Magnetoconvection

The last 30 years have seen great leaps forward in the subject of magnetoconvection: the study of the interplay between magnetic fields and convection. Computational techniques can now explain exotic nonlinear behaviour, transition to chaos and the formation of structures that can be observed on the surface of the Sun.

Here, two leading experts present the current state of knowledge of the subject. They provide a mathematical and numerical treatment of the interactions between electrically conducting fluids and magnetic fields that lead to the complex structures and rich behaviour observed on the Sun and other stars, as well as in the interiors of planets like the Earth. The authors' combined analytical and computational approach provides a model for the study of a wide range of related problems. The discussion includes bifurcation theory, chaotic behaviour, pattern formation in two and three dimensions, and applications to geomagnetism and to the properties of sunspots and other features at the solar surface.

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To Judy and Julia

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Preface

The original motivation for studying magnetoconvection came from the interplay between magnetic fields and convection that is observed in sunspots. Since then this subject has developed into a fascinating and important topic in its own right. We therefore decided to write a comprehensive monograph that would cover all aspects of magnetoconvection from the viewpoint of applied mathematics, and as a branch of astrophysical (or geophysical) fluid dynamics. Thus we shall emphasize the role of nonlinear dynamics, and focus on idealized model problems rather than on ambitious realistic simulations.

The properties of convection in an electrically conducting fluid with an imposed magnetic field are interesting not only in themselves but also as the richest example of double-diffusive behaviour. Linear theory allows both steady and oscillatory solutions, while theoretical descriptions of nonlinear behaviour demonstrate the power of bifurcation theory, with examples of bifurcation sequences that lead to chaos, as well as of group-theoretic applications to pattern selection. These mathematical results can all be related to carefully constructed numerical experiments.

Although we shall adopt an applied mathematical approach, our discussion is particularly relevant to the behaviour of magnetic fields at the surface of the Sun, which are now being observed in unprecedented detail, both from the ground and from space. Convection also interacts with magnetic fields in the solar interior, as it does in other stars, and is a key component of solar and stellar dynamos. In the Earth's molten core, convection is again responsible for the geodynamo that maintains the geomagnetic fields. Magnetoconvection is also relevant to other planets in the solar system, as it must be to exoplanets and in accretion discs. Limited aspects of magnetoconvection have also been investigated in laboratory experiments. Thus our theoretical discussion has a wide range of applications (both actual and potential) and extensions in the real world.

We have made our book reasonably self-contained, with brief introductions to magnetohydrodynamics and to nonlinear dynamics. It also demonstrates how mathematical theory can be extended by judicious use of computation. The monograph is primarily aimed at applied mathematicians and fluid dynamicists but it will also be of interest to experts on nonlinear behaviour, as well as to theoreticians working in astrophysics (including solar physics), geophysics and planetary physics. The potential audience includes graduate students, postdoctoral workers and professional academics. Although the book is designed as a research monograph, it could also form the basis of a graduate lecture course.

Our own experience in this field extends back for half a century and we ourselves have collaborated for more than thirty years. In the beginning, computers were in their infancy and efforts were concentrated on linear problems. Since then, numerical techniques have become enormously powerful, while the whole new subject of nonlinear dynamics has developed. It is the combination of computational and analytical approaches that makes magnetoconvection such an attractive topic.

We wish to thank all those collaborators and close colleagues who, over all these years, have enriched our knowledge of magnetoconvection and helped to make our investigations so enjoyable. They include Nic Brummell, Paul Bushby, Fausto Cattaneo, Stephen Childress, Jon Dawes, Emmanuel Dormy, Thierry Emonet, David Fearn, Dave Galloway, Pascale Garaud, Andrew Gilbert, Douglas Gough, Rebecca Hoyle, David Hughes, Neal Hurlburt, Hiroaki Isobe, Chris Jones, Edgar Knobloch, Willem Malkus, Paul Matthews, Keith Moffatt, Dan Moore, Alastair Rucklidge, Andrew Soward, Steve Tobias and Juri Toomre. We are especially grateful to Ed Spiegel for the inspiration that he has provided over many years. We have benefited too from discussions with Fritz Busse, Leon Mestel, Eugene Parker, Eric Priest, Paul Roberts, Manfred Schüssler, Jack Thomas and George Veronis, and from having been able to talk to George Batchelor, Ludwig Biermann, Edward Bullard, Subrahmanyan Chandrasekhar, Thomas Cowling, Roger Tayler and William Thompson in the past. We are grateful also to Pierre Coullet, Paul Glendinning, John Guckenheimer, Louis Howard, Colin Sparrow and Peter Swinnerton-Dyer for guiding us through the intricacies of nonlinear dynamics and bifurcation theory. In addition, thanks go to our former graduate students and postdocs, Wayne Arter, Andrew Bernoff, Sean Blanchflower, John Edwards, Benjamin Favier, Ann Halford, Steve Houghton, Keith Julien, Masato Nagata, the late Richard Peckover,

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Michael Proctor