

Magnetoconvection

The last 30 years have seen great leaps forward in the subject of magnetoconvection: the study of the interplay between magnetic fields and convection. Computational techniques can now explain exotic nonlinear behaviour, transition to chaos and the formation of structures that can be observed on the surface of the Sun.

Here, two leading experts present the current state of knowledge of the subject. They provide a mathematical and numerical treatment of the interactions between electrically conducting fluids and magnetic fields that lead to the complex structures and rich behaviour observed on the Sun and other stars, as well as in the interiors of planets like the Earth. The authors' combined analytical and computational approach provides a model for the study of a wide range of related problems. The discussion includes bifurcation theory, chaotic behaviour, pattern formation in two and three dimensions, and applications to geomagnetism and to the properties of sunspots and other features at the solar surface.

N. O. WEISS is Emeritus Professor of Mathematical Physics at the University of Cambridge and a Fellow of Clare College. He is a Fellow of the Royal Society and a former President of the Royal Astronomical Society, which awarded him a Gold Medal in 2007. His research has centred on astrophysical fluid dynamics and especially on nonlinear magnetohydrodynamic interactions between convection and magnetic fields in the Sun and other stars, as well as in the Earth.

M. R. E. PROCTOR is Professor of Astrophysical Fluid Dynamics at the University of Cambridge and Provost of King's College, having been for many years a Fellow of Trinity College. He is a Fellow of the Royal Society and of the Institute of Mathematics and Applications. His early research was concerned with nonlinear models of the geomagnetic field. More recently he has contributed to a number of research areas, including dynamical systems and pattern formation, solar dynamo theory and the interactions of magnetic fields and convection in the Sun.

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Magnetoconvection

N. O. WEISS

University of Cambridge

M. R. E. PROCTOR

University of Cambridge



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Contents

	<i>Preface</i>	<i>page ix</i>
1	Introduction	1
	1.1 Background and motivation	1
	1.2 Outline of the book	6
	1.3 General references	7
2	Basic MHD	9
	2.1 The induction equation	9
	2.2 Kinematic MHD	17
	2.3 The Lorentz force	26
	2.4 Kinematic dynamos	31
3	Linearized Boussinesq magnetoconvection	38
	3.1 Onset of convection in a vertical magnetic field	39
	3.2 Inclined magnetic fields	50
	3.3 Other boundary conditions	60
4	The nonlinear regime	64
	4.1 Weakly nonlinear behaviour	66
	4.2 Bifurcations from nonlinear solutions	78
	4.3 Period-doubling and chaos at a heteroclinic bifurcation	92
	4.4 Travelling waves	105
	4.5 Transverse rolls in a horizontal field	109
5	2D Boussinesq magnetoconvection	113
	5.1 Numerical results: Cartesian geometry	113
	5.2 Axisymmetric magnetoconvection	134
	5.3 Transitions to chaos	144
	5.4 Shearing instabilities	152
	5.5 Localized patterns, snaking and convectons	158
	5.6 The strong field limit	172
	5.7 Inclined magnetic fields	176

6	3D Boussinesq magnetoconvection	180
6.1	Pattern selection	181
6.2	Convection and small-scale dynamos	198
6.3	Fully nonlinear magnetoconvection	206
6.4	The strong field regime	211
7	Magnetoconvection, rotation and the dynamo	215
7.1	Linear theory of rotating convection and magnetoconvection	216
7.2	Dynamos due to nonlinear rotating convection in a layer	229
7.3	Dynamos in spherical shells	240
7.4	Laboratory dynamos	246
8	Compressible magnetoconvection	251
8.1	The fully compressible regime	252
8.2	Two-dimensional behaviour	258
8.3	Three-dimensional behaviour	278
8.4	Inclined magnetic fields	307
9	Solar and stellar magnetic fields	320
9.1	Global magnetic activity	321
9.2	Photospheric magnetoconvection	330
9.3	Magnetoconvection in sunspots	338
9.4	From simple idealized models to massive nonlinear computations	347
<i>Appendix A</i> The Boussinesq and anelastic approximations		349
A.1	The Boussinesq approximation	350
A.2	The anelastic approximation	352
A.3	The strong anelastic approximation	353
<i>Appendix B</i> Chaotic systems		355
B.1	The logistic map	355
B.2	The Lorenz equations	356
B.3	The Shilnikov bifurcation	358
<i>Appendix C</i> Double-diffusive convection		360
C.1	Chaos in 2D thermosolutal convection	360
C.2	Low-order model systems	365
<i>Appendix D</i> Magnetic buoyancy and the magneto-Boussinesq approximation		367
<i>References</i>		370
<i>Index</i>		395

Preface

The original motivation for studying magnetoconvection came from the interplay between magnetic fields and convection that is observed in sunspots. Since then this subject has developed into a fascinating and important topic in its own right. We therefore decided to write a comprehensive monograph that would cover all aspects of magnetoconvection from the viewpoint of applied mathematics, and as a branch of astrophysical (or geophysical) fluid dynamics. Thus we shall emphasize the role of nonlinear dynamics, and focus on idealized model problems rather than on ambitious realistic simulations.

The properties of convection in an electrically conducting fluid with an imposed magnetic field are interesting not only in themselves but also as the richest example of double-diffusive behaviour. Linear theory allows both steady and oscillatory solutions, while theoretical descriptions of nonlinear behaviour demonstrate the power of bifurcation theory, with examples of bifurcation sequences that lead to chaos, as well as of group-theoretic applications to pattern selection. These mathematical results can all be related to carefully constructed numerical experiments.

Although we shall adopt an applied mathematical approach, our discussion is particularly relevant to the behaviour of magnetic fields at the surface of the Sun, which are now being observed in unprecedented detail, both from the ground and from space. Convection also interacts with magnetic fields in the solar interior, as it does in other stars, and is a key component of solar and stellar dynamos. In the Earth's molten core, convection is again responsible for the geodynamo that maintains the geomagnetic fields. Magnetoconvection is also relevant to other planets in the solar system, as it must be to exoplanets and in accretion discs. Limited aspects of magnetoconvection have also been investigated in laboratory experiments. Thus

our theoretical discussion has a wide range of applications (both actual and potential) and extensions in the real world.

We have made our book reasonably self-contained, with brief introductions to magnetohydrodynamics and to nonlinear dynamics. It also demonstrates how mathematical theory can be extended by judicious use of computation. The monograph is primarily aimed at applied mathematicians and fluid dynamicists but it will also be of interest to experts on nonlinear behaviour, as well as to theoreticians working in astrophysics (including solar physics), geophysics and planetary physics. The potential audience includes graduate students, postdoctoral workers and professional academics. Although the book is designed as a research monograph, it could also form the basis of a graduate lecture course.

Our own experience in this field extends back for half a century and we ourselves have collaborated for more than thirty years. In the beginning, computers were in their infancy and efforts were concentrated on linear problems. Since then, numerical techniques have become enormously powerful, while the whole new subject of nonlinear dynamics has developed. It is the combination of computational and analytical approaches that makes magnetoconvection such an attractive topic.

We wish to thank all those collaborators and close colleagues who, over all these years, have enriched our knowledge of magnetoconvection and helped to make our investigations so enjoyable. They include Nic Brummell, Paul Bushby, Fausto Cattaneo, Stephen Childress, Jon Dawes, Emmanuel Dormy, Thierry Emonet, David Fearn, Dave Galloway, Pascale Garaud, Andrew Gilbert, Douglas Gough, Rebecca Hoyle, David Hughes, Neal Hurlburt, Hiroaki Isobe, Chris Jones, Edgar Knobloch, Willem Malkus, Paul Matthews, Keith Moffatt, Dan Moore, Alastair Rucklidge, Andrew Soward, Steve Tobias and Juri Toomre. We are especially grateful to Ed Spiegel for the inspiration that he has provided over many years. We have benefited too from discussions with Fritz Busse, Leon Mestel, Eugene Parker, Eric Priest, Paul Roberts, Manfred Schüssler, Jack Thomas and George Veronis, and from having been able to talk to George Batchelor, Ludwig Biermann, Edward Bullard, Subrahmanyan Chandrasekhar, Thomas Cowling, Roger Tayler and William Thompson in the past. We are grateful also to Pierre Couillet, Paul Glendinning, John Guckenheimer, Louis Howard, Colin Sparrow and Peter Swinerton-Dyer for guiding us through the intricacies of nonlinear dynamics and bifurcation theory. In addition, thanks go to our former graduate students and postdocs, Wayne Arter, Andrew Bernoff, Sean Blanchflower, John Edwards, Benjamin Favier, Ann Halford, Steve Houghton, Keith Julien, Masato Nagata, the late Richard Peckover,

Preface

xi

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Nigel Weiss
Michael Proctor