INTRODUCTION TO FINITE ELEMENT VIBRATION ANALYSIS, SECOND EDITION

There are many books on finite element methods but few give more than a brief description of their application to structural vibration analysis. This book presents an introduction to the mathematical basis of finite element analysis as applied to vibrating systems. Finite element analysis is a technique that is very important in modelling the response of structures to dynamic loads. Although this book assumes no previous knowledge of finite element methods, those who do have knowledge will still find the book to be useful. It can be utilised by aeronautical, civil, mechanical and structural engineers as well as naval architects. This second edition includes information on the many developments that have taken place over the last 20 years. Existing chapters have been expanded, where necessary, and three new chapters have been included that discuss the vibration of shells and multi-layered elements and provide an introduction to the hierarchical finite element method.

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Second Edition

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Preface

There are many books on finite element methods but very few give more than a brief description of their application to structural vibration analysis. I have given lecture courses on this topic to undergraduates, postgraduates and those seeking post experience training for many years. Being unable to recommend a single suitable text led me to write this book.

The book assumes no previous knowledge of finite element methods. However, those with a knowledge of static finite element analysis will find a very large proportion of the book useful. It is written in such a way that it can be used by Aeronautical, Civil, Mechanical and Structural Engineers as well as Naval Architects. References are given to applications in these fields.

The text has been written in modular style. This will facilitate its use for courses of varying length and level. A prior knowledge of strength of materials and fundamentals of vibration is assumed. Mathematically, there is a need to be able to differentiate and integrate polynomials and trigonometric functions. Algebraic manipulation is used extensively but only an elementary knowledge of vector methods is required. A knowledge of matrix analysis is essential. The reader should be able to add, subtract, multiply, transpose, differentiate and integrate matrices. Methods of solving linear equations and the existence of a matrix inverse is a prerequisite, and the evaluation of determinants is also required.

This second edition includes information on the many developments that have taken place over the last 20 years. Existing chapters have been expanded where necessary and three new chapters included.

Chapter 1 deals with methods of formulating the equations of motion of a dynamical system. A number of methods are introduced. The advantages and disadvantages of each are discussed and recommendations made. The treatment is simple for ease of understanding, with more advanced aspects being treated in an appendix. The simplest methods derive the equations of motion from the expressions for kinetic and strain energy and the virtual work done by externally applied loads. Expressions for these are derived for various structural elements in Chapter 2.

The response of practical structures cannot be obtained using analytical techniques due to their complexity. This difficulty is overcome by seeking approximate solutions. Chapter 3 begins by describing the technique known as the Rayleigh–Ritz method. The finite element displacement method is then introduced as a generalised Rayleigh–Ritz method. The principal features of the method are introduced by considering rods, shafts, beams and frameworks. In this chapter specific element matrices are evaluated explicitly. However, many of the elements presented in later chapters can only be evaluated using numerical integration techniques. In preparation for this, numerical integration in one dimension is introduced. The extension to two and three dimensions is presented where required. Section 3.11 has been expanded to contain additional information on open and closed thin-walled section beams and curved beams.

In Chapter 4, various membrane elements are derived. These can be used for analysing flat plate structures which vibrate in their plane. Chapter 5 deals with the vibration of solids using both axisymmetric and three-dimensional elements. Details of a 10-node tetrahedron are given. Chapter 6 indicates the difficulties encountered in the development of accurate plate-bending elements. This has led to a large number of elements being developed in attempting to overcome these problems. Details of the IMDKT and MITC4 elements have been added. Chapter 7 describes methods of analysing the vibrations of stiffened plates and folded plate structures. This involves combining the framework, membrane and plate-bending elements described in previous chapters. The problems which arise and how to overcome them are described. This includes further information on how to calculate the real stiffness and inertia coefficients for the drilling degrees of freedom.

There follow three new chapters. The first is on the vibration of shells. This includes thin, thick and axisymmetric elements. The next deals with multi-layered elements. Details are given for plate elements using classical, first-order and third-order shear deformation theory. References are given for shell elements and sandwich plate and shell elements. The final chapter of this trio, Chapter 10, consists of an introduction to the hierarchical finite element method. Beam, plate and shell elements are presented using orthogonal polynomials and trigonometric and mixed functions.

Chapters 11–13 (formerly 8–10) present methods of solving the equations of motion. Chapter 11 considers the equations for free vibration of an undamped structure. These take the form of a linear eigenproblem. The methods of solution to be found in the major finite element systems are discussed, the mathematical details having been excluded. The presentation is designed to give the finite element user an appreciation of the methods. Program developers will need to consult the references given. Methods of reducing the number of degrees of freedom are presented. These consist of making use of symmetry, the analysis of rotationally periodic structures, Guyan reduction and component mode synthesis.

Methods of predicting the response of structures to harmonic, periodic, transient and random loads are described in Chapters 12 and 13. Both direct and modal analysis techniques are presented. Methods of representing damping are discussed. This now includes frequency-dependent damping. Also modal analysis has been expanded to include information on classical and non-classical damping. The prediction of the response to transient loads involves the use of step-by-step integration methods. The stability and accuracy of such methods are discussed. Chapter 13 now includes a section on Ritz vector analysis. However, the sections on fatigue and failure and response spectrum methods have been omitted.

The final chapter on computer analysis techniques assumes that the reader intends to use a commercial program. Those wishing to write programs are referred to suitable texts.

Chapters 3 to 10 present details of the simpler elements. Reference to more advanced techniques are given at the end of each chapter. Each has its own extensive list of references. Throughout the book numerical examples are presented to illustrate the accuracy of the methods described. At the end of several chapters a number of problems are presented to give the reader practice using the techniques described. Many of these can be solved by hand. Those requiring the use of an existing finite element program are indicated. Readers who do not have one available are referred to a suitable one in Chapter 14.

In preparing such a text it is very difficult to acknowledge all the help given to the author. First and foremost I am indebted to the finite element community who have undertaken research and development that have led to the techniques described. Without their publications, many of which are listed, the task would have been all the greater. I should like to thank all my past research students and those of my colleagues who have stimulated my interest in finite element techniques, also, all the students who have taken the courses on which this book is based.

I am indebted to Maureen Mew, whose excellent typing skills speeded up the process of converting my handwritten notes into the final typescript. I should also like to thank Deborah Chase, Marilyn Cramer and Chris Jones for converting my drawings into reproducible form.

Notation

The following is a list of principal symbols used. Those which have local meaning only and may have different meanings in different contexts are defined when used.

Mathematical Symbols

[ ]	A rectangular or square matrix
⌈ ⌋	A diagonal matrix
⌊ ⌋	A row matrix
{}	A column matrix
| |	Matrix determinant
$[\ ]^{\mathrm{T}}$	Matrix transpose
$[\ ]^{-1}$	Matrix inverse
$[\ ]^{-\mathrm{T}}$	Inverse transpose: $[\ ]^{-\mathrm{T}} \equiv ([\ ]^{-1})^{\mathrm{T}} \equiv ([\ ]^{\mathrm{T}})^{-1}$
$[\ ]^{\mathrm{H}}$	Complex conjugate of transposed matrix

Latin Symbols

A	Area
$[\mathbf{B}]$	Strain-displacement matrix
$[\mathbf{C}]$	Structural damping matrix (Global)
D	Dissipation function
$[\mathbf{D}]$	Matrix of material constants
E	Young's modulus
$\{\mathbf{f}\}$	Equivalent nodal forces
G	Shear modulus
h	Plate thickness
I	Second moment of area of beam cross-section
$[\mathbf{I}]$	Unit matrix
J	Torsion constant
$[\mathbf{J}]$	Jacobian matrix
k	Spring stiffness
$[\mathbf{k}]$	Element stiffness matrix
$[\mathbf{K}]$	Structural stiffness matrix (Global)

$[\mathbf{m}]$	Element inertia matrix
$[\mathbf{M}]$	Structural inertia matrix (Global)
$\lfloor \mathbf{N} \rfloor$, $[\mathbf{N}]$	Matrix of assumed displacement functions
$\{\mathbf{q}\}$	Modal coordinates
$\{\mathbf{Q}\}$	Modal forces
r, θ, z	Cylindrical coordinates
t	Time
T	Kinetic energy
u, v, w	Components of displacement
$\{\mathbf{u}\}$	Column matrix of nodal displacements
U	Strain energy
$\boldsymbol{V}$	Volume
$\boldsymbol{W}$	Work done by applied forces
x, y, z	Local Cartesian coordinates
X, Y, Z	Global Cartesian coordinates

Greek Symbols

$[\boldsymbol{\alpha}]$	Receptance matrix
γ	Damping ratio
Δ	Increment operator
δ	Virtual operator
$\{\boldsymbol{\varepsilon}\}$	Strain components
$\theta_x, \theta_y, \theta_z$	Rotations about Cartesian axes
κ	Shear factor
λ	Eigenvalue
ν	Poisson's ratio
ξ, η, ζ	Isoparametric coordinates
ρ	Mass per unit volume
$\{\sigma\}$	Stress components
$\boldsymbol{\phi}$	Eigenvector (mode shape)
$\boldsymbol{\Phi}$	Modal matrix
ω	Circular frequency in radians per second