INTRODUCTION TO STATISTICAL FIELD THEORY

Knowledge of the renormalization group and field theory is a key part of physics, and is essential in condensed matter and particle physics. Written for advanced undergraduate and beginning graduate students, this textbook provides a concise introduction to this subject.

The textbook deals directly with the loop expansion of the free energy, also known as the background field method. This is a powerful method, especially when dealing with symmetries and statistical mechanics. In focusing on free energy, the author avoids long developments on field theory techniques. The necessity of renormalization then follows.

EDOUARD BRÉZIN is Honorary Professor of Physics at the Ecole Polytechnique, and Emeritus Professor at the Ecole Normale Supérieure (Paris). A former President of the French Academy of Sciences, he is also a Foreign Member of the National Academy of Sciences (USA), the American Academy of Arts and Sciences, the Royal Society, and Academia Europaea.

INTRODUCTION TO STATISTICAL FIELD THEORY

EDOUARD BRÉZIN
Ecole Normale Supérieure, Paris

CAMBRIDGE
UNIVERSITY PRESS

CAMBRIDGE
UNIVERSITY PRESS

Shaftesbury Road, Cambridge CB2 8EA, United Kingdom

One Liberty Plaza, 20th Floor, New York, NY 10006, USA

477 Williamstown Road, Port Melbourne, VIC 3207, Australia

314–321, 3rd Floor, Plot 3, Splendor Forum, Jasola District Centre, New Delhi – 110025, India

103 Penang Road, #05–06/07, Visioncrest Commercial, Singapore 238467

Cambridge University Press is part of Cambridge University Press & Assessment,
a department of the University of Cambridge.

We share the University's mission to contribute to society through the pursuit of
education, learning and research at the highest international levels of excellence.

www.cambridge.org
Information on this title: www.cambridge.org/9780521193030

First published 2010

A catalogue record for this publication is available from the British Library

ISBN 978-0-521-19303-0 Hardback

Contents

Cambridge University Press & Assessment
978-0-521-19303-0 — Introduction to Statistical Field Theory
Edouard Brézin
Frontmatter
More Information

Preface

These lecture notes do not attempt to cover the subject in its full extent. There are several excellent books that go much deeper into renormalization theory, or into the physical applications to critical phenomena and related topics. In writing these notes I did not mean either to cover the more recent and exciting aspects of the subject, such as quantum criticality, two-dimensional conformal invariance, disordered systems, condensed matter applications of the AdS/CFT duality borrowed from string theory, and so on.

A knowledge of the renormalization group and of field theory remains a necessary part of today's physics education. These notes are simply an introduction to the subject. They are based on actual lectures, which I gave at Sun Yat-sen University in Guangzhou in the fall of 2008. In order not to scare the students, I felt that a short text was a better introduction. There are even several parts that can be dropped by a hasty reader, such as GKS inequalities or high-temperature series. However, high-T series lead to an easy way of connecting geometrical criticality, such as self-avoiding walks and polymers or percolation to physics. I have chosen not to use Feynman diagrams; not that I think that they are unnecessary, I have used them for ever. But since I did not want to require a prior exposition to quantum field theory, I would have had to deal with a long detour, going through connected diagrams, one-particle irreducibility, and so on. I have chosen instead to base everything on the loop expansion of the free energy, not going here beyond one loop. This method, known nowadays as the background field method, is powerful (specially when dealing with symmetries) and natural from the viewpoint of statistical mechanics. (However, the more technical Chapter 13, on the renormalization of the non-linear sigma model, is aimed at readers who have some familiarity with diagrams.)

In spite of the briefness of these notes I wanted to make it clear why, after K. Wilson's work, not only were critical phenomena understood but the understanding of the meaning of renormalizability in quantum field theory changed drastically. It became clear that a beautiful renormalizable theory, such as quantum electrodynamics, was merely an *effective* theory, rather than a theory able to describe electromagnetism from astronomical distances down to vanishingly small length scales. This does not deprive QED from its exceptional beauty and its astounding agreement with experiment. In the post-Wilson analysis its renormalizability results from the fact that, like critical phenomena, the present day experiments, even at the highest presently available accelerator energies, deal with very large length scales in comparison to those at which new physics must occur. Why 'must'? It is because, unlike QCD, QED lacks 'asymptotic freedom', with the consequence that QED is 'trivial', meaning that it is only for a vanishingly small charge of the electron that it could deal with the smallest length scales. So viewing such theories, in the light of critical phenomena, told us that there has to be new physics at short distance.

Many books overlap part or most of the material of these lectures; among a long list, here is a short selection:

- John Cardy, *Scaling and Renormalization in Statistical Physics* (Cambridge: Cambridge University Press, 1996).
- Giorgio Parisi, *Statistical Field Theory* (New York: Addison-Wesley, 1988).
- J. Zinn-Justin, *Quantum Field Theory and Critical Phenomena*, 3rd edn (Oxford: Oxford University Press, 2002).
- J. Zinn-Justin, *Phase Transitions and the Renormalization Group* (Oxford: Oxford University Press, 2007).
- D. J. Amit and V. Martin-Mayor, *Field Theory, the Renormalization Group and Critical Phenomena* (Singapore: World Scientific, 2005).
- C. Itzykson and J. M. Drouffe, *Statistical Field Theory*, vols 1 and 2. (Cambridge: Cambridge University Press, 1989).

Note also the historical article based on K. Wilson's lectures in Princeton (1971–72): K. G. Wilson and J. Kogut, The renormalization group, *Phys. Rep.*, **12c** (1974) 75. Several books in the long series devoted to *Phase Transitions and Critical Phenomena*, edited by C. Domb and M. Green, and later by C. Domb and J. Lebowitz, will provide additional light on some of the topics of these lectures; see, e.g., vol. 6 of the series.