

GRANULAR PHYSICS

The field of granular physics has burgeoned since its development in the late 1980s, when physicists first began to use statistical mechanics to study granular media. They are prototypical of complex systems, manifesting metastability, hysteresis, bistability and a range of other fascinating phenomena.

This book provides a wide-ranging account of developments in granular physics, and lays out the foundations of the statics and dynamics of granular physics. It covers a wide range of subfields, ranging from fluidisation to jamming, and these are modelled through a range of computer simulation and theoretical approaches. Written with an eye to pedagogy and completeness, this book will be a valuable asset for any researcher in this field.

In addition to Professor Mehta's detailed exposition of granular dynamics, the book contains contributions from Professor Sir Sam Edwards, jointly with Dr Raphael Blumenfeld, on the thermodynamics of granular matter; from Professor Isaac Goldhirsch on granular matter in the fluidised state; and Professor Philippe Claudin on granular statics.

ANITA MEHTA, a former Rhodes scholar, is currently a Radcliffe Fellow at Harvard University. She is well known for being one of the pioneers in granular physics, and is credited with the introduction of many new concepts in this field, in particular to do with the competition of slow and fast modes in granular dynamics.

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With contributions from

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ISAAC GOLDHIRSCH
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Sables

*Il n'est pas de désert si vaste
Que ne puisse traverser
Celui qui porte la musique des étoiles.
Poem on the Paris Underground,
attributed to Michel Le Saint*

Sands

*There is no desert so vast
that it cannot be traversed
by one who carries the music
of the stars.
My translation*

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Preface

This book was commissioned seven years ago, in Oxford, where I was an EPSRC Visiting Fellow at my alma mater, by Cambridge University Press. Its completion in Cambridge, Massachusetts, where I am a Radcliffe Fellow at Harvard University, owes a lot to the tranquillity of my initial and final conditions of work, where I am away from the regular pressures of my permanent position in India.

In the seven years since its conception, many things took priority over its writing, including, to a large extent, the research that has been presented in it. I feel this delay has been largely beneficial. In 1999, many of the developments that now seem obvious, that have now allowed granular media to be the focus of many conferences or multiple sessions at large meetings, were yet to happen. In particular, they changed the conception of the book itself, in my mind.

My initial idea, when I was approached to write a monograph on granular media, was to focus only on those areas where I had some understanding, or where I had myself been active. At that time, it was the so-called statistical mechanics of granular media, pioneered by Edwards, that held centre stage; people like myself were trying to make inroads into the dynamics of these fascinating systems. We focused in particular on what is now known as the jamming limit, which I thought even at the time had fascinating analogies to glasses. So little was known in the late nineties about powders – a feature that was at once attractive and challenging – that doing research on this field was really like stepping on the sand of a pristine beach, unaware of which step would lead to muddied waters, and which would land one on safe ground. I'd thought then of building a book around the new physics of these systems, referring people to traditional tomes on fluid dynamics and chemical engineering for everything else.

The seven years since then have seen a virtuous cycle – people have revisited old and seemingly known issues in the fluidised regime, and questioned the notion of the granular temperature, which had been set in stone by engineers. As always with physicists, people did not destroy an existing idea, but shed light on its fundamentals.

Now we know, for example, that although the kinetic energy of sand in the fluidised state does not yield a true thermodynamic temperature, it can nevertheless be useful in situations where the strict thermodynamics is less important than the use of a variable representing energy input. Additionally, people have embellished what were once only hypotheses; Edwards' compactivity, almost dismissed by many when he first seemed to get it out of thin air, has now been seen to be one of Sir Sam Edwards' many strokes of genius – it has been shown to have the *strict* characteristics of a thermodynamic temperature, despite its derivation from what was seen by many as a 'mere' analogy.

My original idea of focusing on only the dynamics of the jammed state is now simply not possible. What I have therefore done, to add to the modernity of the book, is to ask three distinguished colleagues, Profs. Sir Sam Edwards, Isaac Goldhirsch and Philippe Claudin, to contribute to it. The first of these, in collaboration with Prof. Rafi Blumenfeld, has contributed a chapter (Chapter 13) on his own ideas on the thermodynamics of granular matter, which has been complemented by a chapter (Chapter 14) on theoretical and experimental approaches to granular statics by Prof. Claudin. Prof. Goldhirsch (Chapter 12) has provided an excellent chapter which contains state-of-the-art references on granular media in the fluidised state. To all these colleagues, I owe my warmest thanks for their painstaking efforts, and the excellence of their results.

The plan of the book is as follows: Chapter 1 contains an introduction to many of the subfields that form the subject matter of the book. Chapter 2 contains an introduction to computer simulation approaches, while Chapter 3 expounds in detail on results that we have obtained on the structure of shaken granular material. Some of these results are still predictive and are virgin territory for enterprising experimentalists, while others have already been investigated thoroughly. Chapters 4 and 5 deal with cooperative phenomena in sand – focusing in turn on the dynamics of bridge formation and of the angle of repose – which are unique to such athermal systems. Chapter 6 sets out at length a way to probe the off-lattice and disordered nature of real sand, by setting forth the first of many approaches to model sand via random graphs. Chapters 7 and 8 discuss the shaking of a box of sand, the lattice-based formalism even extending to modelling grain shapes. Chapters 9, 10 and 11 contain very different approaches to the modelling of avalanches, that word from which it all began! – using in turn cellular automata, coupled-map lattice techniques, and the first of many approaches to coupled equations between surface and bulk in a sandpile. Since many of these subjects presented in different chapters are now veritable industries in the far enlarged scope of granular physics today, I make no apologies for presenting in some cases the original versions of current theories – this is done both in the interests of clarity, and because some of the most recent developments have yet to be fully verified in this continually evolving field.

Additionally, since these chapters contain largely my own work on the subject, I take responsibility for any errors, reserving the credit for my collaborators, who have been my constant sources of stimulation in my research. In particular it is to two of them, Dr. Gary Barker and Dr. Jean-Marc Luck, to whom I owe my unreserved thanks – without their active participation at various stages, this book would not have been possible.

It now only remains for me to thank the Editors of Cambridge University Press for their patience; the Service de Physique Théorique at CEA Saclay for allowing me the peace of mind to work on it on my frequent visits there; and of course the Radcliffe Institute of Advanced Study at Harvard University for gifting me the tranquillity of spirit and environment of intellectual stimulation which I so needed to finish this book.