	intensity	W m ⁻²
<i>J</i>	current density	A m ⁻²
<i>K</i>	kinetic energy	J
<i>L</i>	inductance, inductance per unit length	H H m ⁻¹

M	mass per unit length		kg m^{-1}
N	number of grating rulings		
P	pressure		Pa
	degree of polarization		
$\mathcal{P}$	power		W
Q	total charge		C
$\mathcal{Q}$	heat		J
R	radius of curvature		m
	gas constant	8.314 472	$\text{J K}^{-1} \text{mol}^{-1}$
S	surface (area) of integration		m^2
	Stokes vector or parameter		
$\mathcal{S}$	entropy		J K^{-1}
T	period		s
$T(t)$	function of single variable t		
$\mathcal{U}$	potential energy		J
V	voltage, scalar potential		V
	volume		m^3
W	tension		N
$X(x)$	function of single variable x		
$Y(y)$	function of single variable y		
$Z(z)$	function of single variable z		
Z	impedance		Ω
Z_0	impedance of free space	376.730	Ω
α	angle		rad
γ	friction coefficient, specific heat ratio, Lorentz factor		kg s^{-1}
$\delta(x)$	Dirac delta function		
δ_{mn}	Kronecker delta function		
δx	small finite increment of x		
ϵ	energy density		J m^{-3}
ϵ_0	permittivity of free space	$8.854\,188 \times 10^{-12}$	F m^{-1}
ζ	vertical displacement		m
η	refractive index		
ϑ	angle, phase angle		rad
κ	thermal conductivity, radiation enhancement factor		$\text{W m}^{-2} \text{K}^{-1}$
λ	wavelength		m
μ_0	permeability of free space	$4\pi \times 10^{-7}$	H m^{-1}
ν	frequency		Hz
$\bar{\nu}$	spectroscopists' wavenumber		m^{-1}
ξ	horizontal displacement		m
π	Archimedes' constant	3.141 593	

ρ	density (mass per unit length)	kg m^{-1}
	density (mass per unit area)	kg m^{-2}
	density (mass per unit volume)	kg m^{-3}
σ	conductivity,	$\text{A m}^{-2} \text{ V}^{-1}$
	standard deviation	
τ	period, characteristic time, duration	s
φ	angle, phase angle	rad
ψ	wavefunction,	
	angle, phase angle	rad
ω	angular frequency	rad s^{-1}
Δ	finite increment or change; uncertainty	
Δ^2	variance	
∇	vector differential	
Φ	magnetic flux	V s
Σ	sum	
Θ	temperature	K, °C
Ψ	wavefunction superposition	
Re	real part	
Im	imaginary part (a real value)	
Arg	argument	
$ \dots $	modulus	
$\langle \dots \rangle$	mean, expectation value	
$[\dots]$	retarded value	
*	convolution	