Numerical Methods in Engineering with Python

Second Edition

Numerical Methods in Engineering with Python, Second Edition, is a text for engineering students and a reference for practicing engineers, especially those who wish to explore Python. This new edition features 18 additional exercises and the addition of rational function interpolation. Brent's method of root finding was replaced by Ridder's method, and the Fletcher–Reeves method of optimization was dropped in favor of the downhill simplex method. Each numerical method is explained in detail, and its shortcomings are pointed out. The examples that follow individual topics fall into two categories: hand computations that illustrate the inner workings of the method and small programs that show how the computer code is utilized in solving a problem. This second edition also includes more robust computer code with each method, which is available on the book Web site (www.cambridge.org/kiusalaaspython). This code is made simple and easy to understand by avoiding complex bookkeeping schemes, while maintaining the essential features of the method.

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This book is targeted primarily toward engineers and engineering students of advanced standing (juniors, seniors, and graduate students). Familiarity with a computer language is required; knowledge of engineering mechanics (statics, dynamics, and mechanics of materials) is useful, but not essential.

The text attempts to place emphasis on numerical methods, not programming. Most engineers are not programmers, but problem solvers. They want to know what methods can be applied to a given problem, what are their strengths and pitfalls, and how to implement them. Engineers are not expected to write computer code for basic tasks from scratch; they are more likely to utilize functions and subroutines that have been already written and tested. Thus, programming by engineers is largely confined to assembling existing bits of code into a coherent package that solves the problem at hand.

The “bit” of code is usually a function that implements a specific task. For the user the details of the code are unimportant. What matters is the interface (what goes in and what comes out) and an understanding of the method on which the algorithm is based. Since no numerical algorithm is infallible, the importance of understanding the underlying method cannot be overemphasized; it is, in fact, the rationale behind learning numerical methods.

This book attempts to conform to the views outlined above. Each numerical method is explained in detail and its shortcomings are pointed out. The examples that follow individual topics fall into two categories: hand computations that illustrate the inner workings of the method, and small programs that show how the computer code is utilized in solving a problem. Problems that require programming are marked with ■.

The material consists of the usual topics covered in an engineering course on numerical methods: solution of equations, interpolation and data fitting, numerical differentiation and integration, and solution of ordinary differential equations and eigenvalue problems. The choice of methods within each topic is tilted toward relevance to engineering problems. For example, there is an extensive discussion of symmetric, sparsely populated coefficient matrices in the solution of simultaneous equations. In the same vein, the solution of eigenvalue problems concentrates on methods that efficiently extract specific eigenvalues from banded matrices.
Preface to the First Edition

An important criterion used in the selection of methods was clarity. Algorithms requiring overly complex bookkeeping were rejected regardless of their efficiency and robustness. This decision, which was taken with great reluctance, is in keeping with the intent to avoid emphasis on programming.

The selection of algorithms was also influenced by current practice. This disqualified several well-known historical methods that have been overtaken by more recent developments. For example, the secant method for finding roots of equations was omitted as having no advantages over Ridder’s method. For the same reason, the multistep methods used to solve differential equations (e.g., Milne and Adams methods) were left out in favor of the adaptive Runge–Kutta and Bulirsch–Stoer methods.

Notably absent is a chapter on partial differential equations. It was felt that this topic is best treated by finite element or boundary element methods, which are outside the scope of this book. The finite difference model, which is commonly introduced in numerical methods texts, is just too impractical in handling multidimensional boundary value problems.

As usual, the book contains more material than can be covered in a three-credit course. The topics that can be skipped without loss of continuity are tagged with an asterisk (*).

The programs listed in this book were tested with Python 2.5 under Windows XP and Red Hat Linux. The source code is available on the Web site http://www.cambridge.org/kiusalaaspython.
Preface to the Second Edition

The major change in the second edition is the replacement of NumArray (a Python extension that implements array objects) with NumPy. As a consequence, most routines listed in the text required some code changes. The reason for the changeover is the imminent discontinuance of support for NumArray and its predecessor Numeric.

We also took the opportunity to make a few changes in the material covered:

- Rational function interpolation was added to Chapter 3.
- Brent’s method of root finding in Chapter 4 was replaced by *Ridder’s method*. The full-blown algorithm of Brent is a complicated procedure involving elaborate bookkeeping (a simplified version was presented in the first edition). Ridder’s method is as robust and almost as efficient as Brent’s method, but much easier to understand.
- The Fletcher–Reeves method of optimization was dropped in favor of the *downhill simplex method* in Chapter 10. Fletcher–Reeves is a first-order method that requires knowledge of the gradients of the merit function. Because there are few practical problems where the gradients are available, the method is of limited utility. The downhill simplex algorithm is a very robust (but slow) zero-order method that often works where faster methods fail.