Supergravity

Supergravity, together with string theory, is one of the most significant developments in theoretical physics. Although there are many books on string theory, this is the first-ever authoritative and systematic account of supergravity.

Written by two of the most respected workers in the field, it provides a solid introduction to the fundamentals of supergravity. The book starts by reviewing aspects of relativistic field theory in Minkowski spacetime. After introducing the relevant ingredients of differential geometry and gravity, some basic supergravity theories ($D = 4$ and $D = 11$) and the main gauge theory tools are explained. The second half of the book is more advanced: complex geometry and $N = 1$ and $N = 2$ supergravity theories are covered. Classical solutions and a chapter on anti-de Sitter/conformal field theory (AdS/CFT) correspondence complete the text.

Numerous exercises and examples make it ideal for Ph.D. students, and with applications to model building, cosmology, and solutions of supergravity theories, this text is an invaluable resource for researchers. A website hosted by the authors, featuring solutions to some exercises and additional reading material, can be found at www.cambridge.org/supergravity.

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The metric is ‘mostly plus’, i.e. $- + \ldots +$. The curvature is

$$R_{\mu\nu\rho\sigma} = g_{\rho\sigma}(\partial_\mu \Gamma^\rho_{\nu\sigma} - \partial_\nu \Gamma^\rho_{\mu\sigma} + \Gamma^\rho_{\mu\tau} \Gamma^\tau_{\nu\sigma} - \Gamma^\rho_{\nu\tau} \Gamma^\tau_{\mu\sigma})$$

$$= \epsilon^{\rho\sigma}_{\lambda\mu}(\partial_\mu \omega_{\lambda\nuab} - \partial_\nu \omega_{\lambda\muab} + \omega_{\lambda\nuac} \omega^c_{\lambda\mu} - \omega_{\lambda\muac} \omega^c_{\lambda\nu})$$.

Ricci tensor and energy–momentum tensors are defined by

$$R_{\mu\nu} = R^\rho_{\nu\rho\mu}, \quad R = g^{\mu\nu} R_{\mu\nu}.$$  

Covariant derivatives involving the spin connection are, for vectors and spinors,

$$D_\mu V^a = \partial_\mu V^a + \omega^a_{\mu\nu} V^b \eta^{\nu b}, \quad D_\mu \lambda = \partial_\mu \lambda + \frac{1}{4} \omega^a_{\mu\nu} \gamma^{a\nu} \lambda.$$  

We use (anti)symmetrization of indices with ‘weight 1’, i.e.

$$A_{[ab]} = \frac{1}{2} (A_{ab} - A_{ba}) \quad \text{and} \quad A_{(ab)} = \frac{1}{2} (A_{ab} + A_{ba}).$$

The Levi-Civita tensor is

$$\varepsilon_{0123} = 1, \quad \varepsilon^{0123} = -1.$$  

The dual, self-dual, and anti-self-dual of antisymmetric tensors are defined by

$$\tilde{H}^{ab} = -\frac{1}{2} i \varepsilon^{abcd} H_{cd}, \quad H^\pm_{ab} = \frac{1}{2} (H_{ab} \pm \tilde{H}_{ab}), \quad H^\pm_{ab} = (H^\mp_{ab})^*.$$  

Structure constants are defined by

$$[T_A, T_B] = f_{ABC} T_C.$$  

The Clifford algebra is

$$\gamma_\mu \gamma_\nu + \gamma_\nu \gamma_\mu = 2 g_{\mu\nu}, \quad \gamma_{\mu\nu} = \gamma_{[\mu} \gamma_{\nu]}, \ldots,$$

$$(\gamma^\mu)^* = \gamma^0 \gamma^\mu \gamma^0,$$

$$\gamma_a = (-1)^{(D/2)+1} \gamma_0 \gamma_1 \ldots \gamma_D,$$

in four dimensions:

$$\gamma_a = i \gamma_0 \gamma_1 \gamma_2 \gamma_3, \quad \varepsilon_{abcd} \gamma^d = i \gamma_a \gamma_{bc}.$$  

The Majorana and Dirac conjugates are

$$\tilde{\chi} = \chi^T C, \quad \bar{\chi} = i \chi^\dagger \gamma^0.$$  

We mostly use the former. For Majorana fermions the two are equal.

The main SUSY commutator is

$$[\delta(\epsilon_1), \delta(\epsilon_2)] = \frac{1}{2} \varepsilon^2 \gamma^\mu \epsilon_1 \partial_\mu.$$  

$p$-form components are defined by

$$\Phi_p = \frac{1}{p!} \Phi_{\mu_1 \ldots \mu_p} d x^{\mu_1} \wedge \cdots \wedge d x^{\mu_p}. $$

The differential acts from the left:

$$d A = \partial_\mu A_\mu d x^\nu \wedge d x^\mu, \quad A = A_\mu d x^\mu.$$
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The main purpose of this book is to explore the structure of supergravity theories at the classical level. Where appropriate we take a general $D$-dimensional viewpoint, usually with special emphasis on $D = 4$. Readers can consult the Contents for a detailed list of the topics treated, so we limit ourselves here to a few comments to guide readers. We have tried to organize the material so that readers of varying educational backgrounds can begin to read at a point appropriate to their background. Part I should be accessible to readers who have studied relativistic field theory enough to appreciate the importance of Lagrangians, actions, and their symmetries. Part II describes the differential geometric background and some basic physics of the general theory of relativity. The basic supergravity theories are presented in Part III using techniques developed in earlier chapters. In Part IV we discuss complex geometry and apply it to matter couplings in global $\mathcal{N} = 1$ supersymmetry. In Part V we begin a systematic derivation of $\mathcal{N} = 1$ matter-coupled supergravity using the conformal compensator method. The going can get tough on this subject. For this reason we present the final physical action and transformation rules and some basic applications in two separate short chapters in Part VI. Part VII is devoted to a systematic discussion of $\mathcal{N} = 2$ supergravity, including a short chapter with the results needed for applications. Two major applications of supergravity, classical solutions and the AdS/CFT correspondence, are discussed in Part VIII in considerable detail. It should be possible to understand these chapters without full study of earlier parts of the book.

Many interesting aspects of supergravity, some of them subjects of current research, could not be covered in this book. These include theories in spacetime dimensions $D < 4$, higher derivative actions, embedding tensors, infinite Lie algebra symmetries, and the positive energy theorem.

Like many other subjects in theoretical physics, supersymmetry and supergravity are best learned by readers who are willing to ‘get their hands dirty’. This means actively working out problems that reinforce the material under discussion. To facilitate this aspect of the learning process, many exercises for readers appear within each chapter. We give a rough indication of the level of each exercise as follows:
Preface

Level 1. The result of this exercise will be used later in the book.
Level 2. This exercise is intended to illuminate the subject under discussion, but it is not needed in the rest of the book.
Level 3. This exercise is meant to challenge readers, but is not essential.

These levels are indicated respectively by single, double or triple gray bars in the outside margin.

A website featuring solutions to some exercises, errata and additional reading material, can be found at www.cambridge.org/supergravity.

Dan Freedman
Toine Van Proeyen
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