A Student's Guide to Vectors and Tensors

Vectors and tensors are among the most powerful problem-solving tools available, with applications ranging from mechanics and electromagnetics to general relativity. Understanding the nature and application of vectors and tensors is critically important to students of physics and engineering.

Adopting the same approach as in his highly popular *A Student's Guide to Maxwell's Equations*, Fleisch explains vectors and tensors in plain language. Written for undergraduate and beginning graduate students, the book provides a thorough grounding in vectors and vector calculus before transitioning through contra and covariant components to tensors and their applications. Matrices and their algebra are reviewed on the book's supporting website, which also features interactive solutions to every problem in the text, where students can work through a series of hints or choose to see the entire solution at once. Audio podcasts give students the opportunity to hear important concepts in the book explained by the author.

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Preface

This book has one purpose: to help you understand vectors and tensors so that you can use them to solve problems. If you're like most students, you first encountered vectors when you took a course dealing with mechanics in high school or college. At that level, you almost certainly learned that vectors are mathematical representations of quantities that have both magnitude and direction, such as velocity and force. You may also have learned how to add vectors graphically and by using their components in the x-, y- and z-directions.

That's a fine place to start, but it turns out that such treatments only scratch the surface of the power of vectors. You can harness that power and make it work for you if you're willing to delve a bit deeper – to see vectors not just as objects with magnitude and direction, but rather as objects that behave in very predictable ways when viewed from different reference frames. That's because vectors are a subset of a larger class of objects called "tensors," which most students encounter much later in their academic careers, and which have been called "the facts of the Universe." It is no exaggeration to say that our understanding of the fundamental structure of the universe was changed forever when Albert Einstein succeeded in expressing his theory of gravity in terms of tensors.

I believe, and I hope you'll agree, that tensors are far easier to understand if you first establish a stronger foundation in vectors, one that can help you cross the bridge between the "magnitude and direction" level and the "facts of the Universe" level. That's why the first three chapters of this book deal with vectors, the fourth chapter discusses coordinate transformations, and the last two chapters discuss higher-order tensors and some of their applications.

One reason you may find this book helpful is that if you spend a few hours looking through the published literature and on-line resources for vectors and tensors in physics and engineering, you're likely to come across statements such as these:

"A vector is a mathematical representation of a physical entity characterized by magnitude and direction."

"A vector is an ordered sequence of values."

"A vector is a mathematical object that transforms between coordinate systems in certain ways."

"A vector is a tensor of rank one."

"A vector is an operator that turns a one-form into a scalar."

You should understand that every one of these definitions is correct, but whether it's useful to you depends on the problem you're trying to solve. And being able to see the relationship between statements like these should prove very helpful when you begin an in-depth study of subjects that use advanced vector and tensor concepts. Those subjects include Mechanics, Electromagnetism, General Relativity, and others.

As with most projects, a good first step is to make sure you understand the terminology that will be used to attack the problem. For that reason, Chapter 1 provides the basic definitions you'll need to begin understanding vectors and tensors. And if you're ready for more-advanced definitions, you can find those at the beginning of Chapter 5.

You may be wondering how this book differs from other texts that deal with vectors and/or tensors. Perhaps the most important difference is that approximately equal weight is given to vector and tensor concepts, with one entire chapter (Chapter 3) devoted to selected vector applications and another chapter (Chapter 6) dedicated to example tensor applications.

You'll also find the presentation to be very different from that of other books. The explanations in this book are written in an informal style in which mathematical rigor is maintained only insofar as it doesn't obscure the underlying physics. If you feel you already have a good understanding of vectors and may need only a quick review, you should be able to skim through Chapters 1 through 3 very quickly. But if you're a bit unclear on some aspects of vectors and how to apply them to problems, you may find these early chapters quite helpful. And if you've already seen tensors but are unsure of exactly what they are or how to apply them, then Chapters 4 through 6 may provide some insight.

As a student's guide, this book comes with two additional resources designed to help you understand and apply vectors and tensors: an interactive website and a series of audio podcasts. On the website, you'll find the complete solution to every problem presented in the text in interactive format – that means you'll be able to view the entire solution at once, or ask for a series of helpful hints that will guide you to the final answer. So when you see a statement in the text saying that you can learn more about something by looking at the end-of-chapter problems, remember that the full solution to every one of those problems is available to you. And if you're the kind of learner who benefits from hearing spoken words rather than just reading text, the audio podcasts are for you. These MP3 files walk you through each chapter of the book, pointing out important details and providing further explanations of key concepts.

Is this book right for you? It is if you're a science or engineering student and have encountered vectors or tensors in one of your classes, but you're not confident in your ability to apply them. In that case, you should read the book, listen to the accompanying podcasts, and work through the examples and problems before taking additional classes or a standardized exam in which vectors or tensors may appear. Or perhaps you're a graduate student struggling to make the transition from undergraduate courses and textbooks to the more-advanced material you're seeing in graduate school – this book may help you make that step.

And if you're neither an undergraduate nor a graduate student, but a curious young person or a lifelong learner who wants to know more about vectors, tensors, or their applications in Mechanics, Electromagnetics, and General Relativity, welcome aboard. I commend your initiative, and I hope this book helps you in your journey.

Acknowledgments

It was a suggestion by Dr. John Fowler of Cambridge University Press that got this book out of the starting gate, and it was his patient guidance and unflagging support that pushed it across the finish line. I feel very privileged to have worked with John on this project and on my *Student's Guide to Maxwell's Equations*, and I acknowledge his many contributions to these books. A project like this really does take a village, and many others should be recognized for their efforts. While pursuing her doctorate in Physics at Notre Dame University, Laura Kinnaman took time to carefully read the entire manuscript and made major contributions to the discussion of the Inertia tensor in Chapter 6. Wittenberg graduate Joe Fritchman also read the manuscript and made helpful suggestions, as did Carnegie-Mellon undergraduate Wyatt Bridgeman. Carrie Miller provided the perspective of a Chemistry student, and her husband Jordan Miller generously shared his LaTeX expertise. Professor Adam Parker of Wittenberg University and Daniel Ross of the University of Wisconsin did their best to steer me onto a mathematically solid foundation, and Professor Mark Semon of Bates College has gone far beyond the role of reviewer and deserves credit for rooting out numerous errors and for providing several of the better explanations in this work. I alone bear the responsibility for any remaining inconsistencies or errors.

I also wish to acknowledge all the students who have taken a class from me during the two years it took me to write this book. I very much appreciate their willingness to share their claim on my time with this project. The greatest sacrifice has been made by the unfathomably understanding Jill Gianola, who gracefully accommodated the expanding time and space requirements of my writing.