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978-0-521-19323-8 - X-Parameters: Characterization, Modeling, and Design of Nonlinear RF and Microwave Components
David E. Root, Jan Verspecht, Jason Horn and Mihai Marcu
Excerpt
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1 S-parameters – a concise review

1.1 Introduction

This chapter presents a concise treatment of S-parameters, meant primarily as an introduction to the more general formalism of X-parameters. The concepts of *time invariance* and *spectral maps* are introduced at this stage to enable an easier generalization to X-parameters in the ensuing chapters. The interpretations of S-parameters as calibrated measurements, intrinsic properties of the device under test (DUT), IP-secure component behavioral models, and composition rules for linear system design are presented. The cascade of two linear S-parameter components is considered as an example to be generalized to the nonlinear case later. The calculation of S-parameters for a transistor from a simple nonlinear device model is used as an example to introduce the concepts of (static) *operating point* and *small-signal conditions*, both of which must be generalized for the treatment of X-parameters.

1.2 S-parameters

Since the 1950s, S-parameters, or scattering parameters, have been among the most important of all the foundations of microwave theory and techniques.

S-parameters are easy to measure at high frequencies with a vector network analyzer (VNA). Well-calibrated S-parameter measurements represent intrinsic properties of the DUT, independent of the VNA system used to characterize it. Calibration procedures [1] remove systematic measurement errors and enable a separation of the overall values into numbers attributable to the device, independent of the measurement system used to characterize it. These DUT properties (gain, loss, reflection coefficient, etc.) are familiar, intuitive, and important [2]. Another key property of S-parameters is that the S-parameters of a composite system are completely determined from knowledge of the S-parameters of the constituent components and their connectivity. S-parameters provide the complete specification of how a linear component responds to an arbitrary signal. Therefore designs of linear systems with S-parameters are predictable with absolute certainty. S-parameters define a complete behavioral description of the linear component at the external terminals, independent of the detailed physics or specifics of the realization of the component. S-parameters can be shared between component vendors and system integrators freely, without the possibility that the component implementation can be reverse engineered, protecting IP and promoting sharing and reuse. Indeed, one may ask the question, "are S-parameters measurements, or do they constitute a model?" The answer is really "both."

S-parameters need not come only from measurements. They can be calculated from physics by solving Maxwell's equations, by linearizing the semiconductor equations, or computed from matrix analysis of linear equivalent circuits. In this way, the many benefits of S-parameters can be realized, starting from a more detailed representation of the component from first principles or from a complicated linear circuit model.

Graphical methods based around the Smith chart were invented to visualize and interpret S-parameters, and graphical design methodologies soon followed for circuit design [2][3]. These days, electronic design automation (EDA) tools provide simulation components – S-parameter blocks – and design capabilities using the familiar S-parameter analysis mode.

One of the great utilities of S-parameters is the interoperability among the measurement, modeling, and design capabilities they provide. One can characterize the component with measured S-parameters, use them as a high-fidelity behavioral model of the component with complete IP protection, and design systems with them in the EDA environment.

1.3 Wave variables

The term "scattering" refers to the relationship between incident and scattered (reflected and transmitted) traveling waves.

By convention, in this text the circuit behavior is described using generalized power waves [2]. There are alternative wave definitions used in the industry. These are briefly reviewed in Appendix A, together with the general notations used in this text.

The wave variables, A and B, corresponding to a specific port of a network, are defined as simple linear combinations of the voltage and current, V and I, at the same port, according to Figure 1.1 and equations (1.1):

$$A = \frac{V + Z_0 I}{2\sqrt{Z_0}}, \qquad B = \frac{V - Z_0 I}{2\sqrt{Z_0}}. \tag{1.1}$$

The reference impedance for the port, Z_0, is, in general, a complex value. For the purpose of simplifying the concepts presented, the reference impedance is restricted to real values in this text.

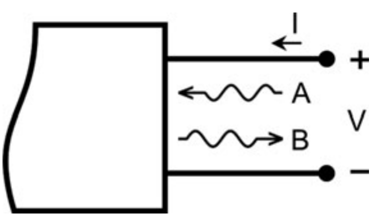

Figure 1.1 Wave definitions.

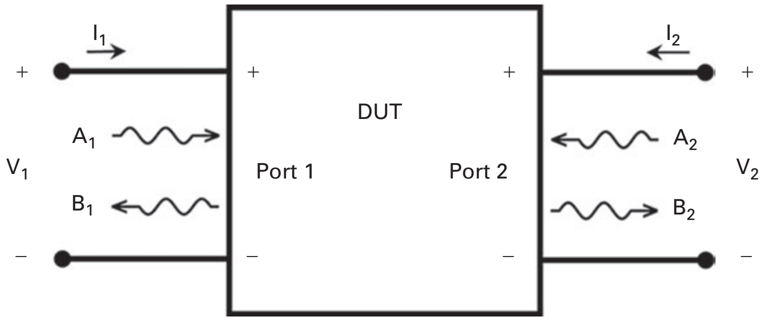

Figure 1.2 Incident and scattered waves of a two-port device.

The currents and voltages can be recovered from the wave variables, according to equations (1.2):

$$\begin{aligned} V &= \sqrt{Z_0}\,(A+B), \\ I &= \frac{1}{\sqrt{Z_0}}\,(A-B). \end{aligned} \tag{1.2}$$

Here, A and B represent the incident and scattered waves, V and I are the port voltage and current, respectively, and Z_0 is the reference impedance for the port. A typical value of Z_0 is 50 Ω by convention, but other choices may be more practical for some applications. A value for Z_0 closer to 1 Ω is more appropriate for S-parameter measurements of power transistors, for example, given that power transistors typically have very small output impedances.

The variables in equations (1.1) and (1.2) are complex numbers representing the RMS-phasor description of sinusoidal signals in the frequency domain (see Appendix A for a more detailed discussion of the notations used). Later this will be generalized to the envelope domain by letting these complex numbers vary in time.

A, B, V, and I can be considered RMS vectors, the components of which indicate the values associated with sinusoidal signals at particular ports labeled by positive integers. Thus A_j is the incident wave RMS phasor at port j and I_k is the current RMS phasor at port k. For now, Z_0 is taken to be a fixed real constant, in particular, 50 Ω.

A graphical representation of the wave description is given in Figure 1.2.

To retrieve the time-dependent sinusoidal voltage signal at the ith port, the complex value of the phasor and also the angular frequency, ω, to which the phasor corresponds, must be known. The voltage is then given by

$$v_i(t) = \mathrm{Re}\{V_i^{(pk)} e^{j\omega t}\}, \tag{1.3}$$

and similarly for the other variables, where $V_i^{(pk)}$ are peak values. Equation (1.3) for the voltage, and a similar equation for the time-dependent current, can be used to define real, time-dependent "wave" quantities using the same linear combinations as in (1.1):

$$\begin{aligned} a(t) &= \frac{1}{2\sqrt{Z_0}}\,\big(v(t) + Z_0 i(t)\big), \\ b(t) &= \frac{1}{2\sqrt{Z_0}}\,\big(v(t) - Z_0 i(t)\big); \end{aligned} \tag{1.4}$$

$$v(t) = \sqrt{Z_0}\,\big(a(t) + b(t)\big),$$
$$i(t) = \frac{1}{\sqrt{Z_0}}\,\big(a(t) - b(t)\big). \tag{1.5}$$

It is convenient to keep track of the frequency associated with a particular set of phasors by rewriting (1.1) according to (1.6), and (1.2) according to (1.7), where the port indexing notation is made explicit:

$$A_i(\omega) = \frac{1}{2\sqrt{Z_0}}\,\big(V_i(\omega) + Z_0 I_i(\omega)\big),$$
$$B_i(\omega) = \frac{1}{2\sqrt{Z_0}}\,\big(V_i(\omega) - Z_0 I_i(\omega)\big); \tag{1.6}$$

$$V_i(\omega) = \sqrt{Z_0}\,\big(A_i(\omega) + B_i(\omega)\big),$$
$$I_i(\omega) = \frac{1}{\sqrt{Z_0}}\big(A_i(\omega) - B_i(\omega)\big). \tag{1.7}$$

For each angular frequency, ω, (1.6) is a set of two equations defined at each port.

The assumption behind the S-parameter formalism is that the system being described is *linear* and therefore there must be a *linear relationship* between the phasor representation of incident and scattered waves. This is expressed in (1.8) for an N-port network as follows:

$$B_i(\omega) = \sum_{j=1}^{N} S_{ij}(\omega) A_j(\omega), \qquad \forall i \in \{1, 2, \ldots, N\}. \tag{1.8}$$

The set of complex coefficients, $S_{ij}(\omega)$, in (1.8) defines the S-parameter matrix or, simply, the S-parameters at that frequency. Equation (1.8), for the fixed set of complex S-parameters, determines the output phasors for any set of input phasors. The summation is over all port indices, so that incident waves at each port, j, contribute in general to the overall scattered wave at each output port, i. For now we consider all frequencies to be positive ($\omega > 0$). Note that contributions to a scattered wave at frequency ω come only from incident waves at the same frequency. This is not the case for the more general X-parameters, where a stimulus at one frequency can lead to scattered waves at different frequencies.

The set of equations (1.8) represents a model of the network under study. However, this model is valid only if the network has the topological connections shown in Figure 1.2. For example, the model might not accurately represent the behavior of the network when connected as shown in Figure 1.3 because potential losses between the reference pins of the two ports are not individually identified in the set of S-parameters in (1.8).

For the purpose of creating a model for the network, all ports should be referenced to the same pin, as shown in Figure 1.4.

Such connectivity is the natural option for the measurement and modeling process of a three-pin network (like a transistor), but it has to be extended in the general case of an arbitrary network, and it is necessary for all networks, linear and/or nonlinear. This connectivity convention is considered by default (unless otherwise specified) for the remainder of this text.

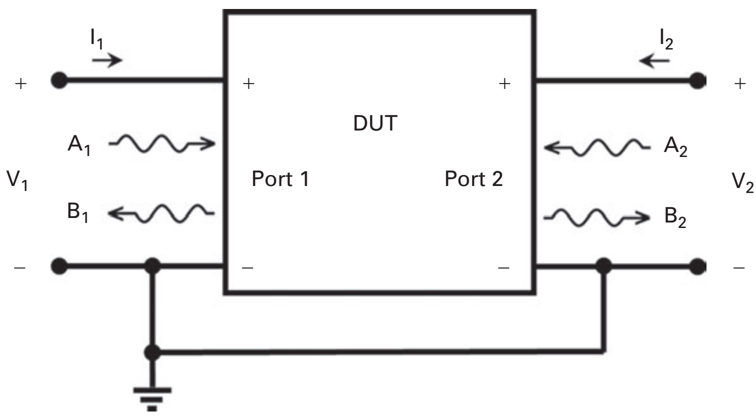

Figure 1.3 Potential losses between reference pins are not individually identified by the model in (1.8).

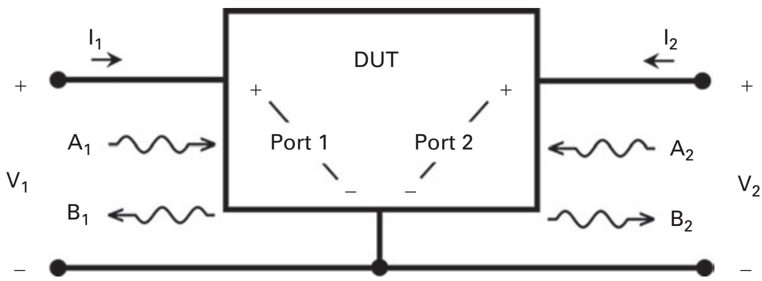

Figure 1.4 All ports should be referenced to the same pin for modeling purposes.

Using the topological connection in Figure 1.4, the set of equations (1.8) represents a complete model of the network under test.

From (1.8) we note that a stimulus (incident wave) at a particular port j will produce a response (scattered wave) at all ports, including the port at which the stimulus is applied.

Equation (1.8) shows that the scattered waves are linear functions of the complex amplitudes (the phasors) of the incident waves. The dependence on frequency of the S-parameter matrix elements, $S_{ij}(\omega)$, can be highly nonlinear, even for a linear device. For example, an ideal band-pass filter response is linear in the incident wave variable, but the filter response is a nonlinear function of the frequency of the incident wave. This is shown in Figure 1.5.

1.4 S-parameter measurement

By setting all incident waves to zero in (1.8), except for A_j, one can deduce the simple relationship between a given S-parameter (S-parameter matrix element) and a particular ratio of scattered to incident waves according to (1.9):

$$S_{ij}(\omega) = \left. \frac{B_i(\omega)}{A_j(\omega)} \right|_{\substack{A_k=0 \\ \forall k \neq j}} . \tag{1.9}$$

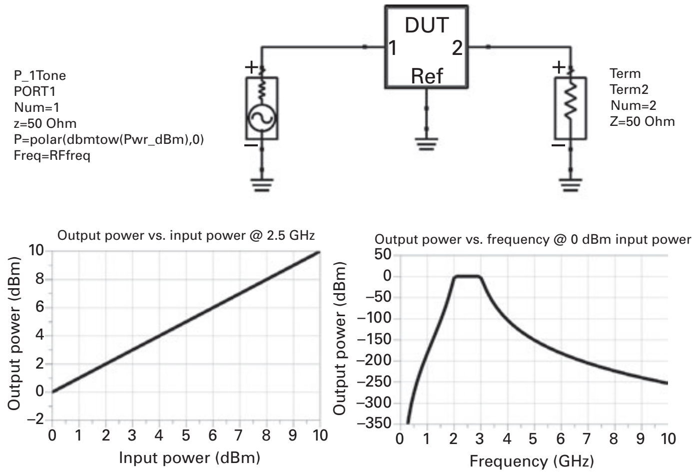

Figure 1.5 A linear network has a linear behavior when plotted versus input power level, but the dependence on frequency is usually not linear.

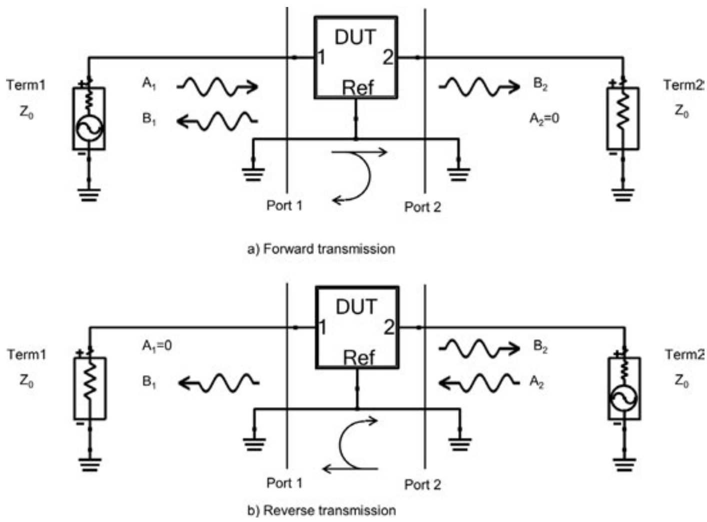

Figure 1.6 S-parameter experiment design: (a) forward transmission; (b) reverse transmission.

Equation (1.9) corresponds to a simple graphical representation shown in Figure 1.6 for the simple case of a two-port component. In Figure 1.6(a), the stimulus is a wave incident at port 1. The fact that A_2 is not present ($A_2 = 0$) is interpreted to mean that the B_2 wave scattered and traveling away from port 2 is not reflected back into the device at port 2. Under this condition, the device is said to be *perfectly matched* at port 2. Two of the four complex S-parameters, specifically S_{11} and S_{21}, can be identified using (1.9) for this case of exciting the device with only A_1. Figure 1.6(b) shows the case where the device is stimulated with a signal, A_2, at port 2, and assumed to be perfectly matched at port 1 ($A_1 = 0$). The remaining S-parameters, S_{12} and S_{22}, can be identified from this ideal experiment.

It is important to note that the ratio on the right-hand side of (1.9) can be computed from independent measurements of incident and scattered waves for actual components corresponding to any non-zero value for the incident wave, A_j. The value of this ratio, however, will generally vary with the magnitude of the incident wave. Therefore, the identification of this ratio with "the S-parameters" of the component is valid for any particular value of incident A_j only if the component behaves linearly, namely according to (1.8). In other words, the values of the incident waves, A_j, need to be in the linear region of operation for this identification to be valid. For nonlinear components, such as transistors biased at a fixed voltage, the scattered waves eventually do not increase as the incident waves become larger in magnitude (this is compression). Therefore, different values of (1.9) result from different values of incident waves. A better definition of S-parameters for a *nonlinear component* is a modification of (1.9), given by (1.10):

$$S_{ij}(\omega) \equiv \lim_{|A_j|\to 0} \frac{B_i(\omega)}{A_j(\omega)}\bigg|_{\substack{A_k=0 \\ \forall k\neq j}}. \tag{1.10}$$

That is, for a general component, bias at a constant DC stimulus, the S-parameters are related to ratios of output responses to input stimuli in the limit of small input signals. This emphasizes that S-parameters properly apply to nonlinear components only in the *small-signal* limit.

1.5 S-parameters as a spectral map

If there are multiple frequencies present in the input spectrum, one can represent the output spectrum in terms of a matrix giving the contributions to each output frequency from each input frequency.

An example in the case of three input frequencies is given by equation (1.11):

$$\begin{bmatrix} B(\omega_1) \\ B(\omega_2) \\ B(\omega_3) \end{bmatrix} = \begin{bmatrix} S(\omega_1) & 0 & 0 \\ 0 & S(\omega_2) & 0 \\ 0 & 0 & S(\omega_3) \end{bmatrix} \begin{bmatrix} A(\omega_1) \\ A(\omega_2) \\ A(\omega_3) \end{bmatrix}. \tag{1.11}$$

Here we assume a single port, for simplicity, and therefore drop the port indices. It is clear from (1.11) that S-parameters are a diagonal map in frequency space. This means that each output frequency contains contributions only from inputs at that same frequency. Or, in other words, each input frequency never contributes to outputs at any different frequency.

A graphical representation is given in Figure 1.7 for the case of forward transmission through a two-port network with matched terminations at both ports.

The interpretation of Figure 1.7, mathematically represented by (1.11), is that S-parameters define a particularly simple *linear spectral map* relating incident to scattered waves. S-parameters are diagonal in the frequency part of the map, namely they predict a response only at the particular frequencies of the corresponding input stimuli. It will be demonstrated in later chapters that X-parameters provide for richer behavior.

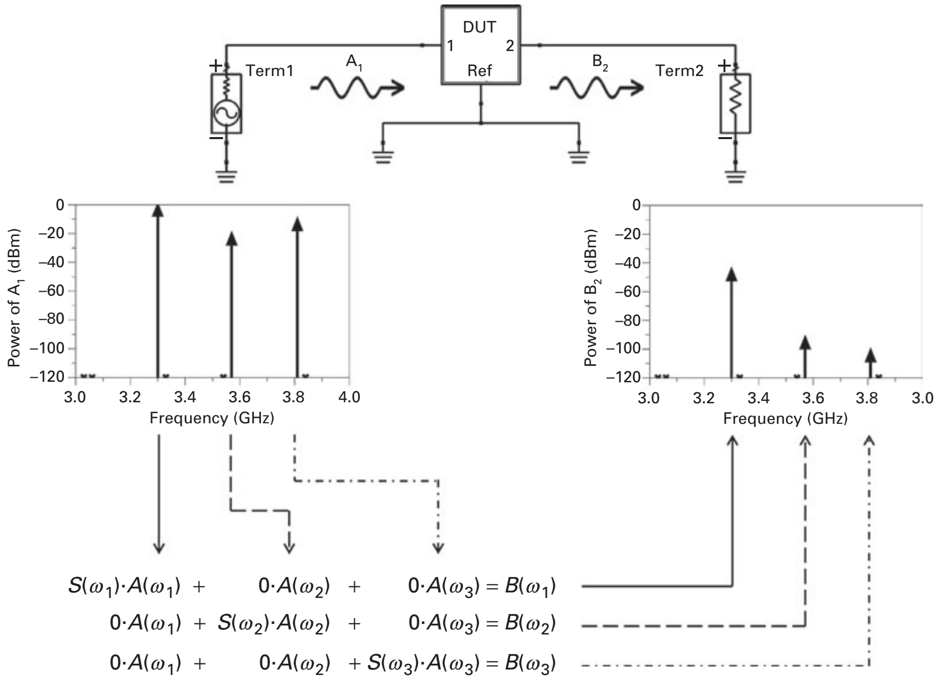

Figure 1.7 Linear spectral map through S-parameters matrix.

For signals with a continuous spectrum, the diagonal nature of the S-parameter spectral map can be written as equation (1.12):

$$B_i(\omega) = \sum_{j=1}^{N} \int S_{ij}(\omega)\delta(\omega - \omega')A_j(\omega')d\omega', \qquad \forall i \in \{1, 2, \ldots, N\}. \tag{1.12}$$

Performing the integral over input frequencies in (1.12) results in equation (1.8), the form usually given for S-parameters.

1.6 Superposition

Any linear theory, such as S-parameters, enables the general response to an arbitrary input signal to be computed by *superposition* of the responses to unit stimuli. Superposition enables great simplifications in analysis and measurement. Superposition is the reason S-parameters can be measured by independent experiments with one sinusoidal stimulus at a time, one stimulus per port per frequency using (1.9). The general response to any set of input signals can be obtained by superposition using (1.8).

An example of superposition is shown in Figure 1.8, with all signals represented in both time and frequency domains.

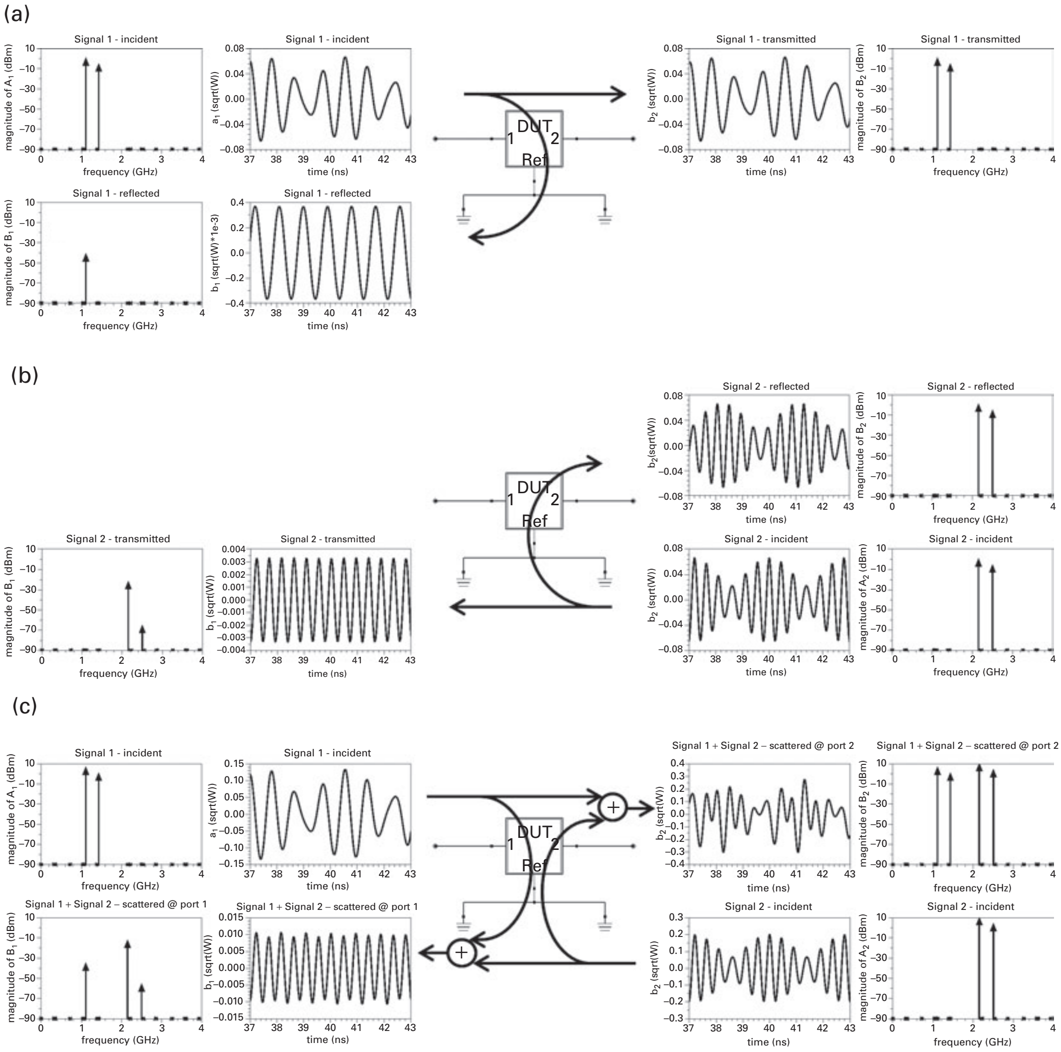

Figure 1.8 Superposition example. (a) Stimulus = signal 1 ⇒ response 1. (b) Stimulus = signal 2 ⇒ response 2. (c) Stimulus = 2 (signal 1) + 3 (signal 2); response = 2 (response 1) + 3 (response 2).

This example uses two signals, each containing two frequency components, as stimuli incident at each port, independently, with the other port perfectly matched. The example shows that the response to a linear combination of the stimuli is the same linear combination of the individual responses.

As always, the caveat is that the component actually behaves linearly over the range of signal levels used to stimulate the device. There is no a-priori way to know whether a component will behave linearly without precise knowledge about its composition or physical measured characteristics.

1.7 Time invariance of components described by S-parameters

A DUT description in terms of S-parameters defined by (1.8) naturally embodies an important principle known as *time invariance.* Time invariance states that if $y(t)$ is the DUT response to an excitation $x(t)$, the DUT response to the time-shifted excitation, $x(t-\tau)$, must be $y(t-\tau)$. This must be true for all time shifts, τ. That is, if the input is shifted in time, the output is shifted by the corresponding amount, but is otherwise identical with the DUT response to the non-shifted input. This is stated mathematically in equation (1.13), where O is the operator taking input to output:

$$\forall \tau \in \mathbb{R} \qquad y(t) = O\big[x(t)\big] \Rightarrow y(t-\tau) = O\big[x(t-\tau)\big]. \tag{1.13}$$

Time invariance is a property of common linear and nonlinear components, such as passive inductors, capacitors, resistors, and diodes, and active devices, such as transistors. Examples of components not time invariant (in the usual sense) are oscillators and other autonomous systems.

The proof follows from elementary properties of the Fourier transform, where a phase shift by $e^{j\omega\tau}$ in the frequency domain corresponds to a time shift of τ in the time domain. The time-domain waves incident at the ports (the stimuli) are $a_k(t)$, and their Fourier transforms are $A_k^{(pk)}(\omega)$, as in equation (1.14):

$$\mathfrak{F}\{a_k(t)\} = A_k^{(pk)}(\omega). \tag{1.14}$$

The time-domain waves scattered from the ports (the response) are $b_i(t)$, and their Fourier transforms are $B_i^{(pk)}(\omega)$, as in equation (1.15):

$$\mathfrak{F}\{b_i(t)\} = B_i^{(pk)}(\omega) = \sum_{k=1}^{N} S_{ik}(\omega) A_k^{(pk)}(\omega) = \sum_{k=1}^{N} S_{ik}(\omega)\, F\{a_k(t)\}. \tag{1.15}$$

If all stimuli are delayed with the same time delay, τ, the response becomes

$$\begin{aligned}\mathfrak{F}^{-1}\left\{\sum_{k=1}^{N} S_{ik}(\omega)\, \mathfrak{F}\{a_k(t-\tau)\}\right\} &= \mathfrak{F}^{-1}\left\{\sum_{k=1}^{N} S_{ik}(\omega)\, \mathfrak{F}\{a_k(t)\} e^{j\omega\tau}\right\} \\ &= \mathfrak{F}^{-1}\left\{B_i^{(pk)}(\omega) e^{j\omega\tau}\right\} = b_i(t-\tau).\end{aligned} \tag{1.16}$$

Equation (1.16) proves that S-parameters are automatically consistent with the principle of time invariance. Therefore, any set of S-parameters describes a time-invariant system.

Unlike the case for S-parameters, a more general (e.g. nonlinear) relationship between incident A waves and scattered B waves is not automatically consistent with the property (1.13) of time invariance. This will be demonstrated in Chapter 2. Therefore, in order to have a consistent representation of a nonlinear time-invariant DUT, the time-invariance property is manifestly incorporated into the mathematical formulation of X-parameters relating input to output waves. A representation of a time-invariant DUT by equations not consistent with (1.13) means the model is fundamentally wrong, and can yield very inaccurate results for some signals, even if the model "fitting" (or identification) appears good at time t.