CAMBRIDGE

Cambridge University Press
978-0-521-18943-9 - Iterative Methods in Combinatorial Optimization
Lap Chi Lau, R. Ravi and Mohit Singh
Frontmatter
More information

Iterative Methods in Combinatorial Optimization

With the advent of approximation algorithms for NP-hard combinatorial optimization problems, several techniques from exact optimization such as the primal-dual method have proven their staying power and versatility. This book describes a simple and powerful method that is iterative in essence and similarly useful in a variety of settings for exact and approximate optimization. The authors highlight the commonality and uses of this method to prove a variety of classical polyhedral results on matchings, trees, matroids, and flows.

The presentation style is elementary enough to be accessible to anyone with exposure to basic linear algebra and graph theory, making the book suitable for introductory courses in combinatorial optimization at the upper undergraduate and beginning graduate levels. Discussions of advanced applications illustrate their potential for future application in research in approximation algorithms.

LAP CHI LAU is an Assistant Professor in the Department of Computer Science and Engineering at The Chinese University of Hong Kong. Lap Chi's main research interests are in combinatorial optimization and graph algorithms. His paper on Steiner tree packing was given the Machtey award in the IEEE Foundations of Computer Science Conference. His Ph.D. thesis was awarded the doctoral prize from the Canadian Mathematical Society and a doctoral prize from the Natural Sciences and Engineering Research Council of Canada.

R. RAVI is Carnegie Bosch Professor of Operations Research and Computer Science at Carnegie Mellon University. Ravi's main research interests are in combinatorial optimization (particularly in approximation algorithms), computational molecular biology, and electronic commerce.

MOHIT SINGH is an Assistant Professor in the School of Computer Science, McGill University. He completed his Ph.D. in 2008 at the Tepper School of Business, Carnegie Mellon University, where his advisor was Professor R. Ravi. His thesis was awarded the Tucker prize by the Mathematical Programming Society. His research interests include approximation algorithms, combinatorial optimization, and models that deal with uncertainty in data.

© in this web service Cambridge University Press www.cambridge.org

CAMBRIDGE

Cambridge University Press
978-0-521-18943-9 - Iterative Methods in Combinatorial Optimization
Lap Chi Lau, R. Ravi and Mohit Singh
Frontmatter
More information

CAMBRIDGE TEXTS IN APPLIED MATHEMATICS

All titles listed below can be obtained from good booksellers or from Cambridge University Press.
For a complete series listing, visit http://www.cambridge.org/uk/series/sSeries.asp?code=CTAM

Complex Variables: Introduction and Applications (2nd Edition),
MARK J. ABLOWITZ & ATHANASSIOS S. FOKAS
Scaling, G. I. R. BARENBLATT
Introduction to Symmetry Analysis, BRIAN J. CANTWELL
Hydrodynamic Instabilities, FRANÇOIS CHARRU
Introduction to Hydrodynamic Stability, P. G. DRAZIN
A First Course in Continuum Mechanics, OSCAR GONZALEZ & ANDREW M. STUART
Theory of Vortex Sound, M. S. HOWE
Applied Solid Mechanics, PETER HOWELL, GREGORY KOZYREFF, & JOHN OCKENDON
Practical Applied Mathematics: Modelling, Analysis, Approximation, SAM HOWISON
A First Course in the Numerical Analysis of Differential Equations (2nd Edition),
ARIEH ISERLES
A First Course in Combinatorial Optimization, JON LEE
Finite Volume Methods for Hyperbolic Problems, RANDALL J. LEVEQUE
Bäcklund and Darboux Transformations, C. ROGERS & W. K. SCHIEF
An Introduction to Parallel and Vector Scientific Computation, RONALD W. SHONKWILER &
LEW LEFTON

© in this web service Cambridge University Press www.cambridge.org

CAMBRIDGE

Cambridge University Press
978-0-521-18943-9 - Iterative Methods in Combinatorial Optimization
Lap Chi Lau, R. Ravi and Mohit Singh
Frontmatter
More information

Iterative Methods in Combinatorial Optimization

LAP CHI LAU
The Chinese University of Hong Kong

R. RAVI
Carnegie Mellon University

MOHIT SINGH
McGill University

CAMBRIDGE
UNIVERSITY PRESS

© in this web service Cambridge University Press www.cambridge.org

CAMBRIDGE

Cambridge University Press
978-0-521-18943-9 - Iterative Methods in Combinatorial Optimization
Lap Chi Lau, R. Ravi and Mohit Singh
Frontmatter
More information

CAMBRIDGE UNIVERSITY PRESS
Cambridge, New York, Melbourne, Madrid, Cape Town,
Singapore, São Paulo, Delhi, Tokyo, Mexico City

Cambridge University Press
32 Avenue of the Americas, New York, NY 10013-2473, USA

www.cambridge.org
Information on this title: www.cambridge.org/9780521189439

© Lap Chi Lau, R. Ravi, and Mohit Singh 2011

This publication is in copyright. Subject to statutory exception
and to the provisions of relevant collective licensing agreements,
no reproduction of any part may take place without the written
permission of Cambridge University Press.

First published 2011

Printed in the United States of America

A catalog record for this publication is available from the British Library.

Library of Congress Cataloging in Publication Data
Lau, Lap Chi.
Iterative methods in combinatorial optimization / Lap Chi Lau, R. Ravi, Mohit Singh.
p. cm. – (Cambridge texts in applied mathematics)
Includes bibliographical references and index.
ISBN 978-1-107-00751-2 (hardback) – ISBN 978-0-521-18943-9 (pbk.)
1. Iterative methods (Mathematics) 2. Combinatorial optimization. I. Ravi, R. (Ramamoorthi), 1969– II. Singh, Mohit. III. Title. IV. Series.
QA297.8.L38 2011
518′.26–dc22 2011003653

ISBN 978-1-107-00751-2 Hardback
ISBN 978-0-521-18943-9 Paperback

Cambridge University Press has no responsibility for the persistence or accuracy of URLs for external or third-party Internet Web sites referred to in this publication and does not guarantee that any content on such Web sites is, or will remain, accurate or appropriate.

© in this web service Cambridge University Press www.cambridge.org

CAMBRIDGE

Cambridge University Press
978-0-521-18943-9 - Iterative Methods in Combinatorial Optimization
Lap Chi Lau, R. Ravi and Mohit Singh
Frontmatter
More information

Contents

Preface *page* ix

1 Introduction **1**
1.1 The assignment problem 1
1.2 Iterative algorithm 3
1.3 Approach outline 5
1.4 Context and applications of iterative rounding 8
1.5 Book chapters overview 8
1.6 Notes 10

2 Preliminaries **12**
2.1 Linear programming 12
2.2 Graphs and digraphs 19
2.3 Submodular and supermodular functions 21

3 Matching and vertex cover in bipartite graphs **28**
3.1 Matchings in bipartite graphs 28
3.2 Generalized assignment problem 32
3.3 Maximum budgeted allocation 35
3.4 Vertex cover in bipartite graphs 40
3.5 Vertex cover and matching: duality 43
3.6 Notes 44

4 Spanning trees **46**
4.1 Minimum spanning trees 46
4.2 Iterative 1-edge-finding algorithm 54
4.3 Minimum bounded-degree spanning trees 57
4.4 An additive one approximation algorithm 60
4.5 Notes 62

© in this web service Cambridge University Press www.cambridge.org

CAMBRIDGE

Cambridge University Press
978-0-521-18943-9 - Iterative Methods in Combinatorial Optimization
Lap Chi Lau, R. Ravi and Mohit Singh
Frontmatter
More information

5 Matroids **65**
5.1 Preliminaries 65
5.2 Maximum weight basis 67
5.3 Matroid intersection 71
5.4 Duality and min–max theorem 74
5.5 Minimum bounded degree matroid basis 77
5.6 k matroid intersection 82
5.7 Notes 85

6 Arborescence and rooted connectivity **88**
6.1 Minimum cost arborescence 89
6.2 Minimum cost rooted k-connected subgraphs 95
6.3 Minimum bounded degree arborescence 101
6.4 Additive performance guarantee 106
6.5 Notes 108

7 Submodular flows and applications **110**
7.1 The model and the main result 110
7.2 Primal integrality 112
7.3 Dual integrality 116
7.4 Applications of submodular flows 117
7.5 Minimum bounded degree submodular flows 124
7.6 Notes 128

8 Network matrices **131**
8.1 The model and main results 131
8.2 Primal integrality 133
8.3 Dual integrality 136
8.4 Applications 139
8.5 Notes 143

9 Matchings **145**
9.1 Graph matching 145
9.2 Hypergraph matching 155
9.3 Notes 160

10 Network design **164**
10.1 Survivable network design problem 164
10.2 Connection to the traveling salesman problem 168
10.3 Minimum bounded degree Steiner networks 172
10.4 An additive approximation algorithm 175
10.5 Notes 179

© in this web service Cambridge University Press www.cambridge.org

CAMBRIDGE

Cambridge University Press
978-0-521-18943-9 - Iterative Methods in Combinatorial Optimization
Lap Chi Lau, R. Ravi and Mohit Singh
Frontmatter
More information

11 Constrained optimization problems **182**
11.1 Vertex cover 182
11.2 Partial vertex cover 184
11.3 Multicriteria spanning trees 187
11.4 Notes 189

12 Cut problems **191**
12.1 Triangle cover 191
12.2 Feedback vertex set on bipartite tournaments 194
12.3 Node multiway cut 197
12.4 Notes 200

13 Iterative relaxation: Early and recent examples **203**
13.1 A discrepancy theorem 203
13.2 Rearrangments of sums 206
13.3 Minimum cost circulation 208
13.4 Minimum cost unsplittable flow 210
13.5 Bin packing 212
13.6 Iterative randomized rounding: Steiner trees 220
13.7 Notes 228

14 Summary **231**

Bibliography 233
Index 241

© in this web service Cambridge University Press www.cambridge.org

CAMBRIDGE

Cambridge University Press
978-0-521-18943-9 - Iterative Methods in Combinatorial Optimization
Lap Chi Lau, R. Ravi and Mohit Singh
Frontmatter
More information

Preface

Audience

As teachers and students of combinatorial optimization, we have often looked for material that illustrates the elegance of classical results on matchings, trees, matroids, and flows, but also highlights methods that have continued application. With the advent of approximation algorithms, some techniques from exact optimization such as the primal-dual method have indeed proven their staying power and versatility. In this book, we describe what we believe is a simple and powerful method that is iterative in essence and useful in a variety of settings.

The core of the iterative methods we describe relies on a fundamental result in linear algebra that the row rank and column rank of a real matrix are equal. This seemingly elementary fact allows us via a counting argument to provide an alternate proof of the previously mentioned classical results; the method is constructive and the resulting algorithms are iterative with the correctness proven by induction. Furthermore, these methods generalize to accommodate a variety of additional constraints on these classical problems that render them NP-hard – a careful adaptation of the iterative method leads to very effective approximation algorithms for these cases.

Our goal in this book has been to highlight the commonality and uses of this method and convince the readers of the generality and potential for future applications. We have used an elementary presentation style that should be accessible to anyone with introductory college mathematics exposure in linear algebra and basic graph theory. Whatever advanced material in these areas we require, we develop from scratch along the way. Some basic background on approximation algorithms such as is provided in the various books and surveys available on this subject will be useful in appreciating the power of the results we prove in this area. Other than the basic definition of an approximation algorithm and the understanding of polynomial-time complexity, no further technical background is required from this typically more advanced subject.

© in this web service Cambridge University Press www.cambridge.org

CAMBRIDGE

Cambridge University Press
978-0-521-18943-9 - Iterative Methods in Combinatorial Optimization
Lap Chi Lau, R. Ravi and Mohit Singh
Frontmatter
More information

An important secondary goal of the book is to provide a framework and material for introductory courses in combinatorial optimization at the upper-class undergraduate and beginning graduate levels. We hope the common approach across the chapters gives a comprehensive way to introduce these topics for the first time. The more advanced applications are useful illustrations for graduate students of their potential for future application in their research.

History

This book is inspired by the application of the iterative method in the field of approximation algorithms and its recent adaptations to prove performance guarantees for problems with two objectives. This adaptation showed us how the proof technique can be used to reprove several classical results in combinatorial optimization and also in approximation algorithms in a unified way. The book owes its origin to the paper by Jain [75] describing a 2-approximation algorithm for a large class of minimum cost network-design problems in undirected networks. There are other earlier illustrations of the method in the literature, but it is Jain's work that inspired the adaptation that led to the results in this monograph.

Jain's result itself was a breakthrough when it appeared, and demonstrated the power of his *iterative rounding* method to prove this result that was conjectured based on a long line of earlier papers that applied a different primal-dual method to these problems. In this sense, his method was a purely primal attack on the problem. His method was extended by Lau et al. [88] to degree-bounded network design problems. The adaptation of this method by Singh and Lau [125] to the degree-bounded minimum cost spanning tree problem surprisingly involves no rounding at all! Instead, variables whose value are set to one in the linear programming relaxation are selected, and the program is modified carefully to continue to yield this property. This explains the title of this monograph and also hints at how this adaptation now allows one to prove *exact* results since we no longer have to round any variables and lose optimality.

Acknowledgments

We are grateful to the many organizations whose support have enabled this work: US National Science Foundation, Research Grants Council of Hong Kong, Microsoft Research, Kyoto University RIMS, the Qatar Foundation, Carnegie Mellon University – Pittsburgh and Doha, McGill University, and the Chinese University of Hong Kong. We are also grateful to our families for their support of this endeavor. We hope you will enjoy reading this monograph as much as we did writing it.

© in this web service Cambridge University Press www.cambridge.org

CAMBRIDGE

Cambridge University Press
978-0-521-18943-9 - Iterative Methods in Combinatorial Optimization
Lap Chi Lau, R. Ravi and Mohit Singh
Frontmatter
More information

Dedications

Lau dedicates this work to his parents, his wife Pui Ming, and their children Ching Lam, Sing Chit, and Ching Yiu. Ravi dedicates this work to the memory of his late brother, R. Balasubramaniam, who encouraged him to write a book. Singh dedicates this work to his parents.

© in this web service Cambridge University Press www.cambridge.org