# Contents

---

<table>
<thead>
<tr>
<th>Preface</th>
<th>page xvii</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nomenclature</td>
<td>xx</td>
</tr>
</tbody>
</table>

## 1 Overview of biomedical image analysis

1.1 Introduction 1

1.2 The scope of the book 2

1.2.1 Signals and systems, image formation, and image modality 2

1.2.2 Stochastic models 3

1.2.3 Computational geometry 3

1.2.4 Variational calculus and level-set methods 3

1.2.5 Image analysis tools 4

1.3 Options for class work 4

1.4 Other references 4

## Part I Signals and systems, image formation, and image modality

---

## 2 Overview of two-dimensional signals and systems

2.1 Definitions 9

2.2 Signal representations 10

2.2.1 Special functions 11

2.2.2 Fourier series 12

2.2.3 Fourier transform 13

2.3 Basic sampling and quantization 16

2.4 Sequence Fourier series 18

2.4.1 Sequence Fourier transform 21

2.4.2 Relationship to the continuous Fourier transform 22

2.5 Discrete Fourier transform 23

2.6 The fast Fourier transform (FFT) algorithm 25

2.6.1 Effect of periodic shifts 26

2.6.2 Circular convolution 27

2.6.3 Linear convolution 28

2.7 The Z-transform 29
Part II Stochastic models

4 Random variables

4.1 Introduction
4.2 Statistical experiments
  4.2.1 Sample space \( \Omega \)
  4.2.2 Field (algebra) \( \sigma_F \)
  4.2.3 Probability measure \( P \)
4.3 Random variables
  4.3.1 Basic concepts
  4.3.2 Properties of the CDF and the PDF of a random variable
  4.3.3 The conditional distribution
  4.3.4 Statistical expectation
  4.3.5 Functions of a random variable
4.4 Two random variables
  4.4.1 Statistical expectation in two dimensions
  4.4.2 Functions of two random variables
  4.4.3 Two functions of two random variables
4.5 Simulation of random variables
4.6 Summary
4.7 Computer laboratory
4.8 Exercises
References

5 Random processes

5.1 Definition and general concepts
  5.1.1 Description of random processes
  5.1.2 Classification of a random process
  5.1.3 Continuity of a random process
  5.1.4 The Kolmogorov consistency conditions
5.2 Distribution functions for a random process
  5.2.1 Definitions
  5.2.2 First- and second-order probability distribution functions
5.3 Some properties of a random process
  5.3.1 Stationarity
  5.3.2 The autocorrelation function
  5.3.3 The autocovariance function
  5.3.4 The cross-correlation function
  5.3.5 Time average
  5.3.6 The power spectrum of a random process
  5.3.7 Cross-spectral density
  5.3.8 Power spectral density of discrete-parameter random process
| 5.4 | Linear systems with random inputs | 126 |
| 5.5 | Two-dimensional random processes | 128 |
| 5.6 | Exercises | 128 |
| References | 130 |

6 Basics of random fields

| 6.1 | Introduction | 131 |
| 6.2 | Graphical models | 136 |
| 6.3 | Markov system | 139 |
| 6.4 | Hidden Markov model | 140 |
| 6.5 | Markov random field | 141 |
| 6.6 | Gibbs model | 143 |
| 6.7 | Markov–Gibbs random field models | 145 |
| 6.7.1 | Auto-models | 146 |
| 6.7.2 | Aura-based GRF model | 147 |
| 6.7.3 | Other models | 148 |
| 6.8 | GRF-based image synthesis | 149 |
| 6.8.1 | Gibbs sampler algorithm | 149 |
| 6.8.2 | Chen algorithm | 149 |
| 6.8.3 | Metropolis algorithm | 151 |
| 6.9 | GRF-based image analysis | 153 |
| 6.9.1 | Coding estimation | 153 |
| 6.9.2 | Least square error method | 155 |
| 6.9.3 | Analytical method for parameter identification | 155 |
| 6.10 | Summary | 159 |
| 6.11 | Exercises | 159 |
| 6.12 | Computer laboratory | 161 |
| References | 162 |

7 Probability density estimation by linear models

| 7.1 | Introduction | 163 |
| 7.2 | Nonparametric methods | 164 |
| 7.2.1 | Kernel-based estimators | 166 |
| 7.2.2 | Parzen window | 167 |
| 7.2.3 | k-NN estimator | 168 |
| 7.3 | Parametric methods | 168 |
| 7.3.1 | Maximum likelihood estimator (MLE) | 169 |
| 7.3.2 | Biased versus unbiased estimator | 170 |
| 7.3.3 | The expectation-maximization (EM) approach | 171 |
| 7.4 | Linear combination of Gaussians model (LCG1) | 172 |
| 7.4.1 | Modifications of the linear model (LCG2) | 174 |
| 7.5 | Modeling the image intensity/appearance through the linear model | 175 |
### Contents

7.6 Exercises 176  
7.7 Computer laboratory 177  
References 179  

## Part III Computational geometry

### 8 Basics of topology and computational geometry 183  
8.1 Introduction 183  
8.2 Shape representation 183  
8.2.1 What is shape? 183  
8.2.2 How should a shape be described? 184  
8.2.3 Criteria for shape representation 184  
8.2.4 Data representation of shape 184  
8.3 Topological equivalence 186  
8.4 Vector spaces 188  
8.5 Surfaces in parameter space 190  
8.5.1 Parametric curves 191  
8.5.2 Parametric surfaces 193  
8.5.3 Surface curvature 196  
8.6 Surfaces as meshes 199  
8.6.1 Manifolds and surfaces 199  
8.6.2 Barycentric coordinates 201  
8.6.3 Triangle local frame 202  
8.6.4 Surface curvature: discrete form 202  
8.7 Summary 204  
8.8 Exercises 205  
8.9 Computer laboratory 207  
References 208  

### 9 Geometric features extraction 213  
9.1 Introduction 213  
9.2 Edges and corners 217  
9.2.1 The Harris detector 217  
9.2.2 The SUSAN corner detector 219  
9.2.3 Harris–Laplace and Harris–affine corner detectors 219  
9.2.4 Blob detectors 221  
9.2.5 Region detectors 223  
9.3 Comparative evaluation of interest points 225  
9.3.1 Multi-scale representations 225  
9.3.2 Scale-space representation 232  
9.3.3 Scale-space and feature detection 233  
9.3.4 Differential singularities and feature detection 234
9.4 Local descriptors 235
9.4.1 Scale-invariant feature transform (SIFT) 235
9.4.2 Case study: Descriptors of small-size lung nodules in chest CT 238
9.4.3 Extensions to the SIFT algorithms 239
9.4.4 Speeded-up robust features (SURF) 241
9.4.5 Multi-resolution local binary pattern (LBP) 241
9.4.6 Image stitching 245
9.5 Three-dimensional local invariant feature descriptors 257
9.5.1 Interest point detection 257
9.5.2 3D descriptor building 261
9.5.3 Descriptor matching 264
9.6 Summary 264
9.7 Exercises 267
9.8 Computer laboratory 267
References 269

Part IV Variational approaches and level sets 273

10 Variational approaches and level sets 275
10.1 Calculus of variation and Euler equation 275
10.1.1 Euler–Lagrange equation for one independent variable 276
10.1.2 Euler–Lagrange equation for multiple independent variables 277
10.1.3 Euler–Lagrange and the gradient descent flow 278
10.2 Curve/surface evolution via classical deformable models 279
10.2.1 Curves and planar differential geometry 279
10.2.2 Geometry of surfaces 280
10.2.3 Geodesic curvature 281
10.2.4 Principal curvatures 281
10.2.5 Planar curves and surface normal 281
10.2.6 Curve/surface evolution as a variational problem 282
10.2.7 Discretization and numerical simulation of snakes 283
10.3 Level sets 284
10.3.1 Implicit representation and the evolution PDE 284
10.3.2 Level-set calculus 286
10.4 Numerical methods for level sets 287
10.4.1 Conservation law and weak solutions 287
10.4.2 Entropy condition and viscosity solutions 288
10.4.3 Upwind direction and discontinuous solutions 288
10.4.4 The Eulerian formulation and the hyperbolic conservation law 289
10.5 Numerical algorithm 290
10.5.1 Need for reinitialization and the distance function 291
10.5.2 Front evolution without reinitialization 292
### 13 Basics of registration

- **13.1 Introduction** 345
- **13.2 Basic concepts and definitions** 346
  - 13.2.1 Components of the registration transformation 349
  - 13.2.2 Choice of transformation 352
  - 13.2.3 Similarity measures 353
- **13.3 Surface registration by the ICP algorithm** 355
  - 13.3.1 Mathematical preliminaries 355
  - 13.3.2 The ICP algorithm 359
- **13.4 Global image registration via mutual information** 366
  - 13.4.1 Imaging model 369
  - 13.4.2 Basics of information theory 371
  - 13.4.3 Registration metric 375
  - 13.4.4 Mutual information registration 377
- **13.5 Applications** 378
- **13.6 Summary** 378
- **13.7 Exercises** 379
- **13.8 Computer laboratory** 380
- **References** 380
- **Appendix 13.1 MATLAB code implementations** 381

### 14 Variational methods for shape registration

- **14.1 Introduction** 387
- **14.2 Shape modeling** 389
  - 14.2.1 Parametric representations 389
  - 14.2.2 Landmark-based representation 390
  - 14.2.3 Medial axes representation 391
  - 14.2.4 Implicit representation using the vector distance function 392
  - 14.2.5 Implicit representation using distance transform 392
- **14.3 Global registration of shapes in implicit spaces** 394
  - 14.3.1 Global matching of shapes 394
  - 14.3.2 VDF-based dissimilarity measure 397
  - 14.3.3 SDF-based dissimilarity measure 398
  - 14.3.4 Examples 400
- **14.4 Local shape registration** 403
  - 14.4.1 Local alignment 405
  - 14.4.2 Gradient descent flows and numerical implementation 408
- **14.5 Summary** 413
- **References** 414

### 15 Statistical models of shape and appearance

- **15.1 Introduction** 417
<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>15.2</td>
<td>Statistical shape models</td>
<td>417</td>
</tr>
<tr>
<td>15.2.1</td>
<td>Construction of statistical shape model using PCA</td>
<td>419</td>
</tr>
<tr>
<td>15.2.2</td>
<td>Fitting a model to new points</td>
<td>421</td>
</tr>
<tr>
<td>15.2.3</td>
<td>Statistical modeling of structures</td>
<td>422</td>
</tr>
<tr>
<td>15.2.4</td>
<td>Modeling shape variations</td>
<td>424</td>
</tr>
<tr>
<td>15.3</td>
<td>Statistical appearance models</td>
<td>428</td>
</tr>
<tr>
<td>15.3.1</td>
<td>Image warping</td>
<td>428</td>
</tr>
<tr>
<td>15.3.2</td>
<td>One-dimensional thin-plate splines</td>
<td>429</td>
</tr>
<tr>
<td>15.3.3</td>
<td>N-dimensional thin-plate splines</td>
<td>429</td>
</tr>
<tr>
<td>15.3.4</td>
<td>Statistical appearance model construction using PCA</td>
<td>431</td>
</tr>
<tr>
<td>15.3.5</td>
<td>Combined appearance models</td>
<td>433</td>
</tr>
<tr>
<td>15.4</td>
<td>Analysis of lung nodules in low-dose CT (LDCT) scans</td>
<td>436</td>
</tr>
<tr>
<td>15.4.1</td>
<td>Lung nodules in low-dose CT</td>
<td>437</td>
</tr>
<tr>
<td>15.5</td>
<td>Appearance-based approach for complete human jaw reconstruction</td>
<td>441</td>
</tr>
<tr>
<td>15.5.1</td>
<td>Jaw prior models</td>
<td>444</td>
</tr>
<tr>
<td>15.5.2</td>
<td>Model-based shape and albedo recovery</td>
<td>445</td>
</tr>
<tr>
<td>15.5.3</td>
<td>Sample results</td>
<td>446</td>
</tr>
<tr>
<td>15.6</td>
<td>Summary</td>
<td>448</td>
</tr>
<tr>
<td></td>
<td>References</td>
<td>448</td>
</tr>
<tr>
<td></td>
<td>Appendix 15.1 Pseudocodes and MATLAB realizations</td>
<td>450</td>
</tr>
</tbody>
</table>

Index

References

Appendix 15.1 Pseudocodes and MATLAB realizations

Index