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Demand and supply in competitive markets

The book centres around the **demand and supply analysis** in **perfectly competitive markets**. To give you an overview of the book, in this chapter these ideas will be briefly explained without the use of mathematics, primarily for readers who have little knowledge of economics. I will also foreshadow various topics that will be covered in the rest of this book and the mathematical techniques that might be used to examine them. I hope it will motivate you to study the subsequent chapters.

Chapter goals By studying this chapter you will

- (1) be able to conduct the demand and supply analysis without the use of mathematics; and
- (2) have a bird's-eye view of this book.

1.1 Markets

Think about what you bought yesterday. The items you bought might include those that are physically tangible, e.g. potato chips, vegetables and books, as well as those that are intangible in nature such as 'watching a movie in a cinema' and 'having a haircut at a hairdresser's'. Economists call tangible products such as potato chips and vegetables **goods**, whereas they call intangible ones **services**.

Goods and services are traded in **markets**. If you have not studied economics before, note that the word 'markets' is a technical term (a jargon) and may be different from how you use it everyday. When you hear the term 'market' you might imagine something like a fish market or a fruit market because we are familiar with these markets. But in economics a 'market' is a much broader idea. For example, consider an ice-cream stand in a football stadium. In economics, it constitutes a market for ice-cream because ice-cream is traded at the stand. Another example of a market is book stores on the Internet (such as Amazon.com) where books are traded. Essentially, the market for a good (or a service) is a set of buyers and sellers who potentially engage in trading it, given a particular time and a location.

Thousands of goods and services are traded in markets. In the following, we shall focus on one of them – say, sausages – and examine how the **price** and **quantity** traded in the sausage market are determined.

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1.1.1 Perfectly competitive markets

Suppose you are selling sausages. Imagine a situation where (a) many other sausage sellers are around you and they all charge the same price; and (b) there are many people in the market who are willing to pay that price to buy sausages. That is, so long as you charge that price, people will buy as many sausage as you want to sell.

To think about how you might decide what price to charge, start with the situation where you charge the same price as others. Now, would you consider charging a higher price? Well, it is not a good idea because if you did, people would buy sausages from other sellers and would not buy any sausages from you. Then, would you consider charging a lower price? It is not a good idea, either. There are many people who buy sausages from you if you charge the same price as others; so why would you want to do worse by charging a lower price?

In this situation, it seems you have no option but to accept the price other sellers charge. Why can you not affect the price of the sausage? It is because there are so many other sausage sellers in the market. When your sausage supply is only a tiny proportion relative to the size of the sausage market, you cannot change the price of the sausage by yourself. Each of the sellers, including you, has no control over the price and hence accepts (takes) the price that stands in the market. Economists call these sellers **price takers**. Similar logic applies to buyers. If each of the sausage buyers is small compared to the size of the entire market, then none of them has influence on the price of a sausage. Each buyer is a price taker in such a situation. You are in this kind of situation when you are shopping in the supermarket.

However, if you are in fresh food (e.g. fruit, vegetable, fish) markets when they are about to close, you find that you have some 'power' to influence the prices of the goods. It is very likely you can bargain the prices down. In such a case, buyers are not price takers. An example of non-price taking behaviour on the seller's side is found in a market where there is only one seller. The seller is called a **monopolist** (or a **monopoly**) and it can set the price as it likes (although the price is likely to be subject to government regulation). Even when there is more than one seller in the market, each of them may have some control over the price they can charge. In the above sausage example, even if there are many more sausage sellers around you, if you supply sausages that are somewhat different from those others provide – e.g. sightly bigger, spicier, healthier – then you can charge a higher price than your competitors (and consumers who have particular tastes will pay more to buy sausages from you).

In any event, in this book we will *not* focus on situations where sellers and/or buyers have control over the price. It means that our focus will be the market where *homogeneous* goods are traded. There are many sellers as well as many buyers in this market and hence each of them behaves as a price taker. The market that satisfies these two conditions – (a) homogeneous goods and (b) price taking sellers and buyers – is called the **perfectly competitive market**. Now let us look at how buyers and sellers might behave when the price of a sausage is given.

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1.2 Demand and supply schedules



Figure 1.1 The market demand for sausages.

1.2 Demand and supply schedules

1.2.1 Market demand

The **market demand schedule** for sausages depicts, for each and every price of a sausage, the quantity of sausages buyers in the market are willing to buy (in a particular time, per day for example).

The market demand schedule for sausages is depicted in Figure 1.1. On the vertical axis of the diagram, the sausage price is shown, whereas on the horizontal axis, the quantity of sausages is shown. The diagram indicates that when the market price of sausage is given as \$1 (remember, buyers take this price as given), buyers wish to buy 2000 sausages (Point *A* in Figure 1.1). In other words, the quantity demanded is 2000 when the price of sausage is \$1. Now suppose the price has risen to \$1.50 (and again buyers must take this new price as given), what will occur to the quantity of sausages buyers would like to buy? You would think it'd go down, yes?

Now that sausages have become more expensive than before, buyers might want to substitute the sausages and buy something else, such as pizzas. Such an effect is called the **substitution effect**. In addition, an increase in the sausage price has effectively made buyers poorer, if other things – prices of other goods and buyers' income levels – are held constant. We say that the buyers' purchasing power has gone down. With lower purchasing power, buyers tend to purchase less sausages.¹ Such an effect is called the **income effect**. Both these effects give rise to a decline in the quantity demanded when the price rises.

1 To avoid confusion for readers who have studied introductory economics before, I note that I am assuming that a sausage is a normal good. If you are unfamiliar with this notion, don't worry about this footnote. It is not essential for following the main text.

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The diagram captures these effects and shows that the quantity demanded has decreased to 1000 when there is a 50-cent increase in the price (Point B in the figure). The demand schedule for sausages can be constructed by going through this exercise for every price, and you will end up obtaining a downward sloping schedule as in Figure 1.1. So an important lesson that we have learnt here is that the demand schedule slopes downwards; this is called the **law of demand**.

By how much does the buyers' demand for sausages change when there is a change in the price? Sausage sellers are interested in the answer to this question because it affects their revenue from selling sausages. When all the sellers decided to raise the price by 50 cents, if the quantity of sausage demanded did *not* change, their revenue would increase. However, as the law of demand suggests, they will not be able to sell as many sausages as before, and hence it is unclear whether the revenue will rise or fall. In Chapter 2 we introduce some basic mathematical concepts, which include functions. An example of a function is the **demand function**, which mathematically represents the demand schedule we have just discussed. The question as to the change in the sellers' revenue is more complex and we will have to put off the investigation until Chapters 4 and 5, where we learn about **differential calculus**. Differential calculus is crucial in conducting economic analysis. As an application of differential calculus, we will introduce **price elasticity of demand**, which is closely related to the sellers' revenue problem we have discussed above.

In fact, what is behind the demand schedule is more complex than you might think. We briefly mentioned in the above discussion how buyers might choose a good over another provided the prices for those goods and their income levels. It is easy to imagine that a change in the price of a sausage will affect the quantity of sausages demanded, but the prices of other goods and income levels are also important determinants of the sausage demand. How does the consumer decides how much of each good to purchase? In Chapter 6, we use this buyers' consumption choice problem as a motivation to learn the mathematical notion called **multivariate calculus**.

1.2.2 Market supply

The **market supply schedule** in Figure 1.2 depicts, for each and every price of a sausage, the quantity sellers in the market are willing to provide (in a particular time, per day for example).

As in the diagram for the demand schedule, we have price on the vertical axis and quantity on the horizontal axis. The diagram indicates that when the market price is \$1 (remember, sellers are price takers), they wish to sell 1000 sausages (Point *C* in Figure 1.2). Another way to put it is that the quantity supplied is 1000 when the price of sausage is \$1. Now suppose the price rises to \$1.50 (and again sellers take this new price as given), what will happen to the quantity of sausages sellers would like to supply, holding other things – such as sellers' sausage production technology, how much it costs to hire a worker, etc. – constant?

When everything else is held constant – in Latin, *ceteris paribus* – an increase in the price of a sausage (by 50 cents in this case) will increase the quantity of sausages supplied in the market. Not only will the existing suppliers want to supply more sausages than before, but some new sellers who did not supply them before might find it worthwhile

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1.3 Market equilibrium



Figure 1.2 The market supply for sausages.

to supply them now that they each sell for \$1.50. In the diagram it is shown that the quantity of sausages supplied rises to 2000 when the price increases to \$1.50 (Point *D* in Figure 1.2). The supply schedule of sausages – the upward sloping schedule in Figure 1.2 – can be constructed by obtaining the quantity supplied for all the other prices. The fact that the supply schedule slopes upwards is called the **law of supply**.

As we learned in the beginning, there are many sellers in a competitive market and each of them supplies sausages. It means that the market supply must be the sum of the supply of these individual sellers. In Chapter 4 we will study an individual seller's decision making problem, which is to choose the quantity of supply given the price so as to maximise **profits**. It is called the **profit maximisation** problem and we will learn how to **differentiate** a function in the course of solving this problem. In Chapter 7 we will demonstrate how we can aggregate the individual seller's supply schedules to obtain the market supply schedule.

1.3 Market equilibrium

Now let us put the two schedules together in Figure 1.3.

Both the market demand and supply schedules for sausages are depicted. Remember that each of the buyers is a price taker and so is each of the sellers. The diagram indicates that when the market price of a sausage is given as \$1, buyers wish to buy 2000 sausages (Point A in Figure 1.3), but sellers wish to sell 1000 sausages (Point C). The horizontal distance CA (measured in sausages) is 1000 and it means the quantity demanded exceeds the quantity supplied by 1000 sausages if the price is \$1. We say that there is an **excess demand** for them. We can also say that there is a **shortage** of sausages. What would you expect to occur when there is excess demand?

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Figure 1.3 The market equilibrium.

Well, you'd probably expect the price to rise. But how do we reconcile it with the price taker assumption? The assumption is that *each* seller cannot raise the price because they will lose all their sales given other sellers are not deviating from the current price. Here the trick is to think as follows; we suppose *all* the sellers agree to increase the price by a little bit *at the same time*. When sellers realise that there are desperate buyers who would pay a bit more to buy sausages (rather than missing out on them), they decide to raise the price as a group, which creates an upward pressure on the price. There will be an upward pressure on the price from the buyers' side as well. Namely, the buyers who do not want to miss out on sausages will offer a higher price (if they can), which the sellers have no reason to decline.

Let us see what might occur if the price is 1.50. Now buyers wish to buy 1000 sausages (Point *B*), but sellers wish to sell 2000 sausages (Point *D*). So there is **excess supply**, which is represented by the horizontal distance *BD*. The quantity supplied exceeds the quantity demanded by 1000 sausages if the price is 1.50. This time there will be a downward pressure on price. That is, realising the excess supply, buyers as a group bid down the price, which sellers tend to accept so as to unload the excess supply.

We have looked at the cases where the amount buyers want to buy does not coincide with the amount sellers would like to supply. Now focus on Point E in Figure 1.3 where the price of a sausage is \$1.25. The demand schedule indicates that buyers wish to buy 1500 whereas the supply schedule shows that sellers would like to provide 1500 to the market. So under this price, the amount buyers want to buy coincides with the amount sellers would like to supply. When this occurs we say that the sausage market **clears** and that the market is in **equilibrium**. It is the situation where (a) each of the buyers and sellers is doing what they want to do; and (b) the quantity demanded equals the quantity supplied, i.e. the market clears. Unlike in the previous cases, buyers and sellers have no reason to change their behaviour under this situation and hence the price and the

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1.4 Rest of this book

quantity of sausages traded stay intact. Point E is called the equilibrium point and the corresponding price and quantity are called the **equilibrium price** and the **equilibrium quantity**, respectively.

What do you think might occur to the equilibrium price and quantity if the government decides to collect a certain amount of a tax from the sellers per sausage sold? It seems that sellers will suffer from this arrangement, but what about the sausage buyers? Will there be any effect on them? It turns out that the buyers also tend to suffer from the tax, despite the fact it is the sellers who pay the tax legally. To study the effect of the taxation, we typically rely on the ideas of **consumer surplus** and **producer surplus**, which measure the welfare of buyers and sellers, respectively, in dollars. To obtain these measures we need to study how to obtain the area under the demand and supply schedules. We will study the mathematical technique that allows us to do it – **integral calculus** – in Chapter 7.

1.4 Rest of this book

As foreshadowed above, the main economic applications that involve the use of calculus will be covered in Chapters 4–7. However, it does not mean the next two chapters are unimportant. Chapter 2 recaps some basic ideas in mathematics – numbers, equations, functions, logic, etc. – and demand and supply analysis will be conducted in a more mathematical fashion. In Chapter 3, some more ideas of basic mathematics – most notably, the exponential function and the logarithmic function – will be studied in the context of introductory finance. These chapters are completely free from the use of calculus and will hopefully consolidate your mathematical background before you delve into the main economic applications that utilise calculus in Chapters 4–7.

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2

Basic mathematics

This chapter deals with some fundamental mathematical rules and ideas we will use in the rest of the book. It is very important that you become comfortable with them. Going through the basic material may be a tedious experience for you, but much of the confusion in studying mathematics that I know appears to stem simply from a lack of appreciation of these mathematical conventions (if so, what a pity...), so I will spend some time on them.¹

To become a good user of a foreign language, we need to know some grammar as well as a bit of slang of that language. Sometimes one gets lost completely during conversations because of the use of slang. For example, if I received a letter saying, 'There is a BBQ party; BYO', I would bring my own drink since I know what BYO means. However, some of you whose native language is not English may have to consult with their dictionaries in order to figure out what BYO means.

Learning mathematics has a similar flavour. You will need to know the basic rules as well as some advanced techniques that stand on them. As you don't expect you can master a foreign language overnight, you also should not expect that you can master mathematics overnight. You will need to work hard in order to learn mathematics. The reward from it, though, should be fairly large. If you have studied a foreign language and have been able to communicate with people – who you otherwise wouldn't have been able to – using it, you know how fun and exciting it is.

OK then, put your head down and let's go through the material together. If you have a sound knowledge of high-school mathematics, you may be able to skip the basic material and jump to Section 2.10 where we see how mathematics can be applied to conduct the demand and supply analysis.

Chapter goals By studying this chapter you will

- (1) recap basic ideas in mathematics that will be used in the rest of the book;
- (2) be able to conduct demand and supply analysis (with mathematics);
- (3) be able to carry out comparative static analysis and interpret its results; and
- (4) be able to interpret implication statements, and explain the difference between necessary and sufficient conditions.

1 In writing this section, I have drawn a lot of material from M. Timbrell, *Mathematics for Economists* (Basil Blackwell, 1985). I have also benefited a lot from R. G. Bartle and D. R. Sherbert, *Introduction to Real Analysis* (Wiley, 1999).

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2.1

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2.1 Numbers

Numbers

To begin with, I will discuss numbers. The numbers we use to count things, i.e. 1, 2, 3, ... are called **natural numbers** (or **positive integers**). If we sum two natural numbers, we have another natural number (say, 5 + 8 = 13). However, if we subtract one natural number from a smaller one (say, 5 - 8), we end up with a negative of a natural number (5 - 8 = -3). The negative of natural numbers, that is, -1, -2, -3... are called **negative natural numbers** (or **negative integers**). The set of these two types of numbers and *zero* are called **integers**.

Now, what occurs if we multiply an integer by another? We obtain an integer. For example, $5 \cdot 8 = 40$ or $3 \cdot (-7) = -21$. Note that \cdot in between two numbers implies a multiplication symbol \times , i.e. $5 \cdot 8 = 5 \times 8 = 40$. Sometimes we even omit \cdot if it is not confusing. For instance, if we multiply two numbers x and y, we can write the product as $x \times y$, $x \cdot y$ or xy. Of course, you should avoid writing 58 to represent $5 \cdot 8$ because people will no doubt interpret it as 'fifty-eight'.

In any event, let's turn to division. If we divide an integer by another (with one exception, which we will see shortly), we obtain a different type of number called **fraction**. For instance, $3 \div 2 = \frac{3}{2}$. The set of fractions and integers is called **rational numbers**. We will introduce other types of numbers in due course.

2.1.1 We cannot divide a number by zero!

Suppose you have \$120 in your pocket and are thinking about going to a rugby game in the local stadium. Tickets cost \$12 per person. How many friends can you invite? Well, 120 divided by 12 gives 10, so you can invite 9 people (10, if you are not going with them). OK, but what if tickets cost \$0? How can we divide 120 by 0? Obviously there is no sensible number for this question, because you can choose any number of people to take (so long as the stadium does not collapse because of overcrowding!). Mathematics disallows imprecision as such by simply excluding the idea of dividing a number by zero. Consider an expression $x \div (y - z)$. This expression is completely fine so long as $y \neq z$. If y = z, then it becomes $x \div 0$, which is not allowed.

2.1.2 Reciprocal

Consider a number x. So long as $x \neq 0$ we can divide 1 by x to create a new number $\frac{1}{x}$. It is called the **reciprocal** of x. You can see that $x \cdot \frac{1}{x} = 1$, which says that the product of a number and its reciprocal is unity. 10

2.2

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Basic mathematics

Fractions, decimal numbers and the use of a calculator

We have seen that all the rational numbers can be represented as the ratio of two integers, but they can be represented in many different ways. For example:

$$\frac{3}{10} = \frac{30}{100},$$
$$\frac{5}{16} = \frac{3125}{10\,000}$$

However, they can also be written as:

$$\frac{3}{10} = 0.3,$$
$$\frac{5}{16} = 0.3125$$

These numbers, 0.3 or 0.3125, are called **decimal numbers**. The advantage of using decimal numbers is that we can compare numbers very easily. It is obvious that $\frac{3}{10}$ is smaller than $\frac{5}{16}$, once we see that the former is 0.3 and the latter is 0.3125. However, it is not really an advantage. We can always find any common denominator of the two numbers and compare their numerators. In the above example, if we use 10 000 as the denominator, the numerator for $\frac{3}{10}$ is 3000, which is smaller than 3125, so we can reach the same conclusion rather easily.

In contrast, the downside of using decimal numbers is quite notable, which I'd like to emphasise. It is important to appreciate that many fractions actually do not have a nice and short decimal number representation. For example:

$$\frac{1}{3} = 0.33333...$$

You can see that 3 is repeating infinitely. This number can be represented as 0.3, meaning that 3 is repeating. The representation is actually nice and short, but that is not the point here, as you will see as you read through the rest of the section. This kind of number is called an **infinite decimal**, whereas numbers such as 0.3 and 0.3125 are called finite decimals.

A rational number is either a finite decimal or an infinite decimal that repeats. There are some infinite decimal numbers that do not repeat. They cannot be represented by any fraction of two integers and are called **irrational numbers**. Examples of irrational numbers include: π , $\sqrt{2}$, $\sqrt{3}$, etc. It will be proven in Section 2.12 that $\sqrt{2}$ is an irrational number.