FRACTURE MECHANICS

Fracture and “slow” crack growth reflect the response of a material (i.e., its microstructure) to the conjoint actions of mechanical and chemical driving forces and are affected by temperature. Therefore, there is a need for quantitative understanding and modeling of the influences of chemical and thermal environments, and of microstructure, in terms of the key internal and external variables and for their incorporation into design and probabilistic implications. This text, which the author has used in a fracture mechanics course for advanced undergraduate and graduate students, is based on the work of the author’s Lehigh University team whose integrative research combined fracture mechanics, surface and electrochemistry, materials science, and probability and statistics to address a range of fracture safety and durability issues on aluminum, ferrous, nickel, and titanium alloys, and ceramics. Examples from this research are included to highlight the approach and applicability of the findings in practical durability and reliability problems.

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Fracture Mechanics

INTEGRATION OF MECHANICS, MATERIALS SCIENCE, AND CHEMISTRY

Robert P. Wei
Lehigh University
To Lee

For her love, counsel, dedication, and support
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Preface

Engineering Fracture Mechanics, as a recognized branch of engineering mechanics, had its beginning in the late 1940s and early 1950s, and experienced major growth through the next three decades. The initial efforts were driven primarily by naval and aerospace interests. By the end of the 1980s, most of the readily tractable mechanics problems had been solved, and computational methods have become the norm in solving practical problems in fracture/structural integrity. On the living (“slow” crack growth) side, the predominant emphasis has been on empirical characterization and usage of data for life prediction and reliability assessments.

In reality, fracture and “slow” crack growth reflect the response of a material (i.e., its microstructure) to the conjoint actions of mechanical and chemical driving forces, and are affected by temperature. The need for quantitative understanding and modeling of the influences of chemical and thermal environments and of microstructure (i.e., in terms of the key internal and external variables), and for their incorporation into design, along with their probabilistic implications, began to be recognized in the mid-1960s.

With support from AFOSR, ALCOA, DARPA, DOE (Basic Energy Sciences), FAA, NSF, ONR, and others, from 1966 to 2008, the group at Lehigh University undertook integrative research that combined fracture mechanics, surface and electrochemistry, materials science, and probability and statistics to address a range of fracture safety and durability issues on aluminum, ferrous, nickel, and titanium alloys and on ceramics. Examples from this research are included to highlight the approach and applicability of the findings in practical problems of durability and reliability. An appended list of publications provides references/sources for more detailed information on research from the overall program.

The title Fracture Mechanics: Integration of Fracture Mechanics, Materials Science, and Chemistry gives tribute to those who have shared the vision and have contributed to and supported this long-term, integrative effort, and to those who recognize the need and value for this multidisciplinary team effort.

The author has used the material in this book in a fracture mechanics course for advanced undergraduate and graduate students at Lehigh University. This book should also serve as a reference for the design and management of engineered systems.
Acknowledgments

The author acknowledges the invaluable contributions and dedication of his colleagues: Dr. Ye T. (Russell) Chou (Materials Science), Dr. Kamil Klier (Surface Chemistry), Dr. Gary Simmons (Surface Chemistry), Dr. D. Gary Harlow (Probability and Statistics/Mechanical Engineering & Mechanics), and Dr. Ming Gao (Materials Science), and the many postdoctoral researchers and graduate students in Mechanical Engineering and Mechanics, Materials Science and Engineering, and Surface Science and Electrochemistry, who made this possible. The author also acknowledges the International Multimedia Resource Center (IMRC) of Lehigh University, under the leadership of Johanna Brams, especially Nyko DePeyer and Dawn Dayawon, for their assistance in graphic arts and manuscript preparation, and Sharon Siegler, Lehigh University librarian, for her counsel and expert assistance.