INTRODUCTION TO STATISTICAL FIELD THEORY

Knowledge of the renormalization group and field theory is a key part of physics, and is essential in condensed matter and particle physics. Written for advanced undergraduate and beginning graduate students, this textbook provides a concise introduction to this subject.

The textbook deals directly with the loop expansion of the free energy, also known as the background field method. This is a powerful method, especially when dealing with symmetries and statistical mechanics. In focusing on free energy, the author avoids long developments on field theory techniques. The necessity of renormalization then follows.

EDOUARD BRÉZIN is Honorary Professor of Physics at the Ecole Polytechnique, and Emeritus Professor at the Ecole Normale Supérieure (Paris). A former President of the French Academy of Sciences, he is also a Foreign Member of the National Academy of Sciences (USA), the American Academy of Arts and Sciences, the Royal Society, and Academia Europaea.

Cambridge University Press & Assessment 978-0-521-19303-0 — Introduction to Statistical Field Theory Edouard Brézin Frontmatter <u>More Information</u>

INTRODUCTION TO STATISTICAL FIELD THEORY

EDOUARD BRÉZIN Ecole Normale Supérieure, Paris



Cambridge University Press & Assessment 978-0-521-19303-0 — Introduction to Statistical Field Theory Edouard Brézin Frontmatter More Information



Shaftesbury Road, Cambridge CB2 8EA, United Kingdom

One Liberty Plaza, 20th Floor, New York, NY 10006, USA

477 Williamstown Road, Port Melbourne, VIC 3207, Australia

314-321, 3rd Floor, Plot 3, Splendor Forum, Jasola District Centre, New Delhi - 110025, India

103 Penang Road, #05-06/07, Visioncrest Commercial, Singapore 238467

Cambridge University Press is part of Cambridge University Press & Assessment, a department of the University of Cambridge.

We share the University's mission to contribute to society through the pursuit of education, learning and research at the highest international levels of excellence.

www.cambridge.org Information on this title: www.cambridge.org/9780521193030

© E. Brézin 2010

This publication is in copyright. Subject to statutory exception and to the provisions of relevant collective licensing agreements, no reproduction of any part may take place without the written permission of Cambridge University Press & Assessment.

First published 2010

A catalogue record for this publication is available from the British Library

ISBN 978-0-521-19303-0 Hardback

Cambridge University Press & Assessment has no responsibility for the persistence or accuracy of URLs for external or third-party internet websites referred to in this publication and does not guarantee that any content on such websites is, or will remain, accurate or appropriate.

Cambridge University Press & Assessment 978-0-521-19303-0 — Introduction to Statistical Field Theory Edouard Brézin Frontmatter <u>More Information</u>

Contents

Preface

page ix

1	A few well-known basic results			
	1.1	The Boltzmann law	1	
		1.1.1 The classical canonical ensemble	1	
		1.1.2 The quantum canonical ensemble	2	
		1.1.3 The grand canonical ensemble	3	
	1.2	Thermodynamics from statistical physics	3	
		1.2.1 The thermodynamic limit	3	
	1.3	Gaussian integrals and Wick's theorem	4	
	1.4	Functional derivatives	6	
	1.5	<i>d</i> -dimensional integrals	6	
	Additional references			
2	Introduction: order parameters, broken symmetries			
	2.1	Can statistical mechanics be used to describe phase		
		transitions?	9	
	2.2	The order-disorder competition	10	
	2.3	Order parameter, symmetry and broken symmetry	12	
	2.4	More general symmetries	16	
	2.5	Characterization of a phase transition through		
		correlations	18	
	2.6	Phase coexistence, critical points, critical exponents	19	
3	Examples of physical situations modelled by the Ising model			
	3.1	Heisenberg's exchange forces	22	
	3.2	Heisenberg and Ising Hamiltonians	24	
	3.3	Lattice gas	26	
	3.4	More examples	28	
	3.5	A first connection with field theory	29	

vi		Contents	
4	A fev	v results for the Ising model	32
	4.1	One-dimensional Ising model: transfer matrix	32
	4.2	One-dimensional Ising model: correlation functions	35
	4.3	Absence of phase transition in one dimension	37
	4.4	A glance at the two-dimensional Ising model	38
	4.5	Proof of broken symmetry in two dimensions (and more)	38
	4.6	Correlation inequalities	42
	4.7	Lower critical dimension: heuristic approach	44
	4.8	Digression: Feynman path integrals, the transfer matrix and	
		the Schrödinger equation	47
5	High	-temperature and low-temperature expansions	52
	5.1	High-temperature expansion for the Ising model	52
		5.1.1 Continuous symmetry	55
	5.2	Low-temperature expansion	56
		5.2.1 Kramers–Wannier duality	57
	5.3	Low-temperature expansion for a continuous symmetry group	58
6	Some	e geometric problems related to phase transitions	60
	6.1	Polymers and self-avoiding walks	60
	6.2	Potts model and percolation	64
7	Phen	omenological description of critical behaviour	68
	7.1	Landau theory	68
	7.2	Landau theory near the critical point: homogeneous case	71
	7.3	Landau theory and spatial correlations	75
	7.4	Transitions without symmetry breaking: the liquid-gas	
		transition	78
	7.5	Thermodynamic meaning of $\Gamma\{m\}$	79
	7.6	Universality	80
	7.7	Scaling laws	82
8	Mear	n field theory	85
	8.1	Weiss 'molecular field'	85
	8.2	Mean field theory: the variational method	87
	8.3	A simpler alternative approach	92
9	Beyo	nd the mean field theory	95
	9.1	The first correction to the mean-field free energy	95
	9.2	Physical consequences	97
10	Intro	duction to the renormalization group	100
	10.1	Renormalized theories and critical points	101
	10.2	Kadanoff block spins	101
	10.3	Examples of real space renormalization groups: 'decimation'	103
	10.4	Structure of the renormalization group equations	109

Cambridge University Press & Assessment 978-0-521-19303-0 — Introduction to Statistical Field Theory Edouard Brézin Frontmatter <u>More Information</u>

		Contents	vii
11	Reno	rmalization group for the φ^4 theory	113
	11.1	Renormalization group without renormalization	114
	11.2	Study of the renormalization group flow in dimension four	116
	11.3	Critical behaviour of the susceptibility in dimension four	118
	11.4	Multi-component order parameters	120
	11.5	Epsilon expansion	122
	11.6	An exercise on the renormalization group: the cubic fixed	
		point	125
12	Reno	rmalized theory	128
	12.1	The meaning of renormalizability	128
	12.2	Renormalization of the massless theory	132
	12.3	The renormalized critical free energy (at one-loop order)	134
	12.4	Away from <i>T</i> _c	136
13	Golds	stone modes	138
	13.1	Broken symmetries and massless modes	138
	13.2	Linear and non-linear $O(n)$ sigma models	142
	13.3	Regularization and renormalization of the $O(n)$ non-linear	
		sigma model in two dimensions	144
		13.3.1 Regularization	144
		13.3.2 Perturbation expansion and renormalization	148
	13.4	Renormalization group equations for the $O(n)$ non-linear	
		sigma model and the $(d-2)$ expansion	150
		13.4.1 Integration of RG equations and scaling	151
	13.5	Extensions to other non-linear sigma models	153
14	Large <i>n</i>		156
	14.1	The linear $O(n)$ model	156
	14.2	O(n) sigma model	161
	Index		165

Cambridge University Press & Assessment 978-0-521-19303-0 — Introduction to Statistical Field Theory Edouard Brézin Frontmatter <u>More Information</u>

Preface

These lecture notes do not attempt to cover the subject in its full extent. There are several excellent books that go much deeper into renormalization theory, or into the physical applications to critical phenomena and related topics. In writing these notes I did not mean either to cover the more recent and exciting aspects of the subject, such as quantum criticality, two-dimensional conformal invariance, disordered systems, condensed matter applications of the AdS/CFT duality borrowed from string theory, and so on.

A knowledge of the renormalization group and of field theory remains a necessary part of today's physics education. These notes are simply an introduction to the subject. They are based on actual lectures, which I gave at Sun Yat-sen University in Guangzhou in the fall of 2008. In order not to scare the students, I felt that a short text was a better introduction. There are even several parts that can be dropped by a hasty reader, such as GKS inequalities or high-temperature series. However, high-T series lead to an easy way of connecting geometrical criticality, such as self-avoiding walks and polymers or percolation to physics. I have chosen not to use Feynman diagrams; not that I think that they are unnecessary, I have used them for ever. But since I did not want to require a prior exposition to quantum field theory, I would have had to deal with a long detour, going through connected diagrams, one-particle irreducibility, and so on. I have chosen instead to base everything on the loop expansion of the free energy, not going here beyond one loop. This method, known nowadays as the background field method, is powerful (specially when dealing with symmetries) and natural from the viewpoint of statistical mechanics. (However, the more technical Chapter 13, on the renormalization of the non-linear sigma model, is aimed at readers who have some familiarity with diagrams.)

In spite of the briefness of these notes I wanted to make it clear why, after K. Wilson's work, not only were critical phenomena understood but the understanding of the meaning of renormalizability in quantum field theory changed

Cambridge University Press & Assessment 978-0-521-19303-0 — Introduction to Statistical Field Theory Edouard Brézin Frontmatter More Information

Х

Preface

drastically. It became clear that a beautiful renormalizable theory, such as quantum electrodynamics, was merely an *effective* theory, rather than a theory able to describe electromagnetism from astronomical distances down to vanishingly small length scales. This does not deprive QED from its exceptional beauty and its astounding agreement with experiment. In the post-Wilson analysis its renormalizability results from the fact that, like critical phenomena, the present day experiments, even at the highest presently available accelerator energies, deal with very large length scales in comparison to those at which new physics must occur. Why 'must'? It is because, unlike QCD, QED lacks 'asymptotic freedom', with the consequence that QED is 'trivial', meaning that it is only for a vanishingly small charge of the electron that it could deal with the smallest length scales. So viewing such theories, in the light of critical phenomena, told us that there has to be new physics at short distance.

Many books overlap part or most of the material of these lectures; among a long list, here is a short selection:

- John Cardy, *Scaling and Renormalization in Statistical Physics* (Cambridge: Cambridge University Press, 1996).
- Giorgio Parisi, Statistical Field Theory (New York: Addison-Wesley, 1988).
- J. Zinn-Justin, *Quantum Field Theory and Critical Phenomena*, 3rd edn (Oxford: Oxford University Press, 2002).
- J. Zinn-Justin, *Phase Transitions and the Renormalization Group* (Oxford: Oxford University Press, 2007).
- D. J. Amit and V. Martin-Mayor, *Field Theory, the Renormalization Group and Critical Phenomena* (Singapore: World Scientific, 2005).
- C. Itzykson and J. M. Drouffe, *Statistical Field Theory*, vols 1 and 2. (Cambridge: Cambridge University Press, 1989).

Note also the historical article based on K. Wilson's lectures in Princeton (1971–72): K. G. Wilson and J. Kogut, The renormalization group, *Phys. Rep.*, **12c** (1974) 75. Several books in the long series devoted to *Phase Transitions and Critical Phenomena*, edited by C. Domb and M. Green, and later by C. Domb and J. Lebowitz, will provide additional light on some of the topics of these lectures; see, e.g., vol. 6 of the series.