

GRAPHENE

Graphene is the thinnest known material: a sheet of carbon atoms arranged in hexagonal cells only a single atom thick, and yet stronger than diamond.

Since it was experimentally isolated in 2004, it has been the object of intense theoretical and experimental research. It has potentially significant applications in nanotechnology, ‘beyond-silicon’ electronics, solid-state realization of high-energy physics phenomena and as a prototype membrane that could revolutionize soft-matter and two-dimensional physics.

In this book, leading graphene research theorist Mikhail I. Katsnelson systematically presents the basic concepts of graphene physics. Topics covered include Berry’s phase, topologically protected zero modes, Klein tunnelling, vacuum reconstruction near supercritical charges and deformation-induced gauge fields. The book also contains an introduction to the theory of flexible membranes relevant to graphene physics and a detailed discussion of electronic transport, optical properties, magnetism and spintronics. Standard undergraduate-level knowledge of quantum and statistical physics and solid-state theory is assumed.

This is an important textbook for graduate students in nanoscience and nanotechnology, and an excellent introduction to the fast-growing field of graphene science for physicists and materials-science researchers working in related areas.

MIKHAIL I. KATSNELSON is a Professor and Head of the Theory of Condensed Matter group at the Institute for Molecules and Materials, Radboud University, in the Netherlands. His fields of expertise are magnetism, electronic structure and quantum many-body theory, and, after the discovery of graphene, he became one of the leaders in this new field. Professor Katsnelson is the author of several books, including *Quantum Solid State Physics* (with S. V. Vonsovsky, Springer, 1989), and has been awarded a Russian State Prize for young researchers. He has also been honoured as a Knight of the Order of the Dutch Lion.

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Carbon in Two Dimensions

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In memory of my teacher Sergey Vonsovsky
and my friend Sasha Trefilov

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Preface

I do not think that I need to explain, in the preface to a book that is all about graphene, what graphene is and why it is important. After the Nobel Prize for physics in 2010, everybody should have heard something about graphene. I do need, however, to explain why I wrote this book and what is special about it.

I hope it will not be considered a disclosure of insider information if I tell you that Andre Geim is a bit sarcastic (especially with theoreticians). Every time I mentioned that I was somewhat busy writing a book on graphene, he always replied ‘Go to amazon.com and search for “graphene”.’ Indeed, there are many books on graphene, many more reviews and infinitely many collections of papers and conference proceedings (well, not really infinitely many ... in the main text I will use the mathematical terminology in a more rigorous way, I promise). Why, nevertheless, has this book been written and why may it be worthwhile for you to read it?

Of course, this is a personal view of the field. I do love it, and it has been my main scientific activity during the last seven years, from 2004 when graphene started to be the subject of intensive and systematic investigations. Luckily, I was involved in this development almost from the very beginning. It was a fantastic experience to watch a whole new world coming into being and to participate in the development of a new language for this new world. I would like to try to share this experience with the readers of this book.

The beauty of graphene is that it demonstrates in the most straightforward way many basic concepts of fundamental physics, from Berry’s phase and topologically protected zero modes to strongly interacting fluctuations and scaling laws for two-dimensional systems. It is also a real testbed for relativistic quantum phenomena such as Klein tunnelling or vacuum reconstruction, ‘CERN on one’s desk’. I was not able to find a book focused on these aspects

of graphene, namely on its role in our general physical view of the world. I have tried to write such a book myself. The price is that I have sacrificed all practical aspects of graphene science and technology, so you will not find a single word here about the ways in which graphene is produced, and there is hardly anything about its potential applications. Well, there is a lot of literature on these subjects. Also, I have said very little about the chemistry of graphene, which is an extremely interesting subject in itself. It certainly deserves a separate book, and I am not chemist enough to write it.

The field is very young, and it is not easy to know what will not be out of date in just a couple of years. My choice is clear from the contents of this book. I do believe that it represents the core of graphene physics which will not be essentially modified in the near future. I do not mean that this is *the most interesting* part; moreover, I am sure that there will be impressive progress, at least, in two more directions that are hardly mentioned in the book: in the *many-body* physics of graphene and in our understanding of electron transport near the neutrality point, where the semiclassical Boltzmann equation is obviously inapplicable. I think, however, that it is a bit too early to cover these subjects in a book, since too many things are not yet clear. Also, the mathematical tools required are not as easy as those used in this book, and I think it is unfair to force the reader to learn something technically quite complicated without a deep internal confidence that the results are relevant for the real graphene.

The way the book has been written is how I would teach a course with the title ‘Introduction to the theory of graphene’. I have tried to make a presentation that is reasonably independent of other textbooks. I have included therefore some general issues such as Berry’s phase, the statistical mechanics of fluctuating membranes, a quick overview of itinerant-electron magnetism, a brief discussion of basic nonequilibrium statistical mechanics, etc. The aims were, first, to show the physics of graphene in a more general context and, second, to make the reading easier.

It is very difficult to give an overview of a field that has developed so quickly as has that of graphene. So many papers appear, literally every day, that keeping permanently up to date would be an enterprise in the style of ancient myths, e.g., those of Sisyphus, the Danaïdes and some of the labours of Hercules. I apologise therefore for the lack of many important references. I tried to do my best.

I cannot even list all of the scientific *reviews* on the basic physics of graphene which are available now (let alone reviews of applications and of popular literature). Let me mention at least several of them, in chronological order: Katsnelson (2007a), Geim & Novoselov (2007), Beenakker (2008),

Castro Neto *et al.* (2009), Geim (2009), Abergel *et al.* (2010), Vozmediano, Katsnelson & Guinea (2010), Peres (2010), Das Sarma *et al.* (2011), Goerbig (2011) and Kotov *et al.* (2011). There you can find different, complementary views on the field (with the possible exception of the first one). Of course, the Nobel lectures by Geim (2011) and Novoselov (2011) are especially strongly recommended. In particular, the lecture by Andre Geim contains a brilliant presentation of the prehistory and history of graphene research, so I do not need to discuss these, unavoidably controversial, issues in my book.

I am very grateful to Andre Geim and Kostya Novoselov, who involved me in this wonderful field before it became fashionable (otherwise I would probably never have dared to join such a brilliant company). I am especially grateful to Andre for regular and long phone conversations; when you have to discuss a theory using just words, without formulas and diagrams, and cannot even make faces, after several years it does improve your understanding of theoretical physics.

It is impossible to thank all my other collaborators in the field of graphene in a short preface, as well as other colleagues with whom I have had fruitful discussions. I have to thank, first of all, Annalisa Fasolino, Paco Guinea, Sasha Lichtenstein and Tim Wehling for especially close and intensive collaboration. I am very grateful to the former and current members of our group in Nijmegen working on graphene: Misha Akhukov, Danil Boukhvalov, Jan Los, Koen Reijnders, Rafa Roldán, Timur Tudorovskiy, Shengjun Yuan and Kostya Zakharchenko, and to my other collaborators and coauthors, especially Mark Auslender, Eduardo Castro, Hans De Raedt, Olle Eriksson, Misha Fogler, Jos Giesbers, Leonya Levitov, Tony Low, Jan Kees Maan, Hector Ochoa, Marco Polini, Sasha Rudenko, Mark van Schilfgaarde, Andrey Shytov, Alyosha Tselik, Maria Vozmediano, Oleg Yazyev and Uli Zeitler.

I am grateful to the Faculty of Science of Radboud University and the Institute for Molecules and Materials for making available to me the time and resources for research and writing.

I am very grateful to Marina Katsnelson and Timur Tudorovskiy for their invaluable help with the preparation of the manuscript and for their critical reading. I am grateful to many colleagues for permission to reproduce figures from their papers and for providing some of the original figures used in the book. I am especially grateful to Annalisa Fasolino for the wonderful picture which is used for the cover.

Of course, the role of my wife Marina in this book amounts to much more than her help with the manuscript. You cannot succeed in such a long and demanding task without support from your family. I am very grateful for her understanding and full support.

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The book is dedicated to the memory of two people who were very close to me, my teacher Sergey Vonsovsky (1910–1998) and my friend Sasha Trefilov (1951–2003). I worked with them for about twenty years, and they had a decisive influence on the formation of my scientific taste and my scientific style. I thought many times during these last seven years how sad it is that I cannot discuss with them some new interesting physics about graphene. Also, in a more technical sense, I would not have been able to write this book without the experience of writing my previous books, Vonsovsky & Katsnelson (1989) and Katsnelson & Trefilov (2002).