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978-0-521-19590-4 - Introduction to Structural Dynamics and Aeroelasticity: Second Edition

Dewey H. Hodges and G. Alvin Pierce

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## INTRODUCTION TO STRUCTURAL DYNAMICS AND AEROELASTICITY, SECOND EDITION

This text provides an introduction to structural dynamics and aeroelasticity, with an emphasis on conventional aircraft. The primary areas considered are structural dynamics, static aeroelasticity, and dynamic aeroelasticity. The structural dynamics material emphasizes vibration, the modal representation, and dynamic response. Aeroelastic phenomena discussed include divergence, aileron reversal, airload redistribution, unsteady aerodynamics, flutter, and elastic tailoring. More than one hundred illustrations and tables help clarify the text, and more than fifty problems enhance student learning. This text meets the need for an up-to-date treatment of structural dynamics and aeroelasticity for advanced undergraduate or beginning graduate aerospace engineering students.

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“Wonderfully written and full of vital information by two unequalled experts on the subject, this text meets the need for an up-to-date treatment of structural dynamics and aeroelasticity for advanced undergraduate or beginning graduate aerospace engineering students.”

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– *AIAA Bulletin*

Dewey H. Hodges is a Professor in the School of Aerospace Engineering at the Georgia Institute of Technology. He is the author of more than 170 refereed journal papers and three books, *Nonlinear Composite Beam Theory* (2006), *Fundamentals of Structural Stability* (2005, with G. J. Simitses), and *Introduction to Structural Dynamics and Aeroelasticity, First Edition* (2002, with G. Alvin Pierce). His research spans the fields of aeroelasticity, dynamics, computational structural mechanics and structural dynamics, perturbation methods, computational optimal control, and numerical analysis.

The late G. Alvin Pierce was Professor Emeritus in the School of Aerospace Engineering at the Georgia Institute of Technology. He is the coauthor of *Introduction to Structural Dynamics and Aeroelasticity, First Edition* with Dewey H. Hodges (2002).

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# Introduction to Structural Dynamics and Aeroelasticity

*Second Edition*

**Dewey H. Hodges**

Georgia Institute of Technology

**G. Alvin Pierce**

Georgia Institute of Technology



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# Foreword

## From First Edition

A senior-level undergraduate course entitled “Vibration and Flutter” was taught for many years at Georgia Tech under the quarter system. This course dealt with elementary topics involving the static and/or dynamic behavior of structural elements, both without and with the influence of a flowing fluid. The course did not discuss the static behavior of structures in the absence of fluid flow because this is typically considered in courses in structural mechanics. Thus, the course essentially dealt with the fields of structural dynamics (when fluid flow is not considered) and aeroelasticity (when it is).

As the name suggests, structural dynamics is concerned with the vibration and dynamic response of structural elements. It can be regarded as a subset of aeroelasticity, the field of study concerned with interaction between the deformation of an elastic structure in an airstream and the resulting aerodynamic force. Aeroelastic phenomena can be observed on a daily basis in nature (e.g., the swaying of trees in the wind and the humming sound that Venetian blinds make in the wind). The most general aeroelastic phenomena include dynamics, but static aeroelastic phenomena are also important. The course was expanded to cover a full semester, and the course title was appropriately changed to “Introduction to Structural Dynamics and Aeroelasticity.”

Aeroelastic and structural-dynamic phenomena can result in dangerous static and dynamic deformations and instabilities and, thus, have important practical consequences in many areas of technology. Especially when one is concerned with the design of modern aircraft and space vehicles—both of which are characterized by the demand for extremely lightweight structures—the solution of many structural dynamics and aeroelasticity problems is a basic requirement for achieving an operationally reliable and structurally optimal system. Aeroelastic phenomena can also play an important role in turbomachinery, civil-engineering structures, wind-energy converters, and even in the sound generation of musical instruments.

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Foreword

Aeroelastic problems may be classified roughly in the categories of response and stability. Although stability problems are the principal focus of the material presented herein, it is not because response problems are any less important. Rather, because the amplitude of deformation is indeterminate in linear stability problems, one may consider an exclusively linear treatment and still manage to solve many practical problems. However, because the amplitude is important in response problems, one is far more likely to need to be concerned with nonlinear behavior when attempting to solve them. Although nonlinear equations come closer to representing reality, the analytical solution of nonlinear equations is problematic, especially in the context of undergraduate studies.

The purpose of this text is to provide an introduction to the fields of structural dynamics and aeroelasticity. The length and scope of the text are intended to be appropriate for a semester-length, senior-level, undergraduate course or a first-year graduate course in which the emphasis is on conventional aircraft. For curricula that provide a separate course in structural dynamics, an ample amount of material has been added to the aeroelasticity chapters so that a full course on aeroelasticity alone could be developed from this text.

This text was built on the foundation provided by Professor Pierce's course notes, which had been used for the "Vibration and Flutter" course since the 1970s. After Professor Pierce's retirement in 1995, when the responsibility for the course was transferred to Professor Hodges, the idea was conceived of turning the notes into a more substantial text. This process began with the laborious conversion of Professor Pierce's original set of course notes to LaTeX format in the fall of 1997. The authors are grateful to Margaret Ojala, who was at that time Professor Hodges's administrative assistant and who facilitated the conversion. Professor Hodges then began the process of expanding the material and adding problems to all chapters. Some of the most substantial additions were in the aeroelasticity chapters, partly motivated by Georgia Tech's conversion to the semester system. Dr. Mayuresh J. Patil,<sup>1</sup> while he was a Postdoctoral Fellow in the School of Aerospace Engineering, worked with Professor Hodges to add material on aeroelastic tailoring and unsteady aerodynamics mainly during the academic year 1999–2000. The authors thank Professor David A. Peters of Washington University for his comments on the section that treats unsteady aerodynamics. Finally, Professor Pierce, while enjoying his retirement and building a new house and amid a computer-hardware failure and visits from grandchildren, still managed to add material on the history of aeroelasticity and on the  $k$  and  $p$ - $k$  methods in the early summer of 2001.

Dewey H. Hodges and G. Alvin Pierce  
Atlanta, Georgia  
June 2002

<sup>1</sup> Presently, Dr. Patil is Associate Professor in the Department of Aerospace and Ocean Engineering at Virginia Polytechnic and State University.

**Addendum for Second Edition**

Plans for the second edition were inaugurated in 2007, when Professor Pierce was still alive. All his colleagues at Georgia Tech and in the technical community at large were saddened to learn of his death in November 2008. Afterward, plans for the second edition were somewhat slow to develop.

The changes made for the second edition include additional material along with extensive reorganization. Instructors may choose to omit certain sections without breaking the continuity of the overall treatment. Foundational material in mechanics and structures was somewhat expanded to make the treatment more self-contained and collected into a single chapter. It is hoped that this new organization will facilitate students who do not need this review to easily skip it, and that students who do need it will find it convenient to have it consolidated into one relatively short chapter. A discussion of stability is incorporated, along with a review of how single-degree-of-freedom systems behave as key parameters are varied. More detail is added for obtaining numerical solutions of characteristic equations in structural dynamics. Students are introduced to finite-element structural models, making the material more commensurate with industry practice. Material on control reversal in static aeroelasticity has been added. Discussion on numerical solution of the flutter determinant via Mathematica™ replaces the method presented in the first edition for interpolating from a set of candidate reduced frequencies. The treatment of flutter analysis based on complex eigenvalues is expanded to include an unsteady-aerodynamics model that has its own state variables. Finally, the role of flight-testing and certification is discussed. It is hoped that the second edition will not only maintain the text’s uniqueness as an undergraduate-level treatment of the subject, but that it also will prove to be more useful in a first-year graduate course.

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