

## GRANULAR PHYSICS

The field of granular physics has burgeoned since its development in the late 1980s, when physicists first began to use statistical mechanics to study granular media. They are prototypical of complex systems, manifesting metastability, hysteresis, bistability and a range of other fascinating phenomena.

This book provides a wide-ranging account of developments in granular physics, and lays out the foundations of the statics and dynamics of granular physics. It covers a wide range of subfields, ranging from fluidisation to jamming, and these are modelled through a range of computer simulation and theoretical approaches. Written with an eye to pedagogy and completeness, this book will be a valuable asset for any researcher in this field.

In addition to Professor Mehta's detailed exposition of granular dynamics, the book contains contributions from Professor Sir Sam Edwards, jointly with Dr Raphael Blumenfeld, on the thermodynamics of granular matter; from Professor Isaac Goldhirsch on granular matter in the fluidised state; and Professor Philippe Claudin on granular statics.

ANITA MEHTA, a former Rhodes scholar, is currently a Radcliffe Fellow at Harvard University. She is well known for being one of the pioneers in granular physics, and is credited with the introduction of many new concepts in this field, in particular to do with the competition of slow and fast modes in granular dynamics.

Cambridge University Press  
978-0-521-18799-2 - Granular Physics  
Anita Mehta  
Frontmatter  
[More information](#)

---

# GRANULAR PHYSICS

ANITA MEHTA

*Harvard University*

*With contributions from*

SIR SAM EDWARDS AND RAPHAEL BLUMENFELD  
ISAAC GOLDBIRSCH  
PHILIPPE CLAUDIN



**CAMBRIDGE**  
UNIVERSITY PRESS

Cambridge University Press  
978-0-521-18799-2 - Granular Physics  
Anita Mehta  
Frontmatter  
[More information](#)

---

CAMBRIDGE UNIVERSITY PRESS  
Cambridge, New York, Melbourne, Madrid, Cape Town,  
Singapore, São Paulo, Delhi, Tokyo, Mexico City

Cambridge University Press  
The Edinburgh Building, Cambridge CB2 8RU, UK

Published in the United States of America by Cambridge University Press, New York

[www.cambridge.org](http://www.cambridge.org)  
Information on this title: [www.cambridge.org/9780521187992](http://www.cambridge.org/9780521187992)

© A. Mehta 2007

This publication is in copyright. Subject to statutory exception  
and to the provisions of relevant collective licensing agreements,  
no reproduction of any part may take place without the written  
permission of the copyright holder.

First published 2007  
First paperback edition 2011

*A catalogue record for this publication is available from the British Library*

ISBN 978-0-521-66078-5 Hardback  
ISBN 978-0-521-18799-2 Paperback

Additional resources for this publication at [www.cambridge.org/9780521187992](http://www.cambridge.org/9780521187992)

---

Cambridge University Press has no responsibility for the persistence or accuracy of  
URLs for external or third-party internet websites referred to in this publication, and  
does not guarantee that any content on such websites is, or will remain, accurate or  
appropriate.

---

**Sables**

*Il n'est pas de désert si vaste  
Que ne puisse traverser  
Celui qui porte la musique des étoiles.  
Poem on the Paris Underground,  
attributed to Michel Le Saint*

**Sands**

*There is no desert so vast  
that it cannot be traversed  
by one who carries the music  
of the stars.  
My translation*

## Contents

	<i>page</i>
<i>Preface</i>	<i>x</i>
1 Introduction	1
1.1 Statistical mechanics framework, packing and the role of friction	2
1.2 Granular flow through wedges, channels and apertures	4
1.3 Instabilities, convection and pattern formation in vibrated granular beds	5
1.4 Size segregation in vibrated powders	8
1.5 Self-organised criticality – theoretical sandpiles?	11
1.6 Cellular automaton models of sandpiles	13
1.7 Theoretical studies of sandpile surfaces	15
2 Computer simulation approaches – an overview	18
2.1 Granular structures – Monte Carlo approaches	18
2.2 Granular flow – molecular dynamics approaches	22
2.3 Simulations of shaken sand – some general remarks	24
3 Structure of vibrated powders – numerical results	27
3.1 Details of simulation algorithm	27
3.2 The structure of shaken sand – some simulation results	29
3.3 Vibrated powders: transient response	40
3.4 Is there spontaneous crystallisation in granular media?	44
3.5 Some results on shaking-induced size segregation	46
4 Collective structures in sand – the phenomenon of bridging	52
4.1 Introduction	52
4.2 On bridges in sandpiles – an overarching scenario	52
4.3 Some technical details	54
4.4 Bridge sizes and diameters: when does a bridge span a hole?	55
4.5 Turning over at the top; how linear bridges form domes	58
4.6 Discussion	61

viii	<i>Contents</i>	
5	On angles of repose: bistability and collapse	63
5.1	Coupled nonlinear equations: dilatancy vs the angle of repose	63
5.2	Bistability within $\delta\theta_B$ : how dilatancy ‘fattens’ the angle of repose	65
5.3	When sandpiles collapse: rare events, activated processes and the topology of rough landscapes	67
5.4	Discussion	69
5.5	Another take on bistability	69
6	Compaction of disordered grains in the jamming limit: sand on random graphs	79
6.1	The three-spin model: frustration, metastability and slow dynamics	81
6.2	How to tap the spins? – dilation and quench phases	82
6.3	Results I: the compaction curve	84
6.4	Results II: realistic amplitude cycling – how granular media jam at densities lower than close-packed	90
6.5	Discussion	93
7	Shaking a box of sand I – a simple lattice model	94
7.1	Introduction	94
7.2	Definition of the model	94
7.3	Results I: on the packing fraction	96
7.4	Results II: on annealed cooling, and the onset of jamming	97
7.5	Results III: when the sandbox is frozen	100
7.6	Results IV: two nonequilibrium regimes	102
7.7	Discussion	103
8	Shaking a box of sand II – at the jamming limit, when shape matters!	104
8.1	Definition of the model	105
8.2	Zero-temperature dynamics: (ir)retrievability of ground states, density fluctuations and anticorrelations	106
8.3	Rugged entropic landscapes: Edwards’ or not?	108
8.4	Low-temperature dynamics along the column: intermittency	113
8.5	Discussion	114
9	Avalanches with reorganising grains	115
9.1	Avalanches type I – SOC	115
9.2	Avalanches type II – granular avalanches	118
9.3	Discussion and conclusions	131
10	From earthquakes to sandpiles – stick–slip motion	132
10.1	Avalanches in a rotating cylinder	132
10.2	The model	133
10.3	Results	135
10.4	Discussion	146

<i>Contents</i>		ix
11	Coupled continuum equations: the dynamics of sandpile surfaces	148
11.1	Introduction	148
11.2	Review of scaling relations for interfacial roughening	150
11.3	Case A: the Edwards–Wilkinson equation with flow	151
11.4	Case B: when moving grains abound	156
11.5	Case C: tilt combined with flowing grains	162
11.6	Discussion	167
11.7	A more complicated example: the formation of ripples	168
11.8	Conclusions	174
12	Theory of rapid granular flows Isaac Goldhirsch	176
12.1	Introduction	176
12.2	Qualitative considerations	177
12.3	Kinetic theory	184
12.4	Boundary conditions	196
12.5	Weakly frictional granular gases	200
12.6	Conclusion	206
13	The thermodynamics of granular materials Sir Sam Edwards and Raphael Blumenfeld	209
13.1	Introduction	209
13.2	Statistical mechanics	211
13.3	Volume functions and forces in granular systems	216
13.4	The stress field	224
13.5	Force distribution	232
14	Static properties of granular materials Philippe Claudin	233
14.1	Statics at the grain scale	233
14.2	Large-scale properties	245
14.3	Conclusion	273
	<i>References</i>	274
	<i>Index</i>	297

*The plate section located between pages 62 & 63 is available for download  
 in colour from [www.cambridge.org/9780521187992](http://www.cambridge.org/9780521187992)*

## Preface

This book was commissioned seven years ago, in Oxford, where I was an EPSRC Visiting Fellow at my alma mater, by Cambridge University Press. Its completion in Cambridge, Massachusetts, where I am a Radcliffe Fellow at Harvard University, owes a lot to the tranquillity of my initial and final conditions of work, where I am away from the regular pressures of my permanent position in India.

In the seven years since its conception, many things took priority over its writing, including, to a large extent, the research that has been presented in it. I feel this delay has been largely beneficial. In 1999, many of the developments that now seem obvious, that have now allowed granular media to be the focus of many conferences or multiple sessions at large meetings, were yet to happen. In particular, they changed the conception of the book itself, in my mind.

My initial idea, when I was approached to write a monograph on granular media, was to focus only on those areas where I had some understanding, or where I had myself been active. At that time, it was the so-called statistical mechanics of granular media, pioneered by Edwards, that held centre stage; people like myself were trying to make inroads into the dynamics of these fascinating systems. We focused in particular on what is now known as the jamming limit, which I thought even at the time had fascinating analogies to glasses. So little was known in the late nineties about powders – a feature that was at once attractive and challenging – that doing research on this field was really like stepping on the sand of a pristine beach, unaware of which step would lead to muddied waters, and which would land one on safe ground. I'd thought then of building a book around the new physics of these systems, referring people to traditional tomes on fluid dynamics and chemical engineering for everything else.

The seven years since then have seen a virtuous cycle – people have revisited old and seemingly known issues in the fluidised regime, and questioned the notion of the granular temperature, which had been set in stone by engineers. As always with physicists, people did not destroy an existing idea, but shed light on its fundamentals.

Now we know, for example, that although the kinetic energy of sand in the fluidised state does not yield a true thermodynamic temperature, it can nevertheless be useful in situations where the strict thermodynamics is less important than the use of a variable representing energy input. Additionally, people have embellished what were once only hypotheses; Edwards' compactivity, almost dismissed by many when he first seemed to get it out of thin air, has now been seen to be one of Sir Sam Edwards' many strokes of genius – it has been shown to have the *strict* characteristics of a thermodynamic temperature, despite its derivation from what was seen by many as a 'mere' analogy.

My original idea of focusing on only the dynamics of the jammed state is now simply not possible. What I have therefore done, to add to the modernity of the book, is to ask three distinguished colleagues, Profs. Sir Sam Edwards, Isaac Goldhirsch and Philippe Claudin, to contribute to it. The first of these, in collaboration with Prof. Rafi Blumenfeld, has contributed a chapter (Chapter 13) on his own ideas on the thermodynamics of granular matter, which has been complemented by a chapter (Chapter 14) on theoretical and experimental approaches to granular statics by Prof. Claudin. Prof. Goldhirsch (Chapter 12) has provided an excellent chapter which contains state-of-the-art references on granular media in the fluidised state. To all these colleagues, I owe my warmest thanks for their painstaking efforts, and the excellence of their results.

The plan of the book is as follows: Chapter 1 contains an introduction to many of the subfields that form the subject matter of the book. Chapter 2 contains an introduction to computer simulation approaches, while Chapter 3 expounds in detail on results that we have obtained on the structure of shaken granular material. Some of these results are still predictive and are virgin territory for enterprising experimentalists, while others have already been investigated thoroughly. Chapters 4 and 5 deal with cooperative phenomena in sand – focusing in turn on the dynamics of bridge formation and of the angle of repose – which are unique to such athermal systems. Chapter 6 sets out at length a way to probe the off-lattice and disordered nature of real sand, by setting forth the first of many approaches to model sand via random graphs. Chapters 7 and 8 discuss the shaking of a box of sand, the lattice-based formalism even extending to modelling grain shapes. Chapters 9, 10 and 11 contain very different approaches to the modelling of avalanches, that word from which it all began! – using in turn cellular automata, coupled-map lattice techniques, and the first of many approaches to coupled equations between surface and bulk in a sandpile. Since many of these subjects presented in different chapters are now veritable industries in the far enlarged scope of granular physics today, I make no apologies for presenting in some cases the original versions of current theories – this is done both in the interests of clarity, and because some of the most recent developments have yet to be fully verified in this continually evolving field.

Additionally, since these chapters contain largely my own work on the subject, I take responsibility for any errors, reserving the credit for my collaborators, who have been my constant sources of stimulation in my research. In particular it is to two of them, Dr. Gary Barker and Dr. Jean-Marc Luck, to whom I owe my unreserved thanks – without their active participation at various stages, this book would not have been possible.

It now only remains for me to thank the Editors of Cambridge University Press for their patience; the Service de Physique Théorique at CEA Saclay for allowing me the peace of mind to work on it on my frequent visits there; and of course the Radcliffe Institute of Advanced Study at Harvard University for gifting me the tranquillity of spirit and environment of intellectual stimulation which I so needed to finish this book.