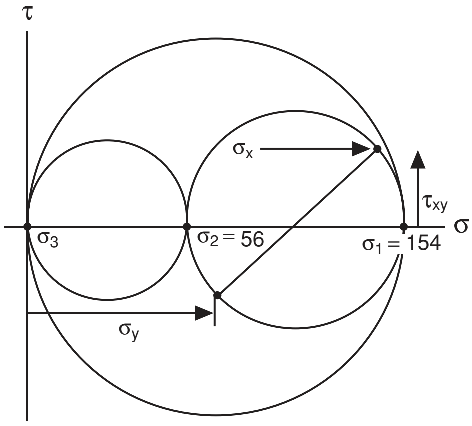
Figure 1.9. Mohr's circles for Example #1.2.



Solution: $\sigma_1, \sigma_2 = (\sigma_x + \sigma_y)/2 \pm \{[(\sigma_x - \sigma_y)/2]^2 + \tau_{xy}^2\}^{1/2} = 154.2, 55.8\,\text{MPa}$, $\sigma_3 = \sigma_z = 0$. Figure 1.9 is the Mohr's circle diagram. Note that the largest shear stress, $\tau_{\max} = (\sigma_1 - \sigma_3)/2 = 77.1$ MPa, is not in the 1–2 plane.

Strains

An infinitesimal normal strain is defined by the change of length, L, of a line:

$$d\varepsilon = dL/L. \tag{1.18}$$

Integrating from the initial length, L_o, to the current length, L,

$$\varepsilon = \int dL/L = \ln(L/L_o) \tag{1.19}$$

This finite form is called *true strain* (or *natural strain, logarithmic strain*). Alternatively, *engineering* or *nominal strain, e*, is defined as

$$e = \Delta L/L_o. \tag{1.20}$$

If the strains are small, then the engineering and true strains are nearly equal. Expressing $\varepsilon = \ln(L/L_o) = \ln(1+e)$ as a series expansion, $\varepsilon = e - e^2/2 + e^3/3!\ldots\ldots$ so as $e \to 0$, $\varepsilon \to e$. This is illustrated in the following example.

EXAMPLE PROBLEM #1.3: Calculate the ratio of e/ε for several values of e.

Solution: $e/\varepsilon = e/\ln(1+e)$. Evaluating:

for $e = 0.001$, $e/\varepsilon = 1.0005$;
for $e = 0.01$, $e/\varepsilon = 1.005$;
for $e = 0.02$, $e/\varepsilon = 1.010$;
for $e = 0.05$, $e/\varepsilon = 1.025$;
for $e = 0.10$, $e/\varepsilon = 1.049$;
for $e = 0.20$, $e/\varepsilon = 1.097$;
for $e = 0.50$, $e/\varepsilon = 1.233$.

Note that the difference e and ε between is less than 1% for $e < 0.02$

There are several reasons that true strains are more convenient than engineering strains:

1. True strains for equivalent amounts of deformation in tension and compression are equal except for sign.
2. True strains are additive. For a deformation consisting of several steps, the overall strain is the sum of the strains in each step.
3. The volume change is related to the sum of the three normal strains. For constant volume, $\varepsilon_x + \varepsilon_y + \varepsilon_z = 0$.

These statements are not true for engineering strains, as illustrated in the following examples.

EXAMPLE PROBLEM #1.4: An element 1 cm long is extended to twice its initial length (2 cm) and then compressed to its initial length (1 cm).

a. Find the true strains for the extension and compression.
b. Find the engineering strains for the extension and compression.

Solution: a. During the extension, $\varepsilon = \ln(L/L_o) = \ln 2 = 0.693$, and during the compression $\varepsilon = \ln(L/L_o) = \ln(1/2) = -0.693$.

b. During the extension, $e = \Delta L/L_o = 1/1 = 1.0$, and during the compression $e = \Delta L/L_o = -1/2 = -0.5$.

Note that with engineering strains, the magnitude of strain to reverse the shape change is different.

EXAMPLE PROBLEM #1.5: A bar 10 cm long is elongated by (1) drawing to 15 cm, and then (2) drawing to 20 cm.

a) Calculate the engineering strains for the two steps and compare the sum of these with the engineering strain calculated for the overall deformation.
b) Repeat the calculation with true strains.

Solution: (a) For step 1, $e_1 = 5/10 = 0.5$; for step 2, $e_2 = 5/15 = 0.333$. The sum of these is 0.833, which is less than the overall strain, $e_{\text{tot}} = 10/10 = 1.00$

(b) For step 1, $\varepsilon_1 = \ln(15/10) = 0.4055$; for step 2, $\varepsilon_1 = \ln(20/15) = 0.2877$. The sum is 0.6931, and the overall strain is $\varepsilon_{\text{tot}} = \ln(15/10) + \ln(20/15) = \ln(20/10) = 0.6931$.

EXAMPLE PROBLEM #1.6: A block of initial dimensions L_{xo}, L_{yo}, L_{zo} is deformed so that the new dimensions are L_x, L_y, L_z. Express the volume strain, $\ln(V/V_o)$, in terms of the three true strains, ε_x, ε_y, ε_z.